MODELLING OF THIRD-ORDER ELEMENTARY CANONICAL STATE MODELS USING CELLULAR NEURAL NETWORKS

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ABSTRACT

The new third-order piecewise-linear systems - elementary canonical systems -that belongs to the C-class can be modelled by cellular neural networks. There is introduced the extended circuit of a cellular neural cell that fits in designing of the elementary canonical systems.

1. INTRODUCTION

The chaotic behaviour and wave phenomena can be observed in many systems. The simplest autonomous systems which can exhibit this behaviour are third-order piecewise-linear (PWL) systems. The known third-order PWL systems are Chua's circuit and Chua's oscillator. Their dynamics is given by two sets of three eigenvalues determining two characteristic polynomials associated with the corresponding regions.

The new third-order PWL models - elementary canonical state models - were introduced in [1]. Elementary canonical state models (ECSM) are topologically conjugated to class C, i.e. Chua's circuit or Chua's oscillator [4].

In [3] is shown that Chua's circuit can be generated by cellular neural networks (CNN). CNN seem to be a universal tool to model any PWL systems. In this paper the synthesis of ECSM using CNN is described. There is introduced new generalised CNN cell that is convenient to implement ECSM.

2. ELEMENTARY CANONICAL STATE MODELS

The elementary canonical state models of the third-order autonomous piecewise-linear dynamical systems are based on the first canonical state model of the third-order non-autonomous linear system completed by a non-linear feedback determined by simple memoryless PWL function, i. e.

$$f(x_1) = \frac{1}{2}\left(|x_1 + 1| - |x_1 - 1|\right) \quad (1)$$

The corresponding state equations of this model are

$$\frac{dx_1}{dt} = q_1.x_1 - x_2 + (p_1 - q_1).f(x_1) \quad (2)$$

$$\frac{dx_2}{dt} = q_2.x_1 - x_3 + (p_2 - q_2).f(x_1) \quad (3)$$

$$\frac{dx_3}{dt} = q_3.x_1 + (p_3 - q_3).f(x_1) \quad (4)$$

This system belongs to class C because PWL function $f(x_1)$ is continuous, odd-symmetric, and R^3 is partitioned by two parallel boundary planes $U_1 : x_1 = 1$ and $U_1 : x_1 = -1$ into the inner region $D_0(-1 \le x_1 \le 1)$ and two outer regions $D_1(x_1 \ge 1)$ and $D_{-1}(x_1 \le -1)$.

The following characteristic polynomials for the individual regions can directly be determined from (2), (3) and (4):

$$D_0: s^3 - p_1.s^2 + p_2.s - p_3 \quad (5)$$

$$D_1, D_{-1}: s^3 - q_1.s^2 + q_2.s - q_3 \quad (6)$$

Coefficients (equivalent eigenvalue parameters) of eq. (5) and (6) are given directly by the coefficients in state equations (2), (3) and (4). This system represents elementary canonical form topologically conjugate to Chua's circuit family.

3. AUTONOMOUS CELLULAR NEURAL CELL MODELS

The CNN having no inputs (i.e., autonomous) are convenient for modelling and generating non-linear wave and chaotic phenomena [3]. Each CNN is defined mathematically by its cell dynamics and synaptic law [2]. In this paper the synaptic law is linear and realised by two layers of linear resistor couplings.

A first-order cell n_j is characterised by

$$\frac{dx_j}{dt} = -g(x_j) + I_j^S \quad (7)$$

where $g(x_j)$ is any scalar function of x_j, so it can equal zero too.

The synaptic law is defined by the synaptic input current, i.e.

$$I_j^S = \sum_{\substack{k \in S_j \\ k \neq 0}} a_k.x_{j+k} + \sum_{\substack{k \in S_j \\ k \neq 0}} b_k.f(x_{j+k}) \quad (8)$$

As shown, this CNN cell is one-dimensional. S_j is a sphere of influence or neighbourhood size.

