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DYNAMICAL BEHAVIOR OF ACOUSTICALLY COUPLED CHAOS OSCILLATORS

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Abstract

We have found, for the first time, that Acoustically Coupling Chaos Oscillator (ACCO) also exhibits a chaotic attractor called the "double scroll Chua's attractor" by using the modified Chua's circuit. When each ACCO is in an oscillating periodic state, the interaction of sound waves with acoustic coupling of two ACCOs can also result in the appearance of chaotic sound waves.

1. INTRODUCTION

It has been generally accepted that Chua's circuit is a very suitable subject for the study of dynamical chaos by means of both laboratory experiments and computer simulations.[1] Chua's circuit[2] is the simplest autonomous circuit that can exhibit bifurcation and chaos. It has been studied extensively and is one of the very few circuits in which a formal proof of the existence of chaos has been accomplished. Moreover, theoretical and simulated behavior of this circuit can be accurately reproduced experimentally. The chaotic nature of double scroll Chua's attractor was proved in Chua et al.[3] and Silva[4] by establishing the existence of a homoclinic loop of the saddle-focus at the origin for some values of parameters and by applying the Shil'nikov theorem.

In this paper, we report the experimental results and the implementation details of acoustically coupled chaos oscillator by using Chua's circuit. We have shown that the acoustically coupled chaos oscillator produces chaotic signals whose attractor is double scroll.

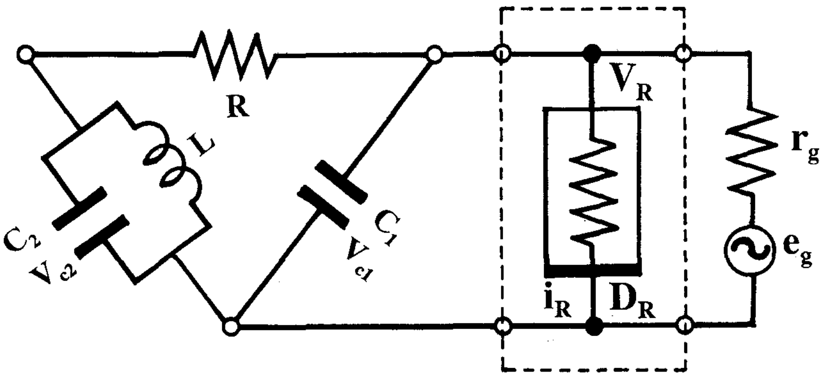

Fig. 1 Construction of our chaos oscillator. The dashed line box is a nonlinear resistor. This part is called as Chua's diode (D_R). We choose a piecewise-linear function for nonlineality which induces period doubling route to chaos. C_1 and C_2 are 0.00068 μF and 0.0068 μF, respectively. L is 3.3 mH. These values are fixed in our all experiments. R is the control parameter in our oscillator.

Chua's circuit is a nonlinear electronic circuit that is the object of much scientific research activities.[1] The circuit contains four linear elements and a nonlinear resistor which can be built using op-amps as shown in Fig. 1. The state equation of Chua's oscillator are as follows:

$$\frac{dv_1}{dt} = \frac{1}{C_1}\{G(v_2 - v_1) - f(v_1)\}, \tag{1}$$

$$\frac{dv_2}{dt} = \frac{1}{C_2}\{G(v_1 - v_2) + i_3\}, \tag{2}$$

$$\frac{dv_3}{dt} = -\frac{1}{L}(v_2 + R_0 i_3), \tag{3}$$

where

$$G = \frac{1}{R}, \tag{4}$$

and

$$f(v_1) = G_b v_1 + \frac{1}{2}(G_a - G_b)\{|v_1 + E| - |v_1 - E|\} \tag{5}$$

is the $v-i$ characteristic of the nonlinear resistor D_R called Chua's diode, with a slope equal to G_a in the inner region and G_b in the outer region. A typical $v-i$ characteristic of D_R is the piecewise-linear function as done by Chua et al. By choosing appropriate values for G_a, G_b and E, any continuous three-segment odd-symmetric piecewise-linear $v-i$ characteristic for Chua's diode can be specified. We choose appropriate values for G_a, G_b and E which induce period doubling bifurcation route to chaos with double scroll attractors.

