ANALYSIS OF CHUA'S CIRCUIT WITH TRANSMISSION LINE

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ABSTRACT

We have analyzed Chua's circuit with a transmission line, where the parallel LC resonator in original Chua's circuit is replaced by a lossless or lossy transmission line. Some papers about Chua's circuit with lossless transmission line (time-delayed Chua's circuit) have been reported, but whose analysis is rather simple because the signal on it is not affected an attenuation due to the line loss. On the other hand, the method of characteristics is developed for the simulation of high-speed VLSI circuits containing lossy transmission lines. In this paper, we will apply this method to analyze Chua's circuit with transmission line. The linear interpolation and the variable stepsize integration techniques are introduced to get the accurate solution around the breakpoint of piecewise linear function. Consequently we found from the numerical experiments that Chua's circuit with lossy transmission line has the complicate and interesting chaotic attractors compared to Chua's circuit with lossless line.

1. INTRODUCTION

Chua's circuit [1] shown in Figure 1(a) is a simple oscillator circuit which exhibits a variety of bifurcation phenomena and attractors. The nonlinear resistor is a piecewise

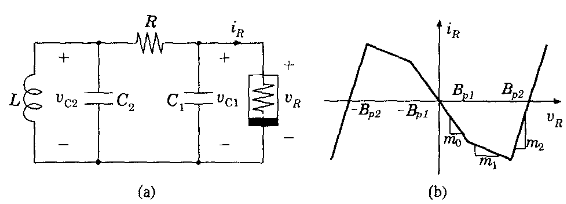

Figure 1. (a) Chua's circuit. (b) $v - i$ characteristic of nonlinear resistor.

linear function as shown in Figure 1(b) and described by:

$$\begin{aligned} i_R &= m_2 v_R + \frac{1}{2}(m_0 - m_1)[|v_R + B_{p1}| - |v_R - B_{p1}|] \\ &+ \frac{1}{2}(m_1 - m_2)[|v_R + B_{p2}| - |v_R - B_{p2}|] \end{aligned} \quad (1)$$

where m_0, m_1, m_2 are the slope in the each segments of this piecewise linear function, and B_{p1}, B_{p2} denote the breakpoints. Many theoretical and experimental studies about this circuit have been reported so far, some studies to utilize this circuit for novel applications in engineering, e.g. secure communication, are also published recently.

By replacing parallel LC resonator in original Chua's circuit by a short-circuited transmission line, a circuit as shown in Figure 2 is obtained. We will call the result-

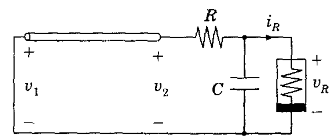

Figure 2. Chua's circuit with transmission line.

ing circuit "Chua's circuit with transmission line". If the transmission line is lossless, this circuit is also referred to as "time-delayed Chua's circuit". Recently, some papers about the time-delayed Chua's circuit have been reported [5]-[10]. But the time-delayed Chua's circuit is analyzed only for the special case of $C = 0$ in most of these literature [5]-[9]. Because the time-delayed Chua's circuit with $C = 0$ can be governed by one-dimensional difference equation, its chaotic behavior can be analyzed by investigating the corresponding one-dimensional map. On the other hand, Hosny *et al.* [10] have presented an analytical method for nonlinear lumped circuit with lossless transmission line. As an example, the chaotic behavior of time-delayed Chua's circuit have analyzed for the case of $C \neq 0$. The chaotic attractor and the bifurcation diagram have been also shown by computer simulation. Unfortunately, this method can not be applied to the analysis of lossy transmission line. Our method can be applied to any kind of lumped circuits with transmission lines. Note that for a lossy transmission line, since the propagation signal is affected the attenuation, time delay, and reflections at both ends, the phenomena of it are much more complicate compared to lossless one.

It is the purpose of this paper to analyze Chua's circuit with lossless or lossy transmission line. In the last decade, many approaches have been proposed for the simulation of high-speed VLSI circuits, and we believe that the method of characteristics [11],[13] are most suitable to analyze transmission lines. We will first apply the classical method of characteristics [11] to Chua's circuit with lossless transmission line, which can get the accurate solutions. Next, the generalized method of characteristics [12] is used to analyze Chua's circuit with lossy transmission line. To get the accurate solution around the breakpoint of piecewise linear function, we introduce the linear interpolation and variable stepsize integration techniques.

Numerical experiments are given in section 3 for various parameters. It seems that the attractors become more complicate compared to the lossless transmission line.

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2. NUMERICAL ANALYSIS

The method of characteristics has been proposed by Branin [11] for the transient analysis of a single lossless transmission line. The lossless transmission line is modeled as disconnected two-port network (henceforth called the characteristic model), shown in Figure 3, consisted of the characteristic impedances and the waveform generators.

