Chaotic Waves and Spatio-Temporal Patterns in Large Arrays of Doubly-Coupled Chua's Circuits

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***Abstract*—We investigate complex dynamic phenomena in arrays composed of interacting chaotic circuits. Such arrays can be thought of as a model of nonlinear phenomena in spatially extended (high-dimensional or infinite-dimensional) systems and active media with potential applications in signal processing. In this paper, we consider a particular structure of the network in which there exists double diffusive interactions between the cells. Such a double interaction can be considered as a paradigm and means for understanding very complex interactions existing in real systems where separate cells can communicate in various ways. We consider two basic cases where separate cells without coupling exhibit two different types of chaotic behavior. Depending on the connection structure, initial conditions imposed in the cells, the array exhibits various kinds of spatially ordered chaotic waves. Patterns of behavior depending on the excitation of the array and the connection structure are studied in this paper. Chua's circuits are taken as standard chaotic cells.**

I. Introduction

Dynamic properties of networks of oscillatory and chaotic elements [1], [2], [4]–[8], [10]–[18] is one of the very lively studied topics. Several special issues of journals are devoted uniquely to studies of spatially extended systems, active media, and coupled lattices [20]–[22] showing important areas where studies of dynamic phenomena in coupled oscillators find potential applications. Studies of dynamic phenomena in arrays composed of chaotic electronic circuits are very important in such investigations as they provide a universal model for phenomena observed in other domains. Electronic circuits are capable of displaying most of the phenomena known so far in such classes of systems. Networks composed of chaotic cells are important as a model for physical systems with many degrees of freedom and also biological signal processing. They offer possible interesting engineering applications eg., in information processing [16]. Due to the high dimensionality of these systems, most of the studies are based on simulation experiments alone. Simulation also poses difficult problems for researchers—solution of 30 000 nonlinear differential equations, as in the case of 100×100 array of third-order oscillators, requires sophisticated problem-oriented software and can be very time-consuming.

Kaneko [7], [8] introduced a classification of dynamic phenomena encountered in coupled systems. The most interesting

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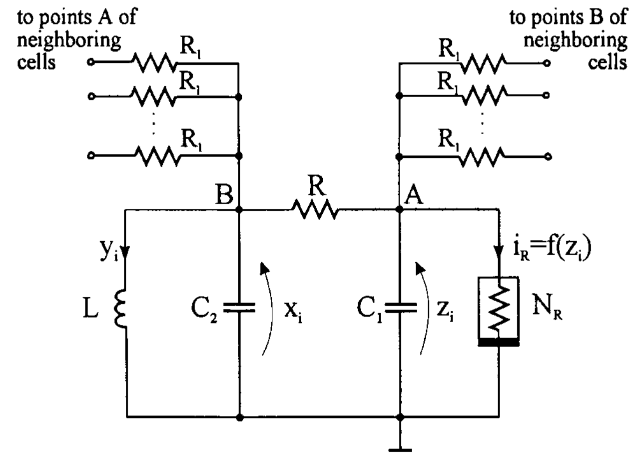

Fig. 1. Schematic diagram of a chaotic cell (Chua's circuit with inputs) used in the numerical experiments.

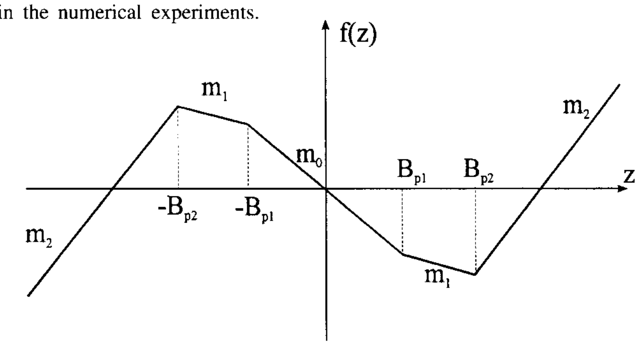

Fig. 2. Eventually passive characteristic of the nonlinear resistor used in the simulation experiments.

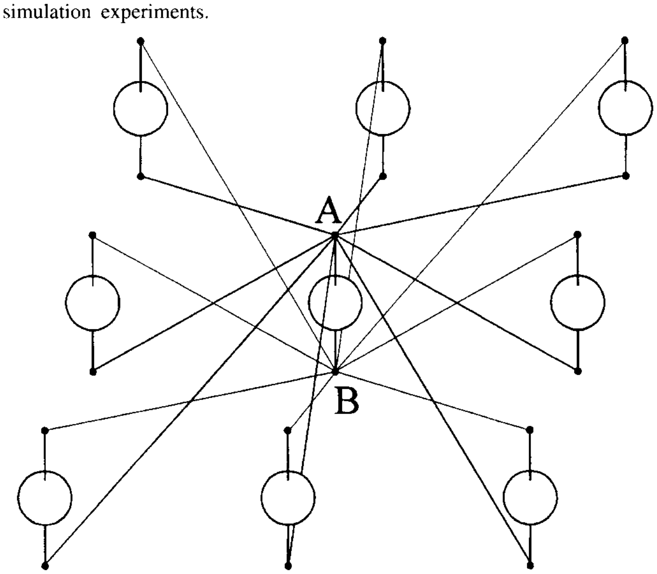

Fig. 3. Schematic diagram of interconnections between neighboring cells. There exists a double diffusive coupling between neighbors. Each line denotes resistive coupling.

