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Adaptive Control of Uncertain Chua's Circuits

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Abstract

In this paper, the adaptive backstepping with tuning functions method is used for the control of uncertain Chua's circuits with all the key parameters unknown. Firstly, we show that several Chua's circuits of different types including the Chua's oscillator, Chua's circuit with cubic nonlinearity, and Murali-Lakshmanan-Chua circuit, can all be transformed into a class of nonlinear systems in the so-called non-autonomous "strict-feedback" form. Secondly, an adaptive backstepping with tuning functions method is extended to the non-autonomous "strict-feedback" system, and then used to control the output of the Chua's circuit to asymptotically track an arbitrarily given reference signal generated from a known, bounded and smooth nonlinear reference model. Both global stability and asymptotic tracking of the closed-loop system are guaranteed. Simulation results are presented to show the effectiveness of the approach.

1 Introduction

Controlling chaotic systems has recently been in the focus of attention in the nonlinear dynamics literature ([6] and the references therein). In particular, many adaptive control schemes have been successfully applied to the control and synchronization of chaotic systems [5][7][14]. All these methods are based on rigorous Lyapunov stability theorem and Lyapunov function methods. But the construction of the Lyapunov functions remains to be a difficult task.

In the past decade, adaptive control of nonlinear systems has undergone rapid developments ([11] and the references therein). By using the backstepping design procedure, Kanellakopoulos *et al.* [8] have presented a systematic approach of globally stable and asymptotically tracking adaptive controllers for a class of nonlinear systems transformable to a parametric strict-feedback canonical form. The overparametrization problem was soon eliminated by Krstić *et al.* [10] by elegantly introducing the concept of tuning functions.

In this paper, by noticing that several Chua's circuits [1] of different types, including the Chua's oscillator [2], Chua's circuit with cubic nonlinearity [15], and Murali-Lakshmanan-Chua circuit [13], which have been used as paradigms in the research of bifurcations and chaos, are actually in the form of non-autonomous strict-feedback system, we extend the adaptive backstepping with tuning functions method to the non-autonomous strict-feedback system in the following form

$$\begin{aligned}\dot{x}_i &= b_i g_i(\bar{x}_i,t)x_{i+1} + \theta^T F_i(\bar{x}_i,t) + f_i(\bar{x}_i,t),\ 1 \le i \le n-1\\ \dot{x}_n &= b_n g_n(\bar{x}_n,t)u + \theta^T F_n(\bar{x}_n,t) + f_n(\bar{x}_n,t)\\ y &= x_1\end{aligned} \tag{1.1}$$

where $\bar{x}_i = [x_1, x_2, \cdots, x_i]^T \in R^i, i = 1,\cdots,n$, $u \in R$, and $y \in R$ are the states, input and output, respectively; $b = [b_1, b_2, \cdots, b_n]^T \in R^n$ and $\theta = [\theta_1, \theta_2, \cdots, \theta_p] \in R^p$ are the vectors of unknown constant parameters of interest; $g_i(\cdot) \neq 0$, $F_i(\cdot)$, $f_i(\cdot)$, $i = 1, \cdots, n-1$ are known, smooth nonlinear functions, $g_n(\cdot) \neq 0$, $F_n(\cdot)$, $f_n(\cdot)$ are known continuous nonlinear functions. We assume that the signs of parameters b_i, $i = 1, \cdots, n$ are known.

This design procedure is then applied to control the output of Chua's circuit to asymptotically track any given reference signal generated from a known, bounded and smooth nonlinear reference model. Simulation studies are conducted to show the effectiveness of the proposed method.