4. CNN CELL DESIGN AND IMPLEMENTATION

To implement ECSM state equations a new generalised circuit of the CNN cell is introduced [5]. It is shown in Fig. 1.

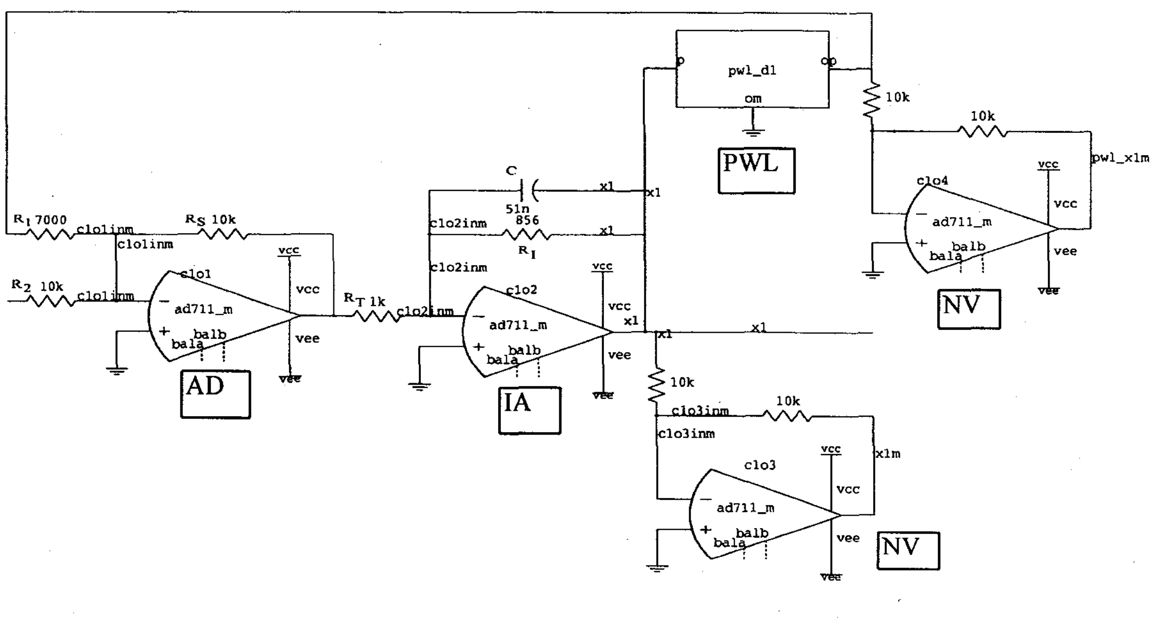

Figure 1. Circuit of the first CNN cell

The generalised circuit is characterised by

$$\frac{dx_j}{dt} = -a_j.x_j + a_{jk}.x_k + a_{jl}.x_{jl} + \\ + b_j.f(x_j) + b_{jk}.f(x_k) + b_{jl}.f(x_l) \quad (9)$$

j being the cell index of the first CNN cell, x_j, x_k and x_l the state variable, $f(x_j), f(x_k)$ and $f(x_l)$ the PWL functions, $a_j, a_{jk}, a_{jk}, b_j, b_{jk}$ and b_{jl} constant parameters. This generalised circuit fits the definition of the CNN cell.