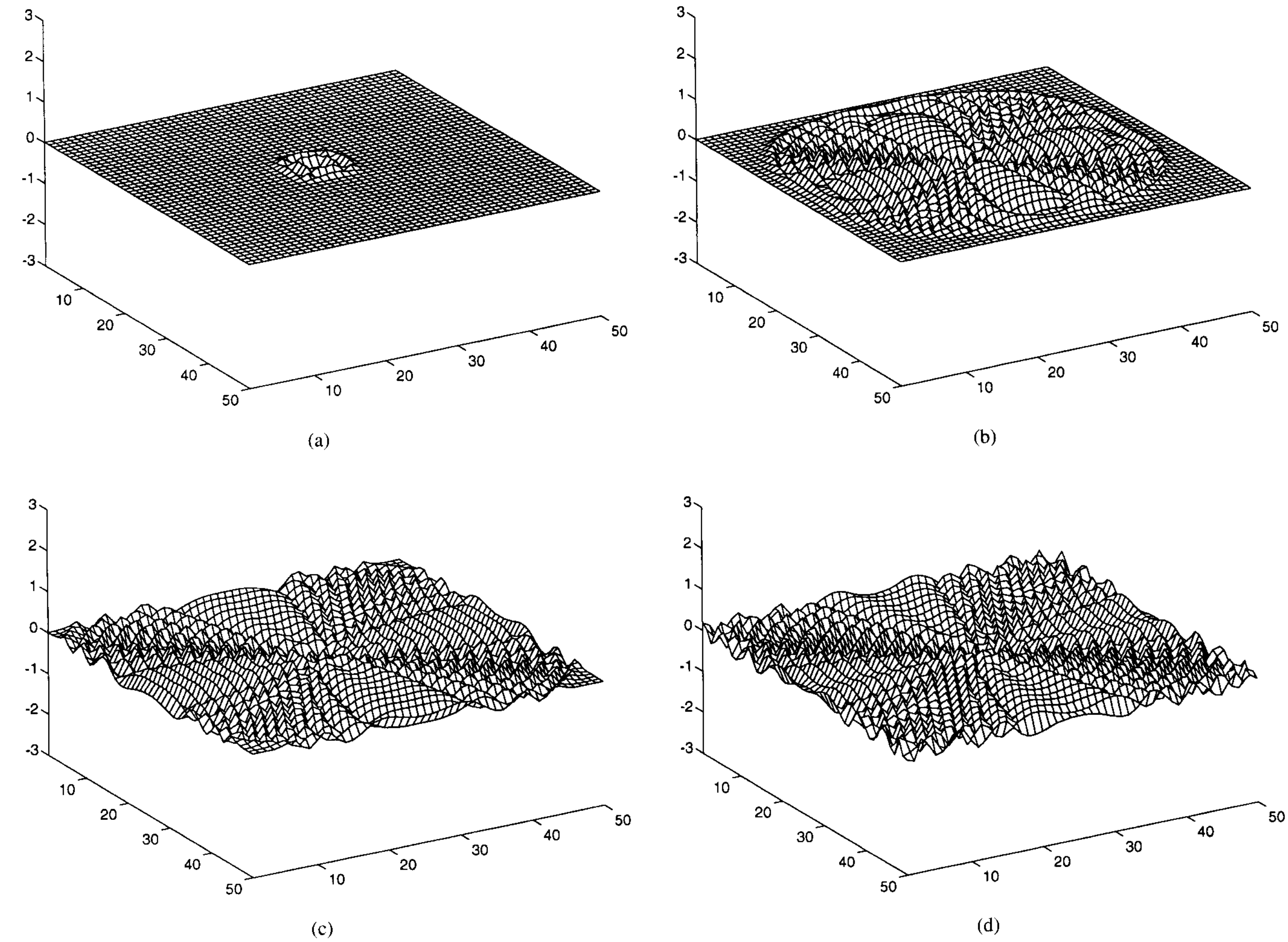

Fig. 4. Propagation of the wavefront through the 50 × 50 CNN array composed of Chua's circuits. Disturbance is initiated at the network center ($x_{25,25} = 0.1$). All other initial conditions across the network are set to zero. Four snapshots are shown. (a) After 100 iterates. (b) 400 iterates. (c) 500 iterates. (d) 900 iterates (timestep 0.1 s). Development of a circular (concentric, target) waves and their propagation is clearly visible.

are "self-organization" phenomena. The simplest type of such an "organized" behavior is synchronization or coherent behavior when the behavior of some cells is identical in time. In some cases, groups of neighboring cells can synchronize forming a "cluster." Oscillator arrays often show universal behavior [10] characterized by Feigenbaum constants. Large array can exhibit waves of different types including solitons and recently discovered in arrays of electronic oscillators, spiral waves [17], and travelling wave fronts [15]–[17].

The most complicated types of dynamics encountered in large interconnections of chaotic oscillators are: hyper-chaos, characterized by more then one positive Lyapunov exponent and the spatio-temporal chaos (sometimes referred to as turbulence [7], [8]) when the observed trajectories exhibit chaotic properties both in time and space (thus characterized by positive Lyapunov exponents in time and space). There are very few tools for studying these types of behavior. Building of experimental laboratory equipment is both expensive and extremely difficult. Electronic circuitry and available simulation tools offer versatility difficult to find in other areas of research. For the purpose of this study, an application-specific software package has been developed. Below, we present some important results of our simulation studies showing that an array composed of chaotic cells is capable of displaying nonlinear chaotic wave phenomena not described up to now in the literature.

II. Structure of the Analyzed Network

Our previous studies carried out in ladder and ring interconnections of Chua's circuits revealed interesting types of oscillatory phenomena [12]. In this paper, we study the dynamic behaviors encountered in planar interconnections of identical simple electronic oscillators (Chua's circuits)—an extension of experiments carried out previously. In our previous studies, we concentrated on interconnections resembling Cellular Neural Networks—each oscillator had a well-defined input and output [14]. Here, the structure is generalized to the case where the oscillators are coupled bidirectionally (diffusively) by means of two resistors (double coupling) cross-connected between the capacitors C_1 and C_2 of the neighboring cells as depicted in Figs. 1 and 3.

In this study, we consider square grids of the size 50 × 50 (2500 cells) only. We employ as a standard chaotic cell the Chua's circuit. The unit cells are interconnected only with