2 Chua's Circuits in Strict-Feedback Form

2.1 Chua's Circuit

The famous Chua's circuit [1] is a simple oscillator circuit which exhibits a rich variety of bifurcations and chaos phenomena. It contains three linear energy storage elements (one inductor L and two capacitors C_1 and C_2), one linear resistor R, and one nonlinear resistor called Chua's diode $g(v_{C_1})$. The dynamic equation of Chua's circuit is described by

$$\begin{aligned}C_1\frac{dv_{C_1}}{dt} &= \frac{1}{R}(v_{C_2} - v_{C_1}) - g(v_{C_1})\\ C_2\frac{dv_{C_2}}{dt} &= \frac{1}{R}(v_{C_1} - v_{C_2}) + i_L\\ L\frac{di_L}{dt} &= -v_{C_2}\end{aligned} \tag{2.1}$$

where C_1, C_2, L and R are all circuit parameters, i_L is the current through the inductor L, v_{C_1} and v_{C_2} are the voltages across C_1 and C_2, respectively, and the piecewise linear function $g(v_{C_1})$ describes the $V-i$ characteristics of the Chua's diode g as follows

$$g(v_{C_1}) = G_b v_{C_1} + \frac{1}{2}(G_a - G_b)(|v_{C_1}+1| - |v_{C_1}-1|) \tag{2.2}$$

with $G_a < 0$ and $G_b < 0$ being some appropriately chosen constants.

By defining $b_1 = 1/L > 0$, $b_2 = 1/RC_2 > 0$, $\theta_1 = 1/C_2$, $\theta_2 = 1/RC_2$, $\theta_3 = 1/RC_1$, $\theta_4 = 1/RC_1 + G_b/C_1$ and $\theta_5 = \frac{G_a - G_b}{2C_1}$, and defining the state variables as

$$x_1 = i_L,\ x_2 = v_{C_2},\ x_3 = v_{C_1} \tag{2.3}$$

then equations (2.1) can be reformulated in the following form

$$\begin{aligned}\dot{x}_1 &= -b_1 x_2 \\ \dot{x}_2 &= b_2 x_3 + \theta_1 x_1 - \theta_2 x_2 \\ \dot{x}_3 &= u + \theta_3 x_2 - \theta_4 x_3 - \theta_5\left[|x_3+1| - |x_3-1|\right]\end{aligned} \tag{2.4}$$

where the control $u(\cdot)$ is assumed to be introduced into the third equation of (2.4) to form the controlled Chua's circuit.

In comparison with the "strict-feedback" system form (1.1), and in the case when all the system parameters are unknown constants, i.e., $\theta = [\theta_1, \theta_2, \cdots, \theta_5]^T$, b_1 and b_2 are unknown (except that the signs of b_1 and b_2 are assumed to be known), we have

$g_1(x_1) = -1,\ g_2(x_1, x_2) = 1,\ g_3(x_1, x_2, x_3) = 1$
$f_1(x_1) = 0,\ f_2(x_1, x_2) = 0,\ f_3(x_1, x_2, x_3) = 0$
$F_1(x_1) = [0\,0\,0\,0\,0]^T,\ F_2(x_1, x_2) = [x_1\ -x_2\ 0\,0\,0]^T,$
$F_3(x_1, x_2, x_3) = [0\,0\,x_2\ -x_3\ -(|x_3+1| - |x_3-1|)]^T$

Following the same procedure, it can be verified that several other kinds of Chua's circuits, such as the Chua's Oscillator and the Chua's circuit with cubic nonlinearity, can all be transformed into the non-autonomous "strict-feedback" form (1.1).

2.2 Murali-Lakshmanan-Chua circuit

The Murali-Lakshmanan-Chua circuit is a simple second order non-autonomous nonlinear circuit, which can exhibit a rich variety of bifurcation and chaos phenomena [13].

The dynamical equation of Murali-Lakshmanan-Chua circuit is described by

$$\begin{aligned}C_1 \frac{dv_{C_1}}{dt} &= i_L - g(v_{C_1}) \\ L\frac{di_L}{dt} &= -v_{C_1} - Ri_L - R_s i_L + F\sin(\Omega t)\end{aligned} \tag{2.5}$$

where $g(v_{C_1})$ is given by (2.2).