2. CHAOS OSCILLATORS — CHUA'S CIRCUIT

In order to generate the acoustic coupling chaos oscillator, we chose the same frequency band from that of the acoustic transducer as discussed in the next section. In this study, adopted nonlinear characteristic was piecewise-linear functions. In our experimental set-up, the resistor value R was varied as a control parameter and the following values were fixed: $C_1 = 0.00068\ \mu\text{F}$, $C_2 = 0.0068\ \mu\text{F}$, $L = 3.3$ mH, $E = 1\ V$ and $G_a < G_b < 0$. Although any circuit which realizes the nonlinear $v - i$ characteristic Chua's diode can be used experimentally, we used two diodes and one op-amp.

Decreasing the value of R, we observed time waveforms (time series) of $V_{c1}(t)$ and $V_{c2}(t)$, attractors projected on the $V_{c1} - V_{c2}$ plane and power spectrum of $V_{c1}(t)$. Until $R = 1.416\ k\Omega$, the attractor was point attractor, i.e., stable equilibrium point. At a threshold value of $R = 1.416\ k\Omega$, an equilibrium point lost stability and stable periodic attractor (limit cycle) emerged through an Hopf bifurcation. Further decreasing of R led to the period doubling bifurcation. We observed period two and period four behavior clearly at $R = 1.388\ k\Omega$ and $R = 1.380\ k\Omega$, respectively. From $R = 1.374\ k\Omega$ to $R = 1.334\ k\Omega$, we observed double scroll attractor which is the typical attractor in the Chua's circuit. The power spectrum of V_{c1} is continuous for the wide range of frequency. This double scroll attractor is constructed by connection of two screw attractors which are symmetric around the original point. Homoclinicity in the double scroll attractor is the origin of the chaotic behavior.[1]

Examples of time waveforms of V_{c1} and V_{c2}, the double scroll attractor and the power spectrum of V_{c1} are shown in Figs. 2, 3 and 4, respectively. From these results, chaotic behavior is confirmed.

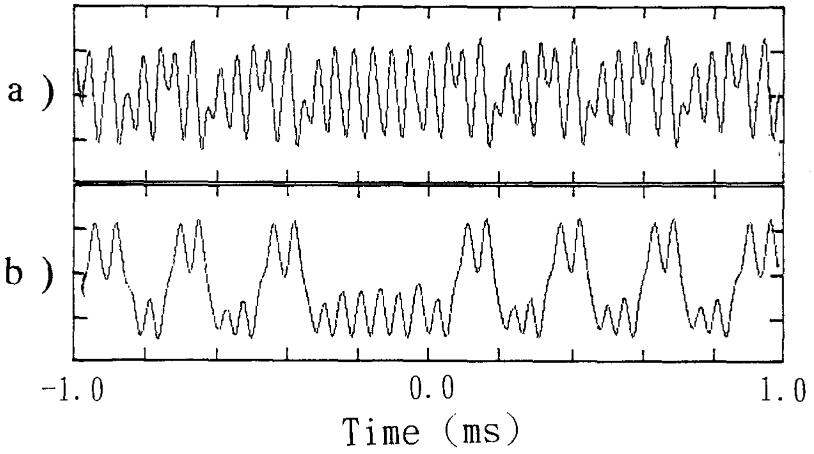

Fig. 2 Time waveforms of: (**a**) V_{c1} and (**b**) V_{c2} ($R = 1.334\ k\Omega$). Chaotic behaviors are found in them. Different waveforms between them are due to the difference in capacitances between C_1 and C_2.