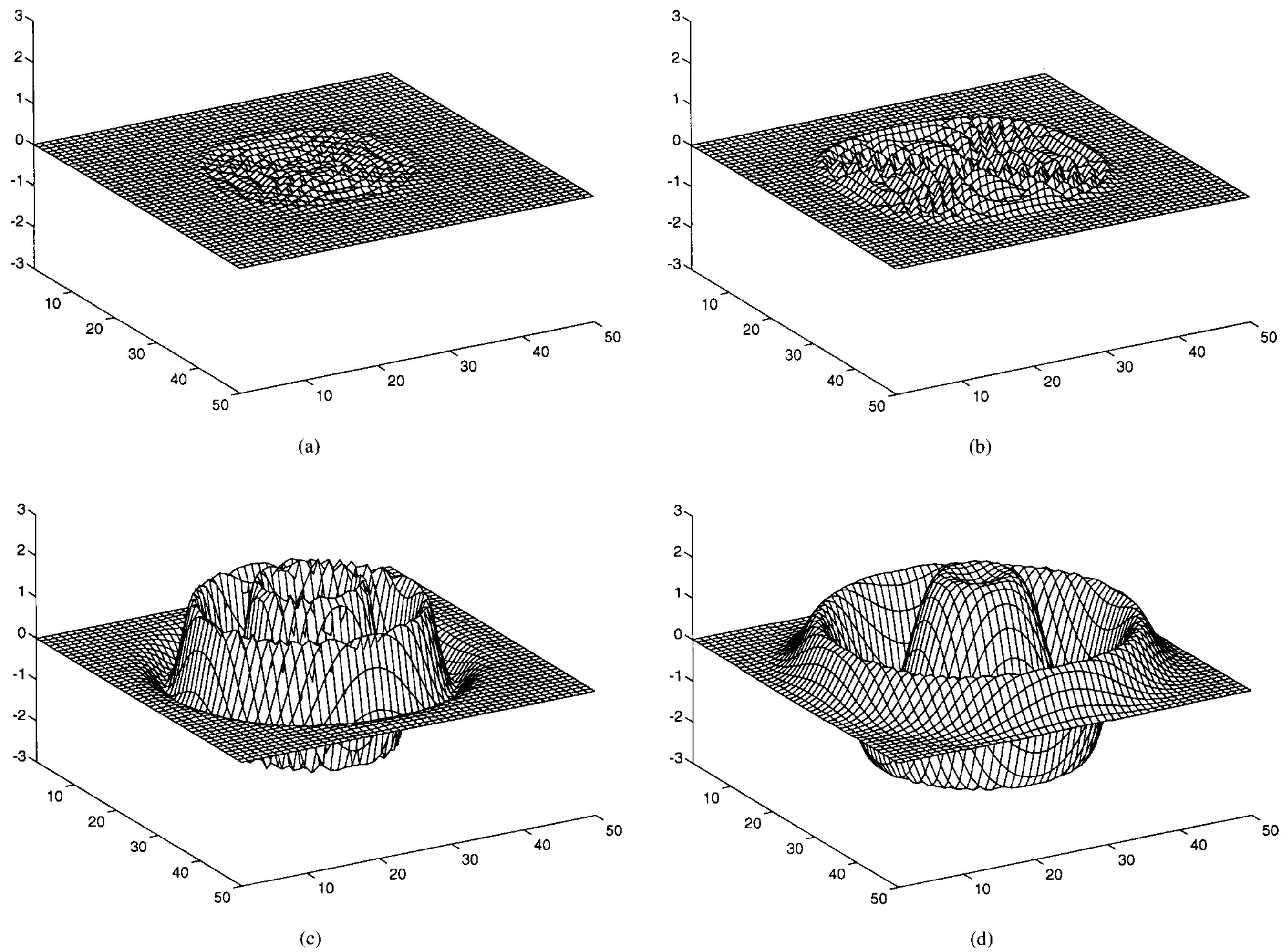

Fig. 5. Comparison of propagation of the circular waves in the 50 × 50 array of doubly-coupled Chua's circuits. Disturbance is initiated at the center ($x_{25,25} = 0.1$). All other initial conditions across the network are set to zero. Snapshots are taken after 300 iterates (timestep 0.1 s) for four different values of G_1. (a) $G_1 = 0.005S$. (b) $G_1 = 0.01S$. (c) $G_1 = 0.05S$. (d) $G_1 = 0.1S$. The wavelength of the circular waves and speed of propagation in the array grow with growing coupling conductance. Above certain threshold for G_1, the amplitude of oscillations grows rapidly—the oscillation are no longer chaotic in time but become periodic.

nearest neighbors (local interconnections). In our experiments, the cells at the corners are connected with three neighbors, the cells at the edges have connections to five neighbors, and internal cells have eight neighbors each. The equations describing system dynamics can be written in a simplified way for each cell as follows:

$$\begin{aligned}\dot{x}_i &= (A_1 + N_i A_7)x_i + A_2 y_i - A_1 z_i - A_7 \sum_{j\in N_i} z_j \\ \dot{y}_i &= A_3 x_i \\ \dot{z}_i &= -A_4 x_i + (A_4 + N_i A_6) z_i + A_5 f(z_i) - A_6 \sum_{j\in N_i} x_j \end{aligned} \tag{1}$$

where

$$\begin{aligned} f(z) = m_2 z &+ \frac{1}{2}(m_1 - m_2)(|z + B_{p_2}| - |z - B_{p_2}|) \\ &+ \frac{1}{2}(m_0 - m_1)(|z + B_{p_1}| - |z - B_{p_1}|) \end{aligned} \tag{2}$$

with $m_0 = -0.8$, $m_1 = -0.5$, $m_2 = 0.8$, $B_{p_1} = 1$, $B_{p_2} = 1.5$. The graph of this piecewise-linear function is shown in Fig. 2.

In (1), we use the following notation: $A_1 = -\frac{G}{C_2}$, $A_2 = -\frac{1}{C_2}$, $A_3 = \frac{1}{L}$, $A_4 = -\frac{G}{C_1}$, $A_5 = -\frac{1}{C_1}$, $A_6 = -\frac{G_1}{C_1}$, $A_7 = -\frac{G_1}{C_2}$. N_i denotes here the set of neighbors of the ith cells (as mentioned above, eight neighbors are being considered).

In simulation experiments, we used typical parameter values for Chua's circuit for which an isolated Chua's circuit generates chaotic oscillations—either the double scroll attractor ($L = 1/7H$, $C_1 = 1/9F$, $C_2 = 1F$, $G = 0.7S$) or the spiral attractor ($L = 1/7H$, $C_1 = 1/9F$, $C_2 = 1F$, $G = 0.78S$). Depending on the actual connection strength (coupling resistor values) and initial conditions imposed in the cells, a variety of wave phenomena could be observed.