By defining $b_1 = 1/L > 0$, $\theta_1 = (R + R_s)/L$, $\theta_2 = F$, $\theta_3 = 1/C_1$, $\theta_4 = G_b/C_1$ and $\theta_5 = \frac{G_a - G_b}{2C_1}$, and defining the state variables as

$$x_1 = i_L,\ x_2 = v_{C_1} \tag{2.6}$$

then equations (2.5) can be transformed as

$$\begin{aligned}\dot{x}_1 &= -b_1 x_2 - \theta_1 x_1 + \theta_2 \sin(\Omega t) \\ \dot{x}_2 &= u + \theta_3 x_1 - \theta_4 x_2 - \theta_5(|x_2+1| - |x_2-1|)\end{aligned} \tag{2.7}$$

where the control $u(\cdot)$ is assumed to be introduced into the second equation of (2.7).

In comparison with the "strict-feedback" system form (1.1), and in the case when all the parameters are unknown constants, we have

$g_1(x_1) = -1,\ g_2(x_1, x_2) = 1,\ f_1(x_1) = 0,\ f_2(x_1, x_2) = 0$
$F_1(x_1) = [\ -x_1\ \sin(\Omega t)\,0\,0\,0\ ]^T,$
$F_2(x_1, x_2) = [\ 0\,0\,x_1\ -x_2\ -(|x_2+1| - |x_2-1|)\ ]^T$

In the next section, we will extend the adaptive backstepping with tuning functions method [Krstič, *et al.*, 1992; Krstič, *et al.*, 1995] to the non-autonomous strict-feedback system in form (1.1).

3 Adaptive Backstepping with Tuning Functions Method

For the controlled system in form (1.1), consider a known, bounded and smooth reference model as follows

$$\begin{aligned}\dot{x}_{ri} &= f_{ri}(x_r, t),\ \ 1 \le i \le m-1 \\ \dot{x}_{rm} &= f_{rm}(x_r, t) \\ y_r &= x_{r1}\end{aligned} \tag{3.1}$$

where $x_r = [x_{r1}, x_{r2}, \cdots, x_{rm}]^T \in R^m (m \ge n)$, $y_r \in R$ are the states and output respectively; $f_{ri}(\cdot), i = 1, 2, \cdots, m-1$ are known smooth nonlinear functions and $f_{rm}(\cdot)$ is a known continuous nonlinear function.

Our objective is to design an adaptive state-feedback controller for system (1.1) that guarantees global stability and force the output $y = x_1(t)$ of system (1.1) to asymptotically track the output $y_r = x_{r1}(t)$ of the reference model, i.e.,

$$|y(t) - y_r(t)| \to 0,\ \text{as } t \to \infty. \tag{3.2}$$

The design procedure is recursive. At ith step, the ith-order subsystem is stabilized with respect to a Lyapunov function V_i by the design of a stabilizing function α_i, and tuning functions τ_i and $\pi_i^{b_1}, \cdots, \pi_i^{b_i}$. For the unknown parameter b_i, we introduce $\hat{b}_i$ and $\hat{\varrho}_i$. $\hat{\varrho}_i$ is the estimate of $\varrho_1 = 1/b_1$ and is introduced to avoid the division by $\hat{b}_1(t)$. The update law for the parameter estimates $\hat{\theta}(t)$ and $\hat{b}_i$, and the feedback control u are designed in the final step.

Step 1. Define $z_1 = x_1 - x_{r1}$. Its derivative is given by

$$\begin{aligned}\dot{z}_1 = {} & \hat{b}_1 g_1 z_2 + \frac{g_1}{\hat{\varrho}_1}\alpha_1 + \hat{\theta}^T F_{1s} + f_{1s} - (\hat{\theta} - \theta)^T F_{1s} \\ & - (\hat{b}_1 - b_1) g_1 z_2 + (b_1\hat{\varrho}_1 - 1)\left(\frac{g_1}{\hat{\varrho}_1}\alpha_1 + g_1 x_{r2}\right)\end{aligned} \tag{3.3}$$

where $z_2 = x_2 - \hat{\varrho}_1 x_{r2} - \alpha_1$, α_1 is an artificial control to be defined later, and $F_{1s} = F_1$, $f_{1s} = f_1 + g_1 x_{r2} - f_{r1}$.