Fig. 3 The attractor projected on a $V_{c1} - V_{c2}$ plane ($R = 1.334\ k\Omega$). This attractor is double scroll attractor which is a typical one in the Chua's circuit ($R = 1.334\ k\Omega$). This attractor is constructed through the connection of two screw attractors.

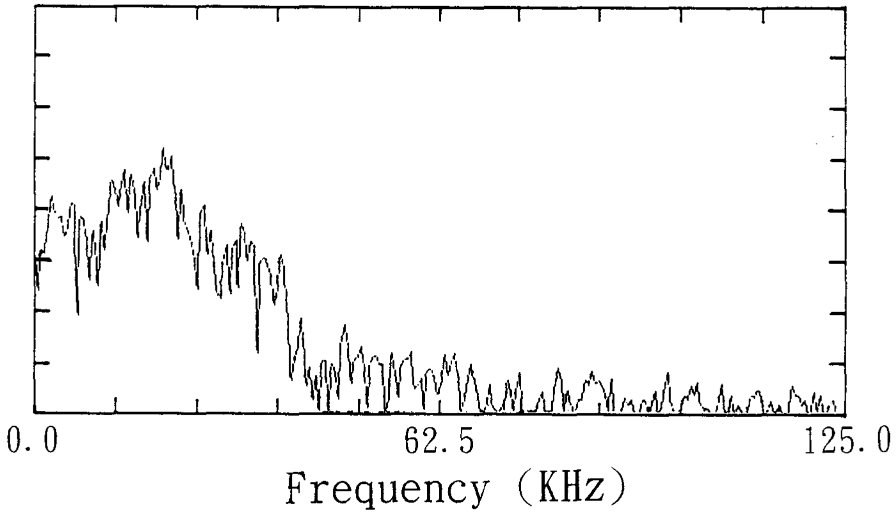

Fig. 4 Power spectrum of V_{c1}. This continuous spectrum is chaotic signal ($R = 1.334\ k\Omega$).

3. ACOUSTICALLY COUPLED CHAOS OSCILLATORS

As a model system for acoustically coupled chaos oscillator, we used an ultrasonic transducer MA23L3. Figure 5 shows the schematic diagram of these experimental circuits. As was done in the case of Chua's circuit, we decreased the value of R and observed waveforms of V_{c1} and V_{c2}, attractors projected on the $V_{c1} - V_{c2}$ plane and the power spectrum of V_{c1}.

Figure 6 shows a typical example of the waveform of V_{c1} and V_{c2} at $R = 1.424\ k\Omega$. This time series of the waveforms shows chaotic behavior. The attractor is double scroll type which resembles that observed in the typical Chua's oscillator as shown in Fig. 7. The double scrolls in a typical Chua's circuit have holes at P^+ and P^- in which two coexisting homoclinic loops originating from a saddle-focus equilibrium point located at the origin of an odd-symmetric vector field.[3] Since P^+ is an "unstable focus" when restricted to

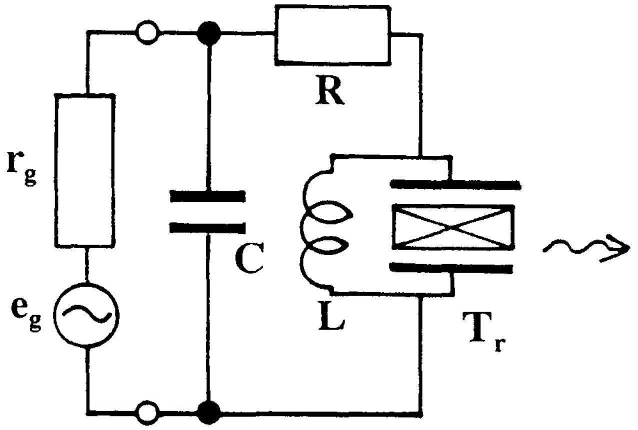

Fig. 5 Our acoustic chaos oscillator which utilize ultrasonic transducer MA23L3.