III. Propagation of Chaotic Waves in the Array

We have carried out an extensive study of wave propagation phenomena in arrays of interconnected chaotic circuits that enabled us to find out some basic properties of these phenomena. In all experiments described below, a 50 × 50 network was considered. In all experiments, we used the fourth-order

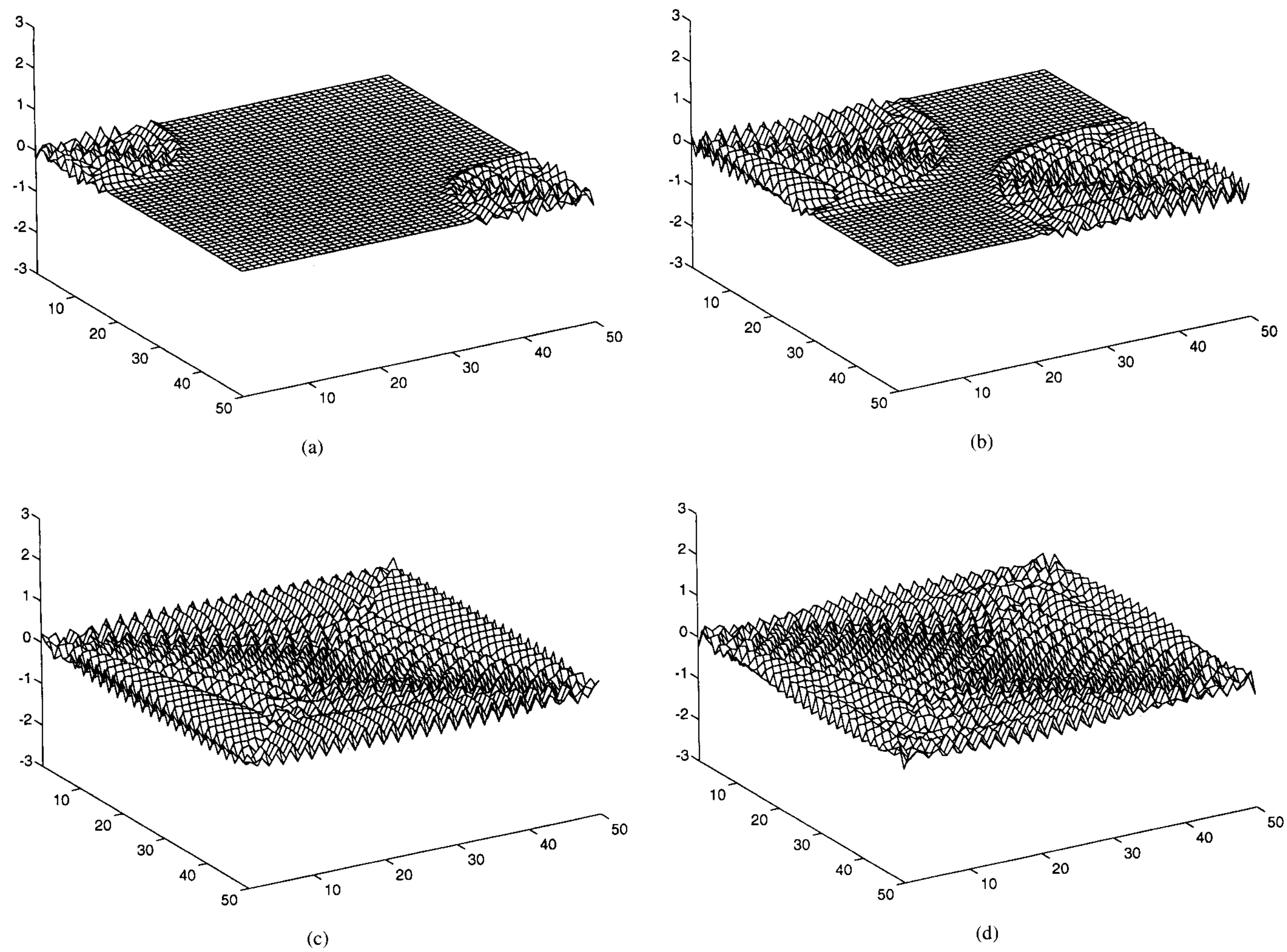

Fig. 6. Experiment showing effects of wave front collisions. Circular waves are initiated at two opposite corners ($x_{1,1} = x_{50,50} = 0.1$). All other initial conditions across the network are set to zero. Four snapshots are shown. (a) After 300 iterates. (b) 500 iterates. (c) 900 iterate. (d) 1700 iterates (timestep 0.1 s). After a long time, spatial structure consisting of lines of coherently oscillating circuits is visible. Along the diagonal line, some reminiscence of concentric waves still exists.

Runge–Kutta integration algorithm with time step 0.1. All experiments assume a uniform structure of the network—a square grid in which each chaotic cell is connected with nearest neighbors only.

Experiment 1: In this experiment, we observed how the disturbance initiated at the center of the network propagates through the array. The experiment is initiated by introducing a nonzero initial condition on C_2 capacitor at the position (25, 25) in the array. All initial conditions in other cells are zero. Fig. 4 shows four snapshots taken after 100, 400, 500, and 900 iterates of the integration routine. The coupling conductances were equal in the network $G_1 = 0.005S$ ($R_1 = 200\ \Omega$). Analysis of the results obtained indicates that a circular wave front that propagates in all directions through the network is generated. Behind the wavefront, concentric waves centered at the cell at which the process was initiated are visible. After hitting the edges, the waves maintain their spatial circular shape—they behave as autowaves—they do not reflect or interfere. The somewhat astonishing effect is that a large number of disordered (chaotic) cells can produce highly ordered "spatially organized" behavior while their temporal behavior remains chaotic.

Experiment 2: In this experiment, we study the influence of the values of coupling resistance on the propagation of the wave in the network. As in previous experiment, the process is initialized at the central cell by introducing a nonzero initial condition on C_2 capacitor. Initial conditions in other cells are set to zero. Fig. 5 shows four snapshots taken after 300 iterates of the integration routine for four different values of G_1: $G_1 = 0.005S$, $G_1 = 0.01S$, $G_1 = 0.05S$, and $G_1 = 0.1S$.