Using α_1 as a control to stabilize (3.3) with respect to the Lyapunov function candidate

$$V_1 = \frac{1}{2}z_1^2 + \frac{1}{2}(\theta - \hat{\theta})^T \Gamma^{-1}(\theta - \hat{\theta}) + \frac{1}{2\gamma}(\hat{b}_1 - b_1)^2 + \frac{|b_1|}{2\gamma}(\hat{\varrho}_1 - \varrho_1)^2 \tag{3.4}$$

The derivative of V_1 is

$$\begin{aligned}\dot{V}_1 = {} & \hat{b}_1 g_1 z_1 z_2 + z_1\left(\frac{g_1}{\hat{\varrho}_1}\alpha_1 + \hat{\theta}^T F_{1s} + f_{1s}\right) \\ & + (b_1\hat{\varrho}_1 - 1) sgn(b_1)\gamma^{-1}\left(\dot{\hat{\varrho}}_1 + sgn(b_1)\gamma z_1 (g_1 x_{r2} \right. \\ & \left. + \frac{g_1}{\hat{\varrho}_1}\alpha_1)\right) + (\hat{\theta} - \theta)^T \Gamma^{-1}(\dot{\hat{\theta}} - \Gamma F_{1s} z_1) \\ & + (\hat{b}_1 - b_1)\gamma^{-1}(\dot{\hat{b}}_1 - \gamma g_1 z_1 z_2)\end{aligned} \tag{3.5}$$

Define the tuning functions τ_1 and $\pi_1^{b_1}$ for $\hat{\theta}$ and $\hat{b}_1$ respectively as $\tau_1 = \Gamma F_{1s} z_1$ and $\pi_1^{b_1} = \gamma g_1 z_1 z_2$. To eliminate the $(b_1\hat{\varrho}_1 - 1)$-term from equation (3.5), we choose the parameter update law for $\hat{\varrho}_1$ as $\dot{\hat{\varrho}}_1 = -sgn(b_1)\gamma z_1(\frac{g_1}{\hat{\varrho}_1}\alpha_1 + g_1 x_{r2})$.

To make the second term in equation (3.5) be equal to $-c_1 z_1^2$, we choose

$$\alpha_1 = \frac{\hat{\varrho}_1}{g_1}(-c_1 z_1 - \hat{\theta}^T F_{1s} - f_{1s}) \tag{3.6}$$

Note that the $(\hat{\theta} - \theta)$-term and $(\hat{b}_1 - b_1)$-term in equation (3.5) would have been eliminated with the choice of update laws $\dot{\hat{\theta}} = \tau_1$ and $\dot{\hat{b}}_1 = \pi_1^{b_1}$. Since this is not the last design step, we postpone the choice of update laws and tolerate the presence of $(\hat{\theta} - \theta)$ and $(\hat{b}_1 - b_1)$ in $\dot{V}_1$ as follows

$$\dot{V}_1 = -c_1 z_1^2 + \hat{b}_1 g_1 z_1 z_2 + (\hat{\theta} - \theta)\Gamma^{-1}(\dot{\hat{\theta}} - \tau_1) + (\hat{b}_1 - b_1)\gamma^{-1}(\dot{\hat{b}}_1 - \pi_1^{b_1}) \quad (3.7)$$