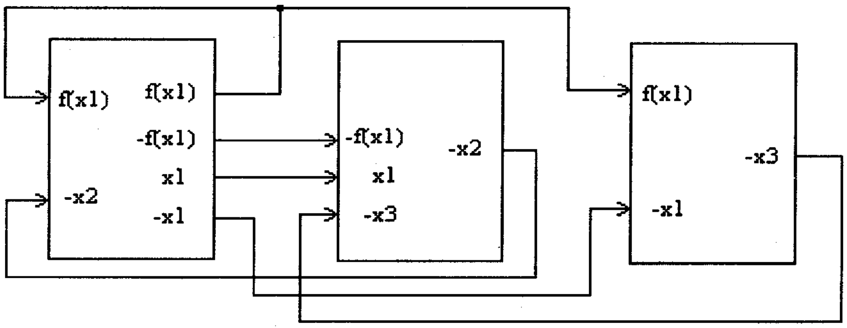

Figure 2. Cell connection scheme

The ECSM can be described by three related equations (9) too. Three fully connected CNN cells, shown in Fig. 2., create the generalised CNN of ECSM. If some parameters $a_j, a_{jk}, a_{jk}, b_j, b_{jk}$ and b_{jl} equal zero, the generalised circuit will be reduced. It can be determined from (3) and (4) that the second and third CNN circuits have no PWL sub-circuits.

A circuit implementation for the generalised model of CNN cell is constituted by four basic blocks:

1) A operational amplifier in AD block ads input signals. Fractions of resistors $\frac{R_A}{R_1}$ form adding constants.

2) The output of the adding amplifier is connected to the input of an integrating circuit in IA block. The resistor R_I in this block makes the inner feedback that is described by the constant a_j in eq. (9). In case of second and third CNN cells in Fig. 2 resistors R_I miss and thus constants a_k and a_l equal to zero.

3) An output of the PWL block is determined by the PWL function. There are a lot of ways how to design this PWL block. The design of the PWL circuit is inspired by [3].

4) Negative values of the outputs of the integral amplifiers and PWL block are made by circuits of two NV blocks. This blocks form negative signs in eq. (2), (3) and (4).

The differential equation that describe the circuit of the first CNN cell in Fig. 1. is equivalent to eq. (9) and is characterised by

$$\frac{dx_j}{dt} = \frac{1}{C.R_T} \begin{pmatrix} -\frac{R_T}{R_I}.x_j + \\ +\frac{R_S}{R_2}.(-x_k) + \\ +\frac{R_S}{R_1}.f(x_j) \end{pmatrix} \qquad (10)$$

5. EXPERIMENTAL RESULTS

A lot of attractors can be obtained with ECSM. If parameters of eq. (2), (3) and (4) are

$$q_1 = -1.168 \quad q_2 = 0.846341$$
$$q_3 = -1.2948 \quad p_1 = 0.09$$
$$p_2 = 0.432961 \quad p_3 = 0.653325$$

ECSM will generate a double scroll, shown in Fig. 3. In accordance with eq. (10) and design considerations, suitable value for the CNN cell components are

$$R_{j1} = 7k\Omega;\ R_{j2} = 10k\Omega;\ R_{jS} = 10k\Omega;$$
$$R_{jT} = 1k\Omega;\ R_{jI} = 856\Omega;\ R_{k1} = 25k\Omega;$$
$$R_{k2} = 12k\Omega;\ R_{k3} = 10k\Omega;\ R_{kS} = 10k\Omega;$$
$$R_{kT} = 1k\Omega;\ R_{l1} = 5k\Omega;\ R_{l2} = 7k7\Omega;$$
$$R_{lS} = 10k\Omega;\ R_{lT} = 1k\Omega;$$
$$C_j = C_k = C_l = 51nF$$