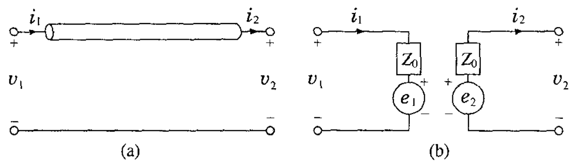


Figure 3. (a) The transmission line and (b) its characteristic model.

Where e_1, e_2 are the waveform generators to simulate the reflection and defined by:

$$\begin{aligned} e_1(t) &= 2v_2(t) - e_2(t-\tau) \\ e_2(t) &= 2v_1(t) - e_1(t-\tau). \end{aligned} \tag{2}$$

Then Chang [12] has extended this method for a lossy transmission line. The lossy transmission line is also modeled by characteristic model shown in Figure 3. The characteristic impedance function Z_0 and the exponential propagation function $e^{-\theta}$ are synthesized as the equivalent lumped circuits by Padé approximation (see [13] for detail). The equivalent circuit for the exponential propagation function consists of ideal delay line to simulate the time delay and lumped-parameter networks to simulate the attenuation loss.

For now on, we apply these methods to Chua's circuit with transmission line. The equivalent circuit of Chua's circuit with transmission line is shown in Figure 4, where the transmission line is replaced by the characteristic model.

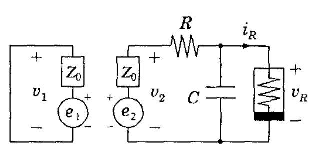


Figure 4. The equivalent circuit of Chua's circuit with transmission line.

Thus, for a lossless transmission line, we have the following circuit equations:

$$\begin{aligned} v_1(t) &= 0 \\ v_2(t) &= \frac{R\, e_2(t-\tau) + Z_0\, v_R(t)}{R+Z_0} \\ C\frac{dv_R(t)}{dt} &= \frac{v_2(t) - v_R(t)}{R} - i_R(v_R) \\ e_1(t) &= 2v_2(t) - e_2(t-\tau) \\ e_2(t) &= -e_1(t-\tau). \end{aligned} \tag{3}$$

Note that for a lossy transmission line, since Z_0 and $e^{-\theta}$ are modeled by the lumped circuit, the second equation and last two equations of (3) must be rewritten by the corresponding state equations. The transient responses can be calculated by the numerical integration method such as Runge-Kutta method. Observe that, since (3) contain the time-delayed waveform generators $e_1(t-\tau)$ and $e_2(t-\tau)$, their values in the period τ need to be stored. To get the accurate solution, we need to estimate the exact state-variables and time passing through a breakpoint of piecewise linear resistor. Therefore, we have applied an interpolation and variable stepsize integration techniques to get their values around the points. In our examples, we used 4th order Runge-Kutta method as the numerical integration method.

3. SIMULATION RESULTS

We first simulate Chua's circuit with lossless transmission line by using the classical method of characteristics [11] and use the following parameters:

$$Z_0 = 424[\Omega], \tau = 0.05[\text{msec}],\ C = 10[\text{nF}],$$
$$m_0 = -0.75[\text{mS}],\ m_1 = -0.41[\text{mS}],\ m_2 = 10[\text{mS}],$$
$$B_{p1} = 1[\text{V}],\ B_{p2} = 8[\text{V}]$$

where we have determined $Z_0 = \sqrt{L/C_2}, \tau = \sqrt{LC_2}\ l, l = 1[\text{m}]$ by using the values $L = 18[\text{mH}], C_2 = 100[\text{nF}]$ used in [4]. With varying the value of linear resistor R, the computer simulation is carried out. The period-doubling phenomenon and Double Scroll attractors similar to original Chua's circuit, as shown in Figure 5, are observed by the transient analysis.

Next, we continue the simulation by changing the parameters. The parameters as listed below are used because the characteristic impedance of a coaxial cable is normally $50[\Omega]$.

$$Z_0 = 50[\Omega], \tau = 14.25[\text{nsec}], C = 1[\text{pF}],$$
$$m_0 = -0.8[\text{mS}],\ m_1 = -0.4[\text{mS}],\ m_2 = 10[\text{mS}],$$
$$B_{p1} = 1[\text{V}],\ B_{p2} = 8[\text{V}]$$

The trajectories in phase plane are shown in Figure 6. We have observed more complex bifurcation phenomenon and Double Scroll attractors compared to the above examples.