Step 2. The derivative of z_2 is expressed as

$$\begin{aligned}\dot{z}_2 &= \dot{x}_2 - \hat{\varrho}_1 \dot{x}_{r2} - \dot{\hat{\varrho}}_1 x_{r2} - \dot{\alpha}_1 \\ &= \hat{b}_2 g_2 z_3 + \frac{g_2}{\hat{\varrho}_2}\alpha_2 + \hat{\theta}^T F_{2s} + f_{2s} + \frac{\partial \alpha_1}{\partial \hat{\theta}}(\tau_2 - \dot{\hat{\theta}}) \\ &\quad - (\hat{b}_2 - b_2) g_2 z_3 + (\hat{b}_1 - b_1)\frac{\partial \alpha_1}{\partial x_1} g_1 x_2 \\ &\quad - (\hat{\theta} - \theta)^T F_{2s} + (b_2 \hat{\varrho}_2 - 1)(g_2 x_{r3} + \frac{g_2}{\hat{\varrho}_2}\alpha_2) \quad (3.8)\end{aligned}$$

where $z_3 = x_3 - \hat{\varrho}_2 x_{3r} - \alpha_2$, α_2 is the virtual control to be defined later, $F_{2s} = F_2 - \frac{\partial \alpha_1}{\partial x_1} F_1$ and $f_{2s} = f_2 - \frac{\partial \alpha_1}{\partial x_1} f_1 + g_2 x_{r3} - \hat{b}_1 \frac{\partial \alpha_1}{\partial x_1} g_1 x_2 - \frac{\partial \alpha_1}{\partial \hat{\theta}} \tau_2 - \hat{\varrho}_1 f_{r2} - \dot{\hat{\varrho}}_1 (x_{r2} + \frac{\partial \alpha_1}{\partial \hat{\varrho}_1}) - \sum_{k=1}^{n} \frac{\partial \alpha_1}{\partial x_{rk}} f_{rk} - \frac{\partial \alpha_1}{\partial t}$.

Using α_2 as a control to stabilize the (z_1, z_2)-subsystem, we choose the following Lyapunov function candidate

$$V_2 = V_1 + \frac{1}{2} z_2^2 + \frac{1}{2\gamma}(\hat{b}_2 - b_2)^2 + \frac{|b_2|}{2\gamma}(\hat{\varrho}_2 - \varrho_2)^2 \quad (3.9)$$

The derivative of V_2 is

$$\begin{aligned}\dot{V}_2 &= -c_1 z_1^2 + \hat{b}_2 g_2 z_2 z_3 + z_2(\hat{b}_1 g_1 z_1 + \frac{g_2}{\hat{\varrho}_2}\alpha_2 + \hat{\theta}^T F_{2s} + f_{2s}) \\ &\quad + z_2 \frac{\partial \alpha_1}{\partial \hat{\theta}}(\tau_2 - \dot{\hat{\theta}}) + (\theta - \hat{\theta})^T \Gamma^{-1}(\dot{\hat{\theta}}_1 - \tau_1 - \Gamma F_{2s} z_2) \\ &\quad + (b_2 \hat{\varrho}_2 - 1) sgn(b_2) \gamma^{-1}(\dot{\hat{\varrho}}_2 + sgn(b_2)\gamma z_2 (g_2 x_{r3} \\ &\quad + \frac{g_2}{\hat{\varrho}_2}\alpha_2)) + (\hat{b}_1 - b_1)\gamma^{-1}(\dot{\hat{b}}_1 - \pi_1^{b_1} + \gamma \frac{\partial \alpha_1}{\partial x_1} g_1 x_2 z_2) \\ &\quad + (\hat{b}_2 - b_2)\gamma^{-1}(\dot{\hat{b}}_2 - \gamma g_2 z_2 z_3) \quad (3.10)\end{aligned}$$