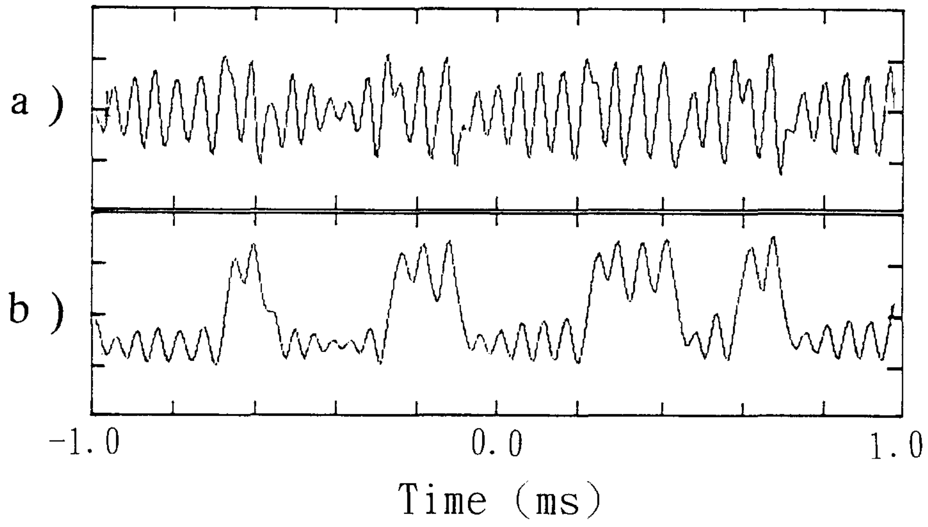

Fig. 6 Chaotic time waveform in the acoustic chaos oscillator. (a) V_{c1} and (b) V_{c2} ($R = 1.424\ k\Omega$).

Fig. 7 Hole-filling Double scroll attractor. The center of scrolls fluctuate in time due to two peaks in the band width of the transducer MA23L3 ($R = 1.424\ k\Omega$).

the eigenspace, it follows that the resulting double scroll will not have a hole and henceforth called a hole-filling orbit. The double scroll shown in Fig. 7 corresponds to a case in hole-filling heteroclinic orbit. The behavior of attractors from a periodic orbit to double scroll attractor for the case of Chua's circuit is clearly different from the hole-filling heteroclinic chaotic orbit. We confirmed that the ACCO exhibits a hole-filling heteroclinic chaotic behavior. The power spectrum closely resembles that in the case of the Chua's circuit and it shows a continuous spectrum as shown in Fig. 8. The behavior of attractors is, limit cycle → screw attractor → double scroll attractor, without period doubling bifurcation. We can conclude that our acoustic oscillator is able to generate chaotic signals. This fact leads to the possibility of the large number of ACCO system.

As a prototype of such coupled systems, we constructed one pair of acoustic chaos oscillators as shown in Fig. 9. As an example, we investigated a coupling between two limit cycles. Each oscillator's R was set at a value which generates simple periodic signal (Fig. 10). If two oscillators were coupled by acoustic periodic waves, both waveforms of V_{c1} became chaotic as in Fig. 11. This chaotic behavior was also recognized by the continuous power spectrum and calculated Lyapunov exponent.[5] When we shut the coupling, limit cycle oscillations were recovered. Therefore chaotic behavior was clearly induced by the acoustic coupling.