Analysis of the results obtained indicates again that circular waves propagating from the center are generated. It is clear however that there exists a threshold value of G_1 above which the character of oscillations changes. Below the threshold, the waves are chaotic in time, spatially ordered—and have low amplitude. Above the threshold value of the coupling conductance, the amplitude of the oscillations grows rapidly, and the time wave forms change—they are no longer chaotic but periodic. This phenomenon can be explained if we realize that strong coupling between the cells causes the trajectories to enter the second linear region (eventual passivity of the nonlinear characteristics in the cells—Fig. 2) and separate cells change their behavior to periodic. This kind of phenomenon of sudden change of type of oscillation and of their amplitude

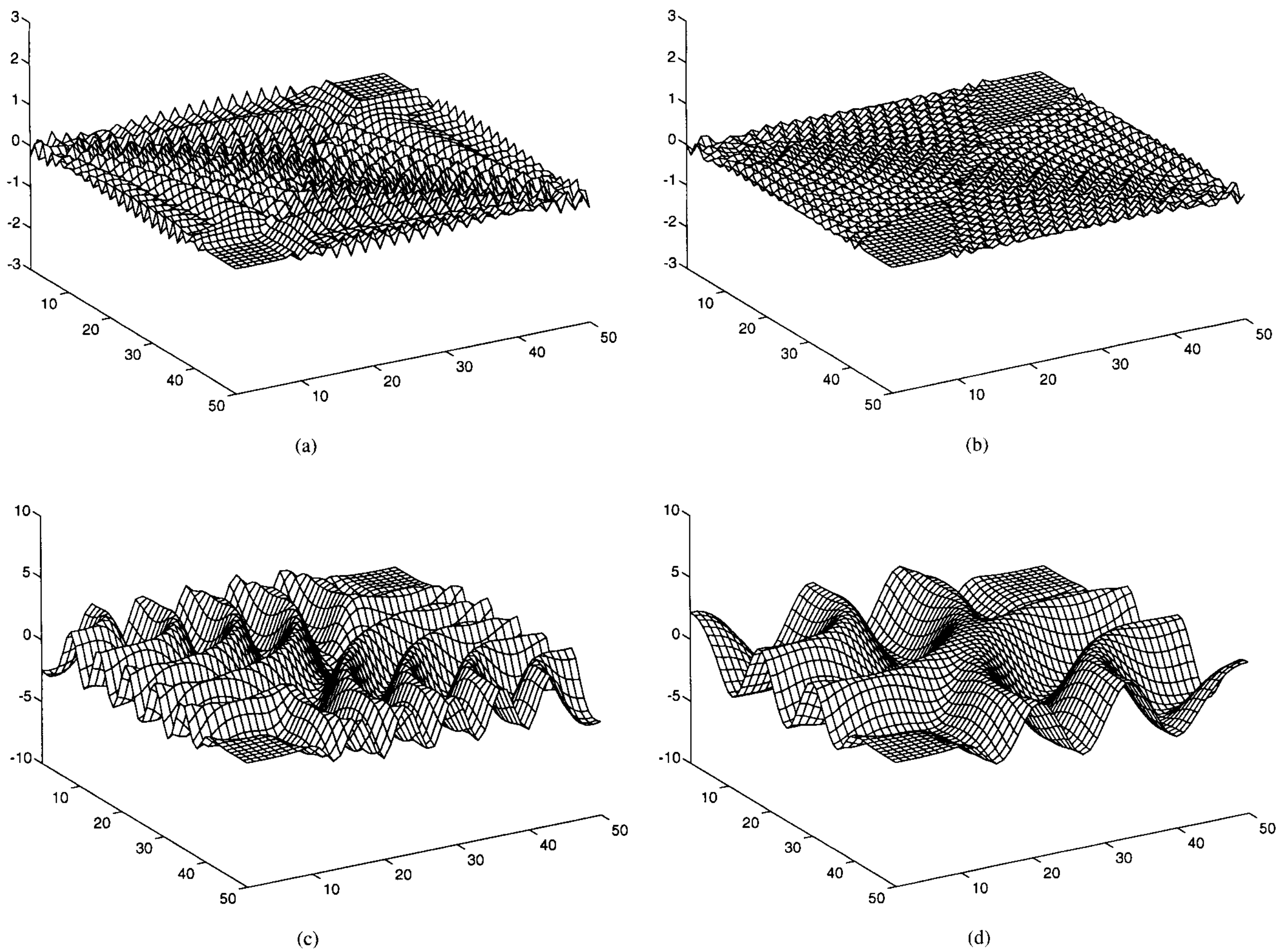

Fig. 7. Influence of coupling coefficients on the wave collision effects in the array. The waves are initialized at opposite corners ($x_{1,1} = x_{50,50} = 0.1$), and all other initial conditions across the network are set to zero. Four snapshots are shown. (a) After 700 iterates, $G_1 = 0.005S$. (b) After 700 iterates, $G_1 = 0.01S$. (c) After 500 iterates, $G_1 = 0.05S$. (d) After 500 iterates, $G_1 = 0.1S$ (timestep 0.1 s).

could be called "chaotic resonance" between the cells—two (or more) cells operating in a chaotic mode when strongly coupled amplify mutually their oscillations eventually producing large-amplitude periodic oscillations.

The second property that is clearly visible is that the wavelength of the circular waves and speed of propagation in the array grow with growing coupling conductance.

Experiment 3: In the third experiment, we studied collisions of waves initiated at two distinct points in the network—namely at two opposite corners. In this case, circular concentric waves (so-called target waves) are generated from these two points and propagate through the array. These chaotic in time and circularly ordered in space waves collide eventually at the center of the array. It seems that the collision of two waves has a different effect to the "edge" effects observed in experiment 1—at the center (collision point) both chaotic waves do not annihilate or interfere, however, the "side" effects become visible—after a long time, part (near the edges) of the array becomes perfectly synchronized—there are groups (lines) of circuits oscillating in a perfectly coherent way while along the diagonal some reminiscence of concentric waves is still visible (Fig. 6(d)).

Experiment 4: This experiment has been carried out to show the influence of the coupling conductance on the effects of collisions of chaotic waves. Fig. 7 presents four snapshots showing the collisions for four different values of G_1: $G_1 = 0.005S$, $G_1 = 0.01S$, $G_1 = 0.05S$, and $G_1 = 0.1S$.

For larger coupling conductance value, as in Experiment 2, the wave forms observed are of high amplitude and time-periodic.