Define tuning functions τ_2, $\pi_2^{b_1}$ and $\pi_2^{b_2}$ for $\hat{\theta}$, $\hat{b}_1$ and $\hat{b}_2$ respectively as $\tau_2 = \tau_1 + \Gamma F_{2s} z_2$, $\pi_2^{b_1} = \pi_1^{b_1} - \gamma \frac{\partial \alpha_1}{\partial x_1} g_1 x_2 z_2$ and $\pi_2^{b_2} = \gamma g_2 z_2 z_3$. To eliminate the $(b_2 \hat{\varrho}_2 - 1)$-term from equation (3.10), we choose the parameter update law for $\hat{\varrho}_2$ as $\dot{\hat{\varrho}}_2 = -sgn(b_2)\gamma z_2 (\frac{g_2}{\hat{\varrho}_2}\alpha_2 + g_2 x_{r3})$.

To make the third term in equation (3.10) be equal to $-c_2 z_2^2$, we choose

$$\alpha_2 = \frac{\hat{\varrho}_2}{g_2}(-c_2 z_2 - \hat{b}_1 g_1 z_1 - \hat{\theta}^T F_{2s} - f_{2s}) \quad (3.11)$$

which yields

$$\begin{aligned}\dot{V}_2 &= -c_1 z_1^2 - c_2 z_2^2 + \hat{b}_2 g_2 z_2 z_3 + z_2 \frac{\partial \alpha_1}{\partial \hat{\theta}}(\tau_2 - \dot{\hat{\theta}}) \\ &\quad + (\hat{\theta} - \theta)\Gamma^{-1}(\dot{\hat{\theta}} - \tau_2) + (b_1 - \hat{b}_1)\gamma^{-1}(\dot{\hat{b}}_1 - \pi_2^{b_1}) \\ &\quad + (b_2 - \hat{b}_2)\gamma^{-1}(\dot{\hat{b}}_2 - \pi_1^{b_2}) \quad (3.12)\end{aligned}$$

Step 3. The derivative of z_3 is expressed as

$$\begin{aligned}\dot{z}_3 &= \hat{b}_3 g_3 z_4 + \frac{g_3}{\hat{\varrho}_3}\alpha_3 + \hat{\theta}^T F_{3s} + f_{3s} + (\hat{b}_1 - b_1)\frac{\partial \alpha_2}{\partial x_1} g_1 x_2 \\ &\quad + (\hat{b}_2 - b_2)\frac{\partial \alpha_2}{\partial x_2} g_2 x_3 - (\hat{b}_3 - b_3) g_3 z_4 - (\hat{\theta} - \theta)^T F_{3s} \\ &\quad + \frac{\partial \alpha_2}{\partial \hat{\theta}}(\tau_3 - \dot{\hat{\theta}}) + \frac{\partial \alpha_2}{\partial \hat{b}_1}(\pi_3^{b_1} - \dot{\hat{b}}_1) \\ &\quad + (b_3 \hat{\varrho}_3 - 1)(g_3 x_{r4} + \frac{g_3}{\hat{\varrho}_3}\alpha_3) \quad (3.13)\end{aligned}$$

where $z_4 = x_4 - \hat{\varrho}_3 x_{r4} - \alpha_3$, α_3 is the virtual control to be defined later, $F_{3s} = F_3 - \frac{\partial \alpha_2}{\partial x_1} F_1 - \frac{\partial \alpha_2}{\partial x_2} F_2$ and $f_{3s} = f_3 - \frac{\partial \alpha_2}{\partial x_1} f_1 - \frac{\partial \alpha_2}{\partial x_2} f_2 + g_3 x_{r4} - \hat{b}_1 \frac{\partial \alpha_2}{\partial x_1} g_1 x_2 - \hat{b}_2 \frac{\partial \alpha_2}{\partial x_2} g_2 x_3 - \frac{\partial \alpha_2}{\partial \hat{\theta}} \tau_3 - \frac{\partial \alpha_2}{\partial \hat{b}_1} \pi_3^{b_1} - \sum_{k=1}^{n} \frac{\partial \alpha_2}{\partial x_{rk}} f_{rk} - \frac{\partial \alpha_2}{\partial \hat{\varrho}_1} \dot{\hat{\varrho}}_1 - \frac{\partial \alpha_2}{\partial t} - \hat{\varrho}_2 f_{r3} - \dot{\hat{\varrho}}_2 (x_{r3} + \frac{\partial \alpha_2}{\partial \hat{\varrho}_2})$.