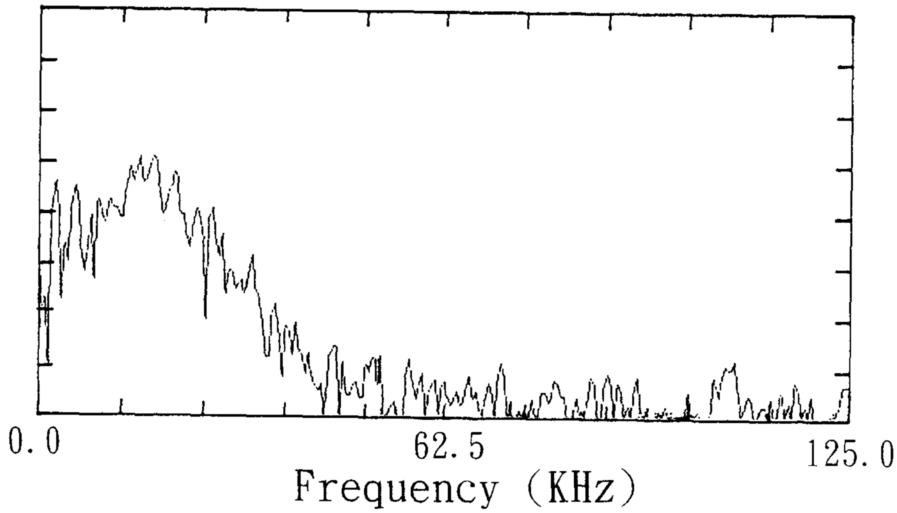

Fig. 8 Power spectrum of V_{c1}. Continuous spectrum typical in chaotic signals is apparent ($R=1.424$ $k\Omega$).

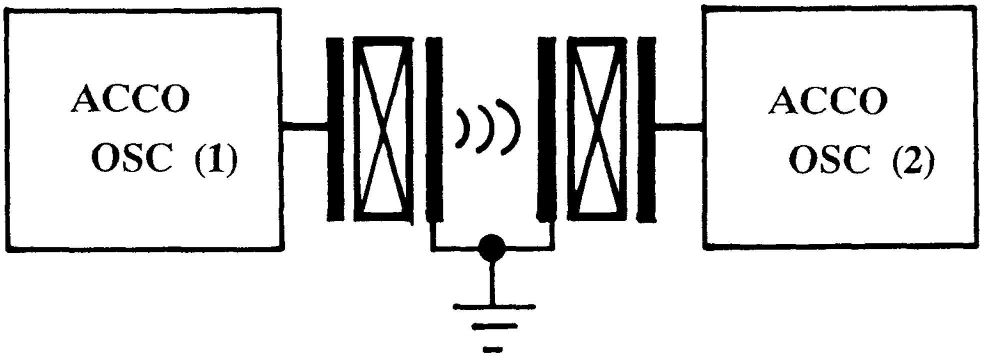

Fig. 9 Schematic figures of Acoustically Coupled Chaos Oscillator (ACCO). Two chaos oscillators are coupled by ultrasonic sound.

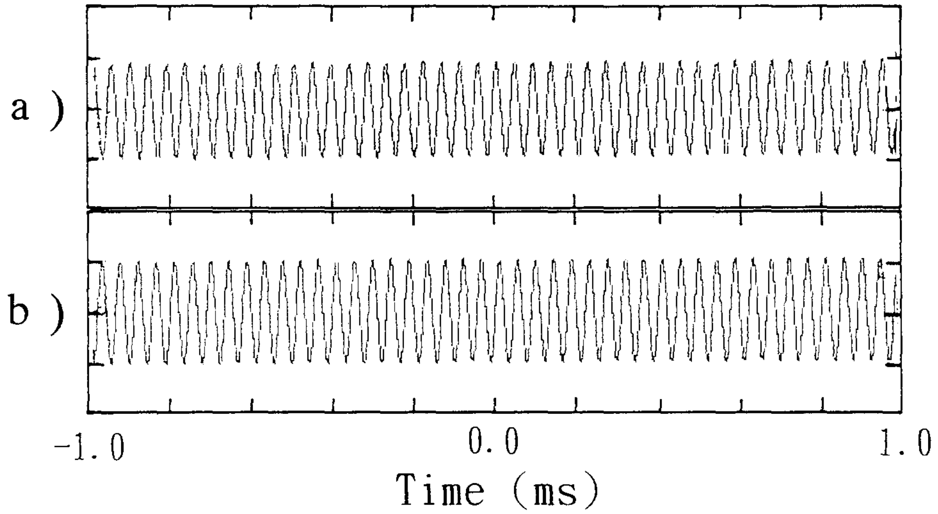

Fig. 10 Time waveforms of V_{c1} in (a) oscillator 1 and (b) oscillator 2 without coupling between two oscillators. Simple periodic states are realized.