Experiment 5: In all previous experiments, the structure of the network and the position of initialized cells were perfectly symmetrical. Experiment 5 has been carried out in order to investigate how the oscillations develop in the case of asymmetrically excited network. Nonzero initial conditions were applied at three cells—one on the edge $x_{1,10} = 0.1$, and two locations inside the network: $x_{19,7} = 0.1$ and $x_{35,42} = 0.01$. As can be expected, circular wave fronts are initiated at all three cells—as the third initial condition was 10 times smaller then the two others for some time its influence is hardly visible (Fig. 8(a)). After 300 iterates, the three developing wave fronts are of comparable size. The coupling coefficient was chosen above the "chaotic resonance" threshold, thus the waves have a large amplitude. The experiments have

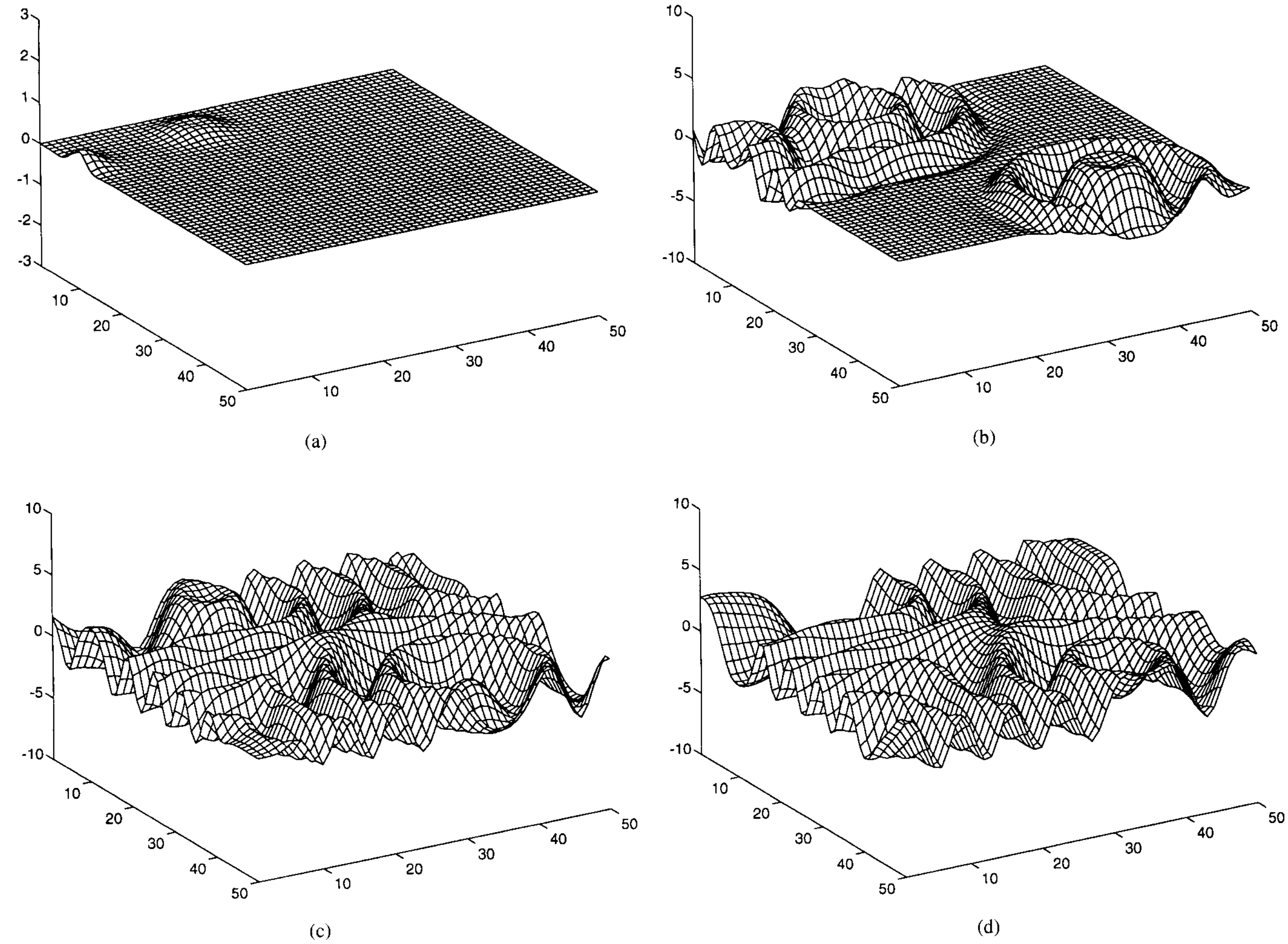

Fig. 8. Dynamic behavior in the array initialized in an asymmetric way—nonzero initial conditions were applied at locations: $x_{1,10} = 0.1$, $x_{19,7} = 0.1$, and $x_{35,42} = 0.01$. All other initial conditions in the network are set to zero. Four snapshots are shown: (a) After 100 iterates. (b) 300 iterates. (c) 500 iterates. (d) 800 iterates (timestep 0.1 s). Development of wave fronts from all three initialized cells is visible. (c) shows the collision of the wave fronts. An interesting fact is that the systems have a "memory effect"—in (c), (d), the centers were the nonzero initial conditions were applied are still visible.

shown that the chaotic network maintains for a long time a kind of memory effect—cells where the waves were initiated can easily be identified (Fig. 8(c) and (d)). This kind of memory effect of initial condition could offer interesting applications—but requires further thorough study.

Experiment 6: In this experiment, we observed again how the disturbance initiated at the center of the network propagates through the array, but this time the parameters of each cells have been changed in order that it produced a different type of chaotic behavior—namely a Rössler spiral-type attractor ($G = 0.78S$). The experiment is initiated by introducing a nonzero initial condition on C_2 capacitor at the position (25, 25) in the array. All initial conditions in other cells are zero. Fig. 9 shows four snapshots taken after 100, 1600, 1700, and 2500 iterates of the integration routine. The coupling conductances were equal in the network $G_1 = 0.005S$ ($R_1 = 200\ \Omega$). Analysis of the results obtained indicates that for a long time there is a very small propagation effect—a wavefront of very small amplitude can be seen (Fig. 9(a) and (b)). A very interesitng effect is the change of the wave front shape. Initially, it is almost rectangular (Fig. 9(a)) and then before the qualitative change of the behavior, it becomes circular (Fig. 9(b)). After about 1700 iterates, suddenly a burst of chaotic oscillations at the center appears which develops very slowly in all directions (Fig. 9(c) and (d)). This phenomenon was highly intriguing for the authors, and for a long time, we believed that it is an integration routine artifact. Thorough studies using different integration routines and different timesteps carried out also with networks of various sizes have convinced us that the network composed of spiral-chaos oscillators behaves in a very unpredictable manner—very different to nice (although chaotic) spatial structures observed in the case of double scroll attractors. In this case, we observed in many cases a sort of self-organization—bursts of high-amplitude oscillations at very particular positions in the network—depending on the coupling, initial conditions and size of the network.