Using α_3 as a control to stabilize the (z_1, z_2, z_3)-subsystem, we choose the following Lyapunov function candidate

$$V_3 = V_2 + \frac{1}{2} z_3^2 + \frac{1}{2\gamma}(\hat{b}_3 - b_3)^2 + \frac{|b_3|}{2\gamma}(\hat{\varrho}_3 - \varrho_3)^2 \quad (3.14)$$

The derivative of V_3 is

$$\begin{aligned}\dot{V}_3 &= -c_1 z_1^2 - c_2 z_2^2 + \hat{b}_3 g_3 z_3 z_4 + z_2 \frac{\partial \alpha_1}{\partial \hat{\theta}}(\tau_2 - \dot{\hat{\theta}}) + z_3(\hat{b}_2 g_2 z_2 \\ &\quad + \frac{g_3}{\hat{\varrho}_3}\alpha_3 + \hat{\theta}^T F_{3s} + f_{3s}) + z_3 \frac{\partial \alpha_2}{\partial \hat{\theta}}(\tau_3 - \dot{\hat{\theta}}) + z_3 \frac{\partial \alpha_2}{\partial \hat{b}_1}(\pi_3^{b_1} \\ &\quad - \dot{\hat{b}}_1) + (\theta - \hat{\theta})^T \Gamma^{-1}(\dot{\hat{\theta}} - \tau_2 - \Gamma F_{3s} z_3) + (\hat{b}_3 - b_3)\gamma^{-1}(\dot{\hat{b}}_3 \\ &\quad - \gamma g_3 z_3 z_4) + (\hat{b}_2 - b_2)\gamma^{-1}(\dot{\hat{b}}_2 - \pi_2^{b_2} + \gamma z_3 \frac{\partial \alpha_2}{\partial x_2} g_2 x_3) \\ &\quad + (\hat{b}_1 - b_1)\gamma^{-1}(\dot{\hat{b}}_1 - \pi_2^{b_1} + \gamma z_3 \frac{\partial \alpha_2}{\partial x_1} g_1 x_2) \\ &\quad + (b_3 \hat{\varrho}_3 - 1) sgn(b_3)\gamma^{-1}(\dot{\hat{\varrho}}_3 + sgn(b_3)\gamma z_3 (g_3 x_{r4} + \frac{g_3}{\hat{\varrho}_3}\alpha_3)) \quad (3.15)\end{aligned}$$

Define tuning functions τ_3, $\pi_3^{b_1}$, $\pi_3^{b_2}$ and $\pi_3^{b_3}$ for $\hat{\theta}$, $\hat{b}_1$, $\hat{b}_2$ and $\hat{b}_3$ respectively as $\tau_3 = \tau_2 + \Gamma F_{3s} z_3$, $\pi_3^{b_1} = \pi_2^{b_1} - \gamma z_3 \frac{\partial \alpha_2}{\partial x_1} g_1 x_2$, $\pi_3^{b_2} = \pi_2^{b_2} - \gamma z_3 \frac{\partial \alpha_2}{\partial x_2} g_2 x_3$ and $\pi_3^{b_3} = \gamma g_3 z_3 z_4$. To eliminate the $(b_3 \hat{\varrho}_3 - 1)$-term from equation (3.15), we choose the parameter update law for $\hat{\varrho}_3$ as $\dot{\hat{\varrho}}_3 = -sgn(b_3)\gamma z_3 (\frac{g_3}{\hat{\varrho}_3}\alpha_3 + g_3 x_{r4})$.