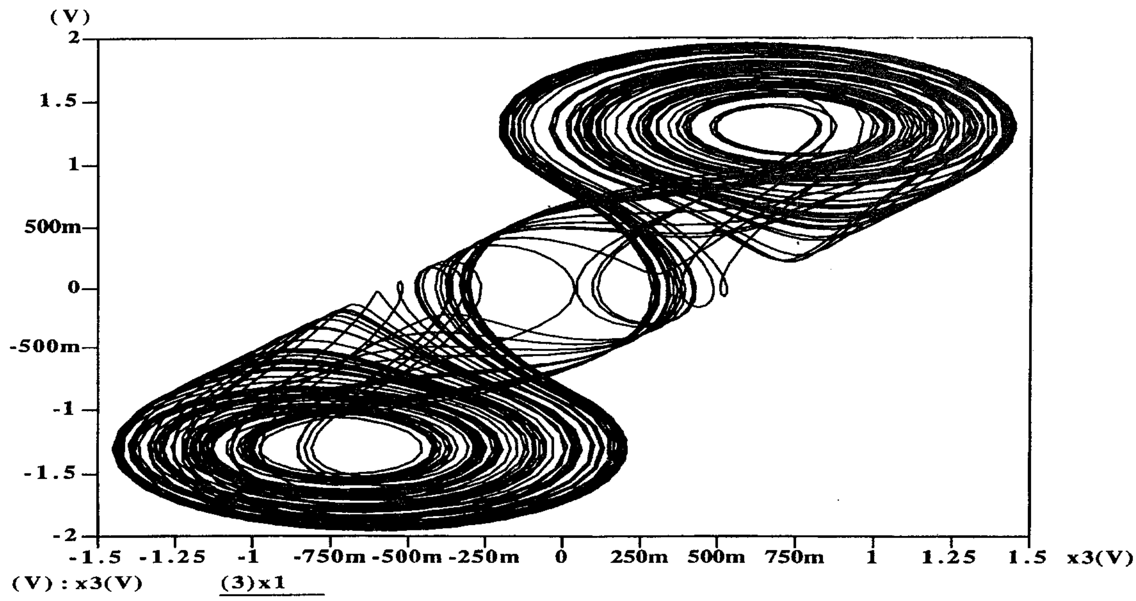

Figure 3. Double scroll

The circuit of CNN is simulated by the SABER system. The SABER is general purpose simulator and provides a way to build hierarchical designs made up of primitive symbols at the lowest level of the design and hierarchical symbols representing sub-circuits at higher levels.

6. CONCLUSIONS

Cellular neural networks are universal tools to study, model and simulate analogue electronic circuits. In this paper it is designed the generalised circuit of CNN cell and the CNN, that realises ECSM, is implemented. Using generalised CNN the chaotic behavioural of ECSM is observed.

The system of CNN based on ECSM employs:

1. The ECSM belongs to Class C, the characteristic polynomials (5) and (6) for the individual regions can directly be determined from (2), (3) and (4). So that this system represents elementary canonical form topologically conjugated to Chua's family.

2. The CNNs are very studied and there are presented a lot of practical implementations of CNN circuits. Any systems of differential and partial differential equations can be simulated by the CNN by synthesising the appropriate a CNN cell and synaptic coupling.

REFERENCES

[1] Pospíšil J., Brzobohatý J., Kolka Z.: „Elementary canonical state models of the third-order autonomous piecewise-linear dynamical systems.“, Proc. ECCTD'95, Istanbul, vol. 1, pp. 463 - 466, 1995

[2] Chua L. O., Yang L.: „Cellular neural networks: Theory.“, IEEE Trans. Circuits Syst., vol. 35, pp. 1257 - 1272, 1988

[3] Arena P., Baglio S., Fortuna L., Manganaro G.: „Chua's circuit can be generated by CNN cells.“, IEEE Trans. Circuits Syst., vol. 42, pp. 123 - 125, 1995

[4] Chua L. O.: „Global unfolding Chua's circuit“, IEEE Trans. Fundamentals, vol. E76-A, pp. 704 - 734, May 1993

[5] Chua L. O., Hasler M., Moschytz G. S., Neirynck J.: „Autonomous Cellular Neural Networks: A Unified Paradigm for Pattern Formation and Active Wave Propagation.“, IEEE Trans. Circuits Syst., vol. 42, pp. 559 - 577, October 1995

[6] Kaderka J.: „Using PWL circuits for analog synthesis of approximate identity neural networks“, Proc. Workshop 96, part 3., pp. 871 - 872, January 1996