Experiment 7: In this experiment, we study the influence of the values of coupling resistance on the propagation of the wave in the network of Chua's circuits generating a spiral attractor (as in the previous experiment). As in the previous experiment, the process is initialized at the central cell by introducing a nonzero initial condition on the C_2 capacitor.

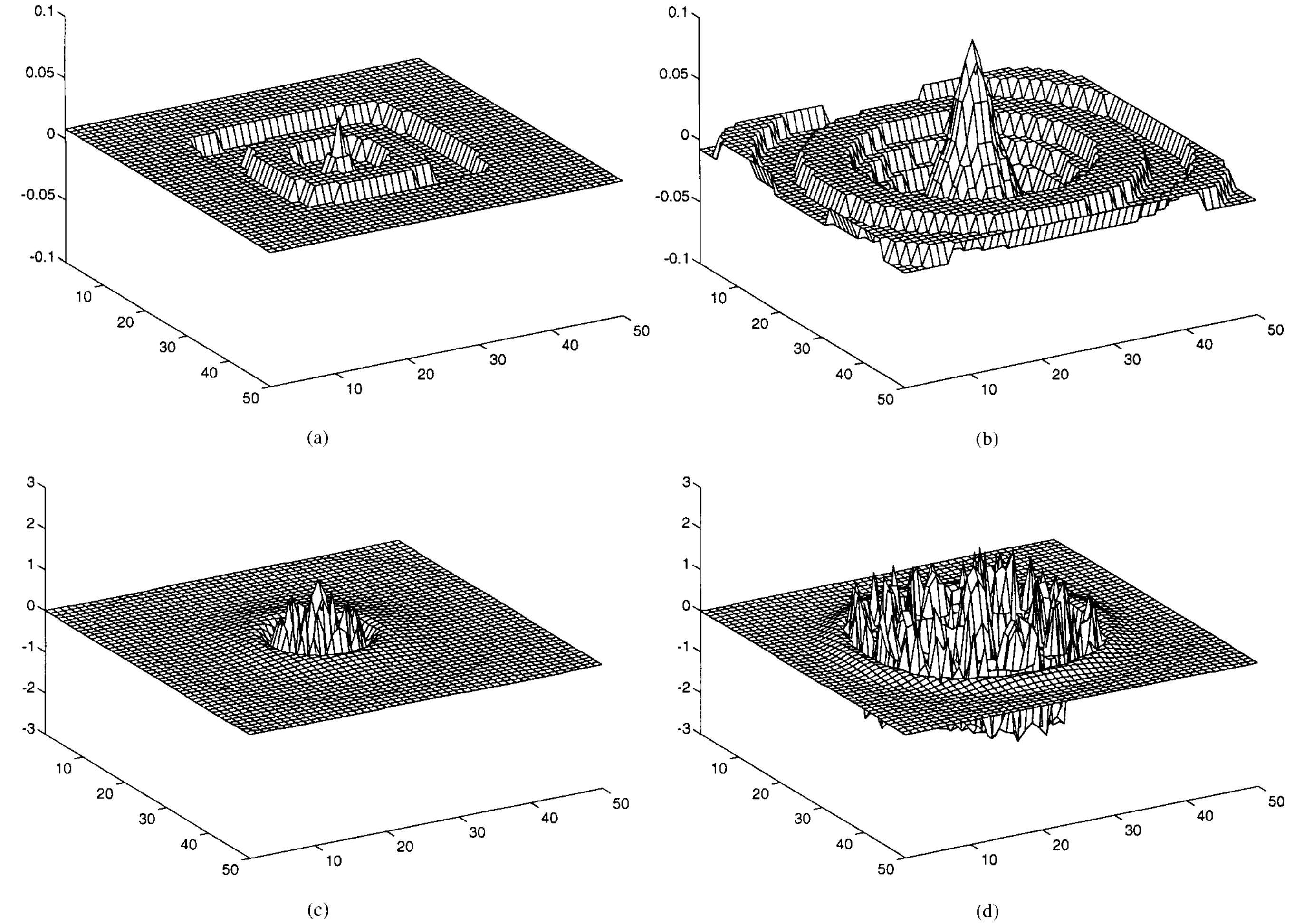

Fig. 9. Dynamic behavior in the 50 × 50 network composed of Chua's circuits operating at a spiral type chaotic attractor. Disturbance is initiated at the network center ($x_{25,25} = 0.1$). All other initial conditions across the network are set to zero. Four snapshots are shown. (a) After 100 iterates. (b) 1600 iterates. (c) 1700 iterates. (d) 2500 iterates (timestep 0.1 s). For a long time, only very small amplitude waves are visible. After around 1700 iterates, a burst of high-amplitude oscillation can be observed which persists for a long time.

Initial conditions in other cells are set to zero. Fig. 10 shows four snapshots taken after 300 iterates of the integration routine for four different values of G_1: $G_1 = 0.005S$, $G_1 = 0.01S$, $G_1 = 0.05S$, and $G_1 = 0.1S$.

Analysis of the results shows that in all four cases circular waves propagating from the center are generated. For different values of G_1, we observe, however, qualitatively different effects. In the first case, ($G_1 = 0.005S$) for a long time we observe a very low-amplitude oscillations, which then converts into high-amplitude ones, traveling slowly from the center to the edges. In Fig. 10(a), a moment before chaotic burst is shown.

In the second case, we observe a very short low-amplitude period. The amplitude of oscillations increases, and high-amplitude chaotic oscillations are born in the center of the network. The wave front of these oscillations moves quicker than in the first case, but their amplitude is approximately the same. In Fig. 10(b), the state of the network after high-amplitude chaotic burst is presented. In the last two cases (Fig. 10(c) and (d)) when the coupling conductance is large, the amplitude of the oscillations grows rapidly, and the time wave forms change—they are no longer chaotic but periodic. An intriguing effect is that in the cases of high coupling conductances, we observe in time a "stiffening effect"—the wavelength changes in time becoming longer and longer, the oscillations of neighboring cells get better and better synchronized—as if a sheet of soft material waving in the air was getting stiffer due to "starching"—after a very long transient, this stiffening effect leads to complete synchronization—all cells oscillate simultaneously—in the picture, the whole square moves up-and-down.