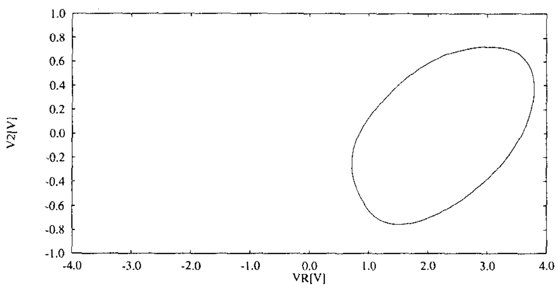
Finally, we carry out the simulation for Chua's circuit with lossy transmission line. We used the following parameters:

$$R_d = 0.55[\Omega/\text{m}],\ L_d = 0.2375[\mu\text{H/m}],\ C_d = 95[\text{pF/m}],$$
$$G_d = 0.1[\text{mS/m}],\ l = 3[\text{m}],\ C = 1[\text{pF}],$$
$$m_0 = -0.9[\text{mS}],\ m_1 = -0.5[\text{mS}],\ B_{p1} = 1[\text{V}]$$

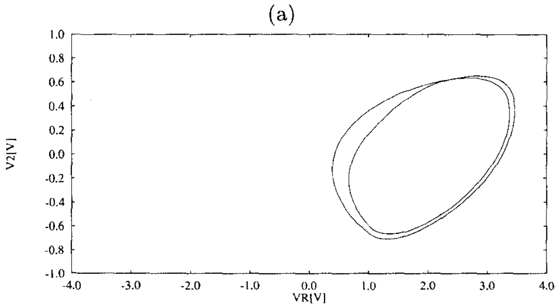
where R_d, L_d, C_d, G_d denote the per-unit-length line parameters and the outer segments with negative slope m_1 of nonlinear resistor are extended to infinity (i.e., $\pm B_{p2} = \pm\infty$). Note that this transmission line is relatively close to the distortionless line satisfying the condition of $(R_dC_d/L_dG_d = 2.2)$. In this case, 4th order Padé approximation of Z_0 and $e^{-\theta}$ has good agreement [14]. The trajectories in phase plane are shown in Figure 7. Chua's circuit with lossy transmission line has more complex Double Scroll attractors compared to lossless one.

4. CONCLUSION

In this paper, we have presented an efficient analysis method for a chaotic circuit with both transmission line and piecewise linear element. Chua's circuit with transmission line was efficiently analyzed by the proposed method. We



(a)



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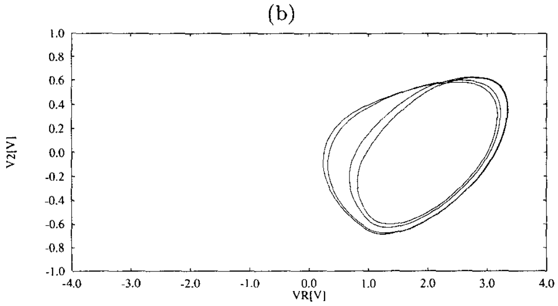
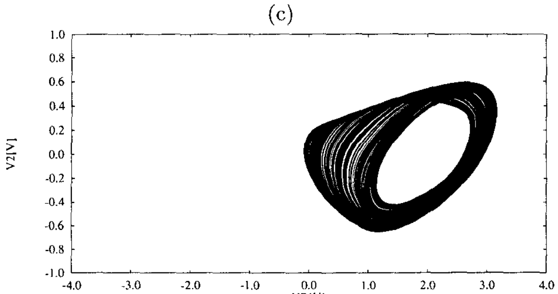
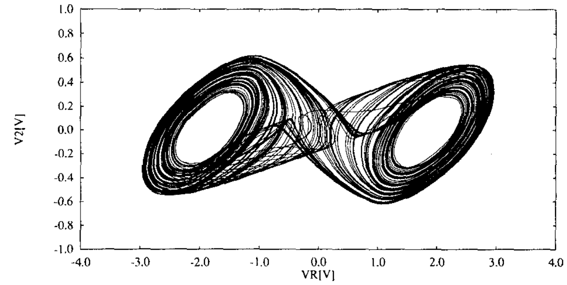


(c)



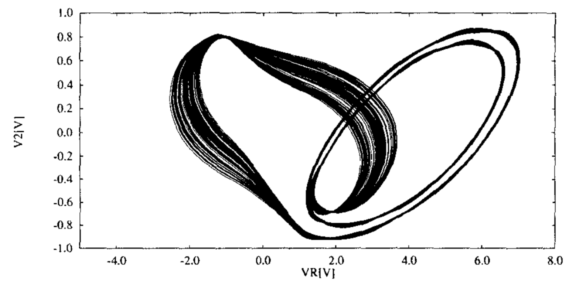
(d)

Figure 5. Phase planes of Chua's circuit with lossless transmission line for (a) $R = 1800[\Omega]$, (b) $R = 1760[\Omega]$, (c) $R = 1745[\Omega]$, (d) $R = 1720[\Omega]$, (e) $R = 1690[\Omega]$.

