An Electrically Small Dipole Antenna Loaded With Chua's Oscillator: Time-Retarded Chaos

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1. Introduction

Recently an electrically small dipole antenna loaded with the nonlinear circuit known as *Chua's oscillator* was analyzed. It was assumed that the dipole could be modeled as a pure capacitance, and an equivalent circuit of the combined antenna/load system was determined (Figure 1) [1]. It was demonstrated numerically that by assuming certain values for the linear resistive and reactive elements of this equivalent circuit, various types of periodic, period-doubled, and chaotic behavior which occur for the voltage at the antenna input terminals could also be shown to occur for the associated radiated electromagnetic field components. Thus for a dipole modeled as above, it was demonstrated numerically that when the antenna voltage function was temporally chaotic, the antenna radiated temporally chaotic electromagnetic fields with the same dynamical behavior referenced to some later time. Similarly when the antenna voltage was periodic, the radiated fields were periodic but referenced to some later time.

In the present work, a more realistic dipole model is introduced and again combined with Chua's oscillator as the nonlinear load. This more complicated model consists of a capacitance in series with a resistance and an inductance which are themselves in parallel (Figure 2) [2]. The equivalent circuit of this new model is shown in Figure 3. The coupled ordinary differential equations (ODEs) of this equivalent circuit are obtained, and the new model is compared to the simpler model used previously.

2. The Pure Capacitance Model

Previously the pure capacitance antenna model resulted in three coupled ODEs given by [1]

$$L\dot{I}_3 = -\left(R_0 I_3 + V_a\right) \tag{1a}$$

$$\left(C_2 + C_a\right)\dot{V}_a = I_3 - \left(V_a - V_1\right)G \tag{1b}$$

$$C_1\dot{V}_1 = \left(V_a - V_1\right)G - f\left(V_1\right) \quad . \tag{1c}$$

Equations (1) describe a three-dimensional phase space (or state space) given by

$$\vec{X}(t) = \left\{I_3(t), V_a(t), V_1(t)\right\} \tag{2}$$

which is the state of the system at time t. A solution, $\vec{X}(t)$, starting from some initial state, $\{I_3(t_0), V_a(t_0), V_1(t_0)\}$, at some beginning time, t_0, is called a trajectory of (1). In Equation (1c), $f(V_1)$ denotes the piecewise continuous linear approximation of the driving-point characteristic of the Chua diode. The function $f(V_1)$ is given by [3]

$$f(V_1) = \begin{cases} G_b V_1 + (G_b - G_a)E & ; \ V_1 < -E \\ G_a V & ; \ V_1 < |E| \\ G_b V_1 - (G_b - G_a)E & ; \ V_1 > E \end{cases} \quad . \tag{3}$$

Once $V_a(t)$, the voltage at the antenna input terminals is known from solution of (1), the antenna current function is given by $I_a = C_a \dot{V}_a(t)$ or

$$I_a(t) = \frac{C_a}{(C_a + C_2)} \{ I_3(t) - [V_a(t) - V_1(t)]G \} \quad . \tag{4}$$

Using a vector potential formulation, the electromagnetic field components can be obtained using Equations (1), (3), and (4) as in Reference 1.

3. A More Realistic Model

Using the dipole model shown in Figure 2 (without including the effects of the inductance, L_a, for now) and the resulting antenna/load equivalent circuit of Figure 3, the coupled ODEs for this situation are given by two alternate forms. If the phase space variables are assumed to be $\vec{Y}_1(t) = \{I_3, V_1, V_2, I_a\}$ then the ODEs are

$$L\dot{I}_3 = -(R_0 I_3 + V_2) \tag{5a}$$

$$C_1 \dot{V}_1 = (V_2 - V_1)G - f(V_1) \tag{5b}$$

$$C_2 \dot{V}_2 = I_3 - I_a - (V_2 - V_1)G \tag{5c}$$

$$C_2 R_a \dot{I}_a = I_3 - \left(1 + \frac{C_2}{C_a}\right) I_a - (V_2 - V_1)G \quad . \tag{5d}$$

If the phase space variables are $\vec{Y}_2(t) = \{I_3, V_1, V_a, I_a\}$, then the resulting ODEs are

$$L\dot{I}_3 = -(R_0 I_3 + R_a I_a + V_a) \tag{6a}$$

$$C_1 \dot{V}_1 = G(V_a + R_a I_a - V_1) - f(V_1) \tag{6b}$$

$$C_a \dot{V}_a = I_a \tag{6c}$$

$$C_2 R_a \dot{I}_a = I_3 - \left(1 + \frac{C_2}{C_a} + GR_a\right) I_a - (V_a - V_1)G \quad . \tag{6d}$$

Both sets of Equations (5) and (6), reduce to the pure capacitance model when the limit as R_a tends to zero is taken.

4. Conclusion

In the present work, a more realistic antenna/load model for the dipole combined with the nonlinear load known as Chua's oscillator has been considered. The simpler pure capacitance model results in a three-dimensional phase space and a corresponding set of equations that are very similar to the ODEs representing an isolated Chua's oscillator [1]. The more realistic dipole model results in two possible formulations, depending on the phase space variables chosen, and in both formulations a four-dimensional phase space results. Numerical results based on the two alternative four-dimensional phase space representations will be shown.

References

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3. M. P. Kennedy, "Three steps to chaos — Part II: A Chua's circuit primer," *IEEE Trans. Circuits Syst.*, CAS-40, 657-674 (October 1993).

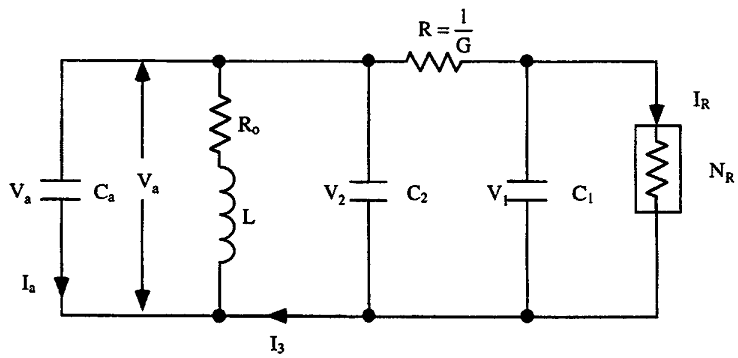

Figure 1. Equivalent Circuit of a Purely Capacitive Dipole Antenna Loaded With Chua's Oscillator.