IV. Conclusion

The main results of our study can be summarized as follows:

- Our study reveals extremely interesting phenomena not described so far in the literature. Several interesting types of dynamic behaviors have been confirmed to exist in the considered arrays of Chua's circuits. These include: spatially synchronized states, spatio-temporal chaos, temporal hyperchaos, chaotic wave propagation, chaotic bursts, "stiffening effect."
- For asymmetrically applied initial conditions, a memory effect has been observed—i.e., the system maintains for

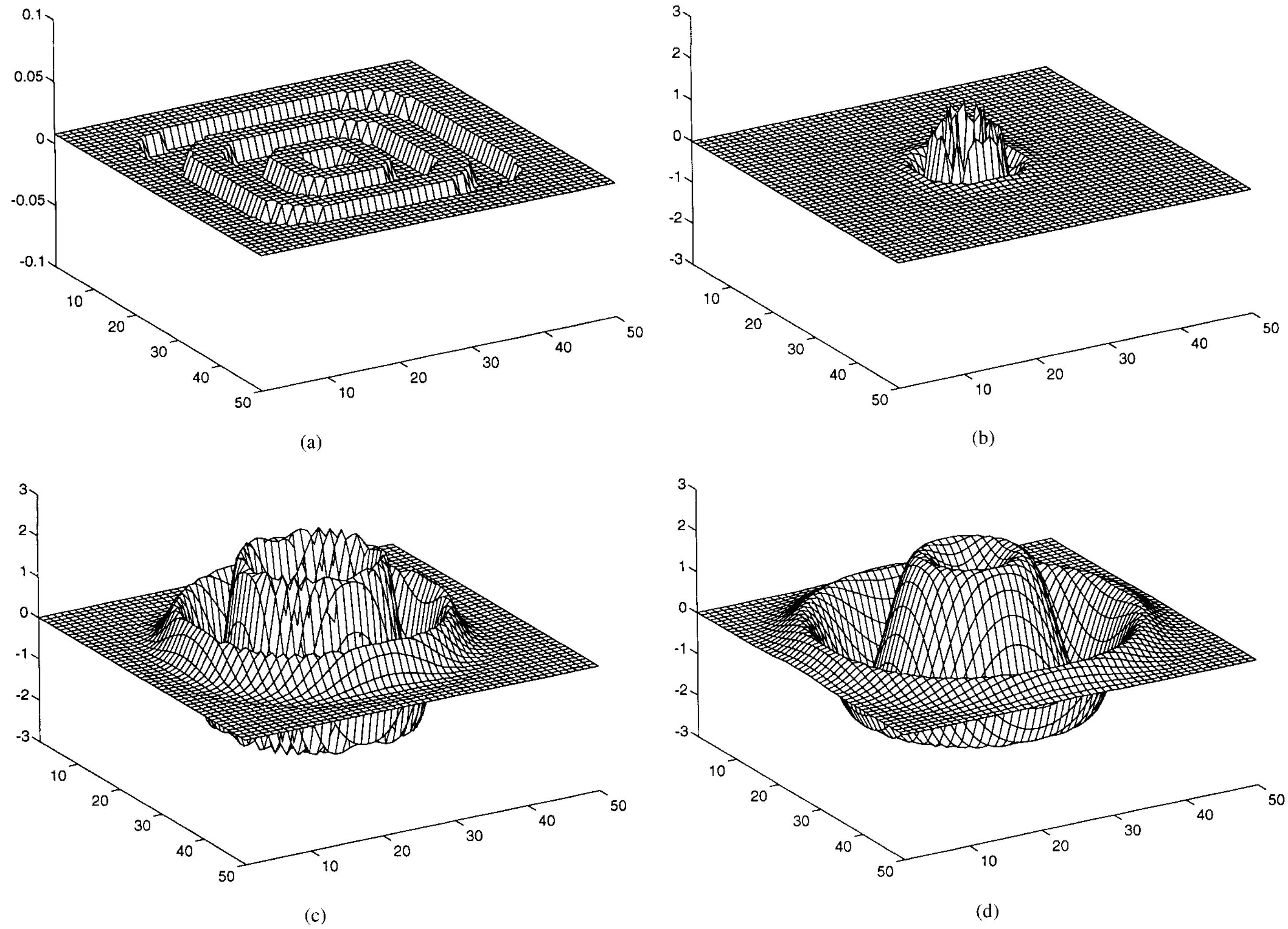

Fig. 10. Comparison of propagation of the circular waves in the 50 × 50 array of doubly-coupled Chua's circuits. Disturbance is initiated at the center ($x = 0.1$). All other initial conditions across the network are set to zero. Snapshots are taken after 300 iterates (timestep 0.1 s) for four different values of G_1. (a) $G_1 = 0.005S$. (b) $G_1 = 0.01S$. (c) $G_1 = 0.05S$. (d) $G_1 = 0.1S$. The wavelength of the circular waves and speed of propagation in the array grow with growing coupling conductance. Above a certain threshold for G_1, the amplitude of oscillations grows rapidly—the oscillation are no longer chaotic in time but become periodic.

a long time the information where the initial conditions were applied.

- A particular kind of cluster formation (coherent oscillations of several cells) is development of wave fronts and target waves. These chaotic waves show properties resembling autowaves—they do not interfere or disperse. The wavelength grows with growing coupling conductance. Above some threshold value of the coupling conductance, the character of oscillation in the network changes from chaotic time wave forms of small amplitude to periodic waves of high amplitude.
- Several intriguing behaviors have been observed in arrays of circuits producing spiral-type attractors. These phenomena recall observations reported in biology and physics literature—self-organization and "edge of chaos." These phenomena require further thorough study.
- Changes of the coupling resistances and initial conditions offer a possibility of controlling the dynamic behavior. One can choose between different types of dynamics. To be able to take advantage of this kind of control and possibly find useful applications for the arrays of chaotic elements, more investigations must be carried out.

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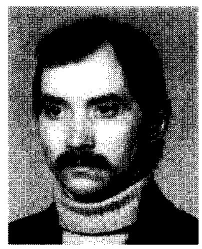

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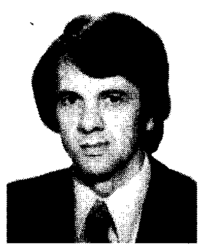

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