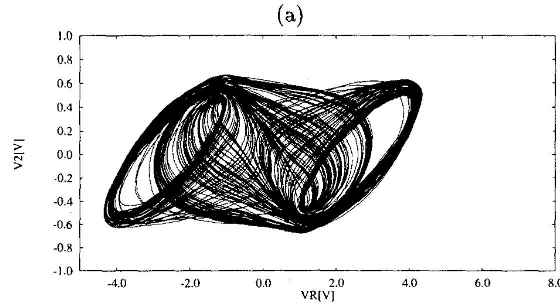


(e)

Figure 5. *continued.*



(a)



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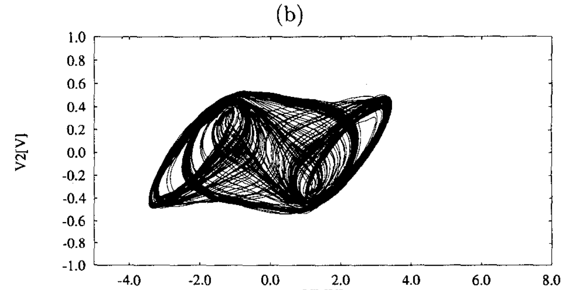

(c)

Figure 6. Phase planes of Chua's circuit with lossless transmission line for (a) $R = 2000[\Omega]$, (b) $R = 1800[\Omega]$, (c) $R = 1300[\Omega]$.

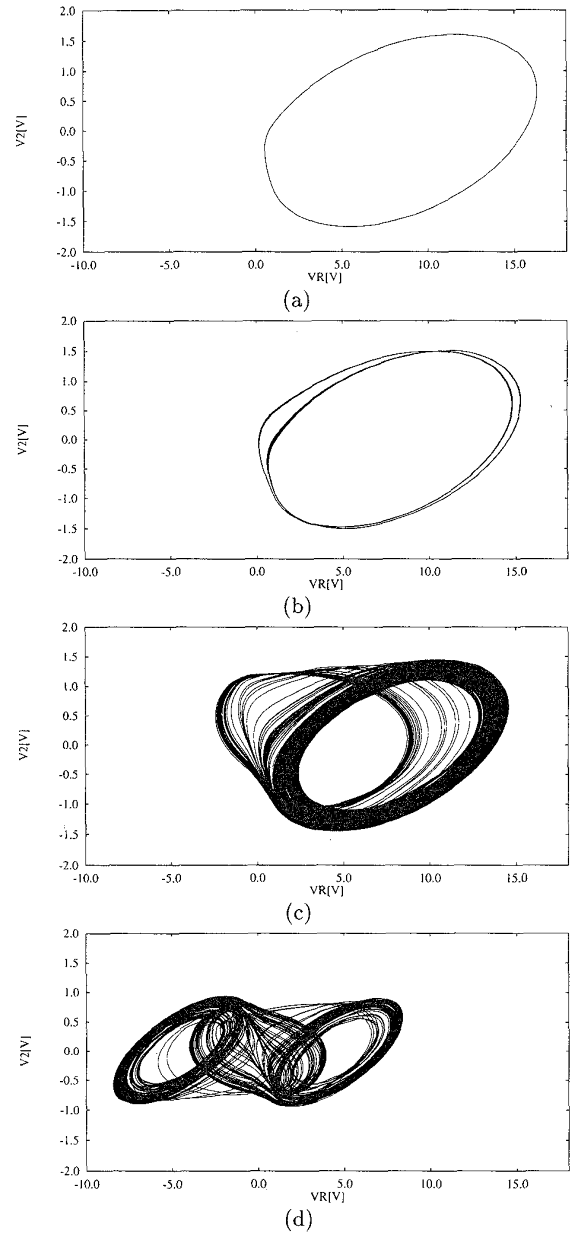

Figure 7. Phase planes of Chua's circuit with lossy transmission line for (a) $R = 1830[\Omega]$, (b) $R = 1820[\Omega]$, (c) $R = 1810[\Omega]$, (d) $R = 1700[\Omega]$, (e) $R = 1600[\Omega]$.

have confirmed the generation of various chaotic phenomena including Double Scroll attractor by computer simulation.

In future, we intend to confirm these results experimentally. Further, we are going to simulate this circuit in detail by varying the circuit parameters and try the qualitative analysis of the bifurcation route to chaos and stability of attractors. Chua's circuit with transmission line may also has the different types of chaotic phenomena depending on the line length. Also, we intend to confirm the generation of period-adding phenomenon in time-delayed Chua's circuit without C.

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