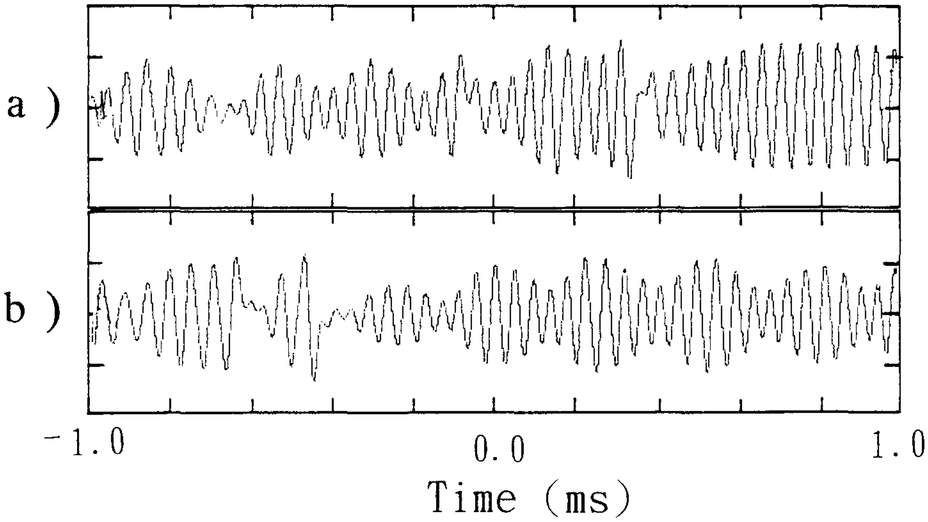

Fig. 11 Time waveforms of V_{c1} in (a) oscillator 1 and (b) oscillator 2 with acoustic coupling. In both oscillators chaotic signals are realized. If we shut the coupling physically, simple periodic waveforms are recovered for both oscillators.

As in other types of coupling, we are investigating the case of the periodic-chaos and chaos-chaos couplings now.

4. CONCLUSIONS

In this study, we have realized an acoustic chaos oscillator and its coupling utilizing transducers MA23L3 on Chua's circuit. A variety of interesting features were confirmed as follows:

(1) The route to chaos in ACCO is, limit cycle → screw attractor → double scroll attractor.
(2) Acoustic oscillator can exhibit chaotic behavior and the attractor is hole-filling double scroll attractor, due to the electro-mechanical effect.
(3) Due to the acoustic interactions between two ACCOs, two periodic oscillations change to chaotic behaviors.

Our ACCO would lead to wide applications such as electro-mechanical devices, chaos communications[6] and a typical model of large numbers of globally coupled chaos oscillators.[7]

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REFERENCES

1. L. P. Shil'nikov, *Int. J. Bifurcation and Chaos* **3**, 489 (1994).
2. L. O. Chua, C. W. Wu, A. S. Huang and G. Q. Zhong, *IEEE Trans. Circuits and Systems I* **40**, 732 (1993).
3. L. O. Chua, M. Komuro and T. Matsumoto, *IEEE Trans. Circuit Syst. CAS* **33**, 1072 (1986).
4. C. P. Silva, in *Chua's Circuit: A Paradigm for Chaos*, ed. R. N. Madan (Singapore, World Scientific, 1993), p. 671.
5. A. Wolf, J. B. Swift, H. L. Swinny and J. A. Vastano, *Physica* **D16**, 285 (1985).
6. K. M. Cuomo and A. V. Oppenheim, *Phys. Rev. Lett.* **71**, 65 (1993).
7. K. Kaneko, *Physica* **D34**, 1 (1989).