Noting that $\tau_2 - \dot{\hat{\theta}} = \tau_3 - \dot{\hat{\theta}} + \tau_2 - \tau_3 = \tau_3 - \dot{\hat{\theta}} - \Gamma F_{3s} z_3$, equation (3.15) can be written as

$$\begin{aligned}\dot{V}_3 &= -c_1 z_1^2 - c_2 z_2^2 + \hat{b}_3 g_3 z_3 z_4 + (z_2 \frac{\partial \alpha_1}{\partial \hat{\theta}} + z_3 \frac{\partial \alpha_2}{\partial \hat{\theta}})(\tau_3 - \dot{\hat{\theta}}) \\ &\quad + z_3(\hat{b}_2 g_2 z_2 + \frac{g_3}{\hat{\varrho}_3}\alpha_3 + \hat{\theta}^T F_{3s} + f_{3s} - z_2 \frac{\partial \alpha_1}{\partial \hat{\theta}}\Gamma F_{3s}) \\ &\quad + (\theta - \hat{\theta})^T \Gamma^{-1}(\dot{\hat{\theta}} - \tau_3) + (\hat{b}_1 - b_1)\gamma^{-1}(\dot{\hat{b}}_1 - \pi_3^{b_1}) \\ &\quad + (\hat{b}_2 - b_2)\gamma^{-1}(\dot{\hat{b}}_2 - \pi_3^{b_2}) + (\hat{b}_3 - b_3)\gamma^{-1}(\dot{\hat{b}}_3 - \pi_3^{b_3}) \\ &\quad + z_3 \frac{\partial \alpha_2}{\partial \hat{b}_1}(\pi_3^{b_1} - \dot{\hat{b}}_1) \quad (3.16)\end{aligned}$$

To make the bracketed term multiplying z_3 in equation (3.16) be equal to $-c_3 z_3^2$, we choose

$$\alpha_3 = \frac{\hat{\varrho}_3}{g_3}(-c_3 z_3 - \hat{b}_2 g_2 z_2 - \hat{\theta}^T F_{3s} - f_{3s} + z_2 \frac{\partial \alpha_1}{\partial \hat{\theta}}\Gamma F_{3s}) \quad (3.17)$$

which yields

$$\begin{aligned}\dot{V}_3 &= -c_1 z_1^2 - c_2 z_2^2 - c_3 z_3^2 + \hat{b}_3 g_3 z_3 z_4 + (z_2 \frac{\partial \alpha_1}{\partial \hat{\theta}} + z_3 \frac{\partial \alpha_2}{\partial \hat{\theta}}) \\ &\quad (\tau_3 - \dot{\hat{\theta}}) + z_3 \frac{\partial \alpha_2}{\partial \hat{b}_1}(\pi_3^{b_1} - \dot{\hat{b}}_1) + (\theta - \hat{\theta})^T \Gamma^{-1}(\dot{\hat{\theta}} - \tau_3) \\ &\quad + (\hat{b}_1 - b_1)\gamma^{-1}(\dot{\hat{b}}_1 - \pi_3^{b_1}) + (\hat{b}_2 - b_2)\gamma^{-1}(\dot{\hat{b}}_2 - \pi_3^{b_2}) \\ &\quad + (\hat{b}_3 - b_3)\gamma^{-1}(\dot{\hat{b}}_3 - \pi_3^{b_3}) \quad (3.18)\end{aligned}$$

Step i. The derivative of z_i is expressed as

$$\dot{z}_i = \hat{b}_i g_i z_{i+1} + \frac{g_i}{\hat{\varrho}_i}\alpha_i + \hat{\theta}^T F_{is} + f_{is} - (\hat{\theta}-\theta)^T F_{is} + \frac{\partial \alpha_{i-1}}{\partial \hat{\theta}}(\tau_i - \dot{\hat{\theta}}) + \sum_{k=1}^{i-2}\frac{\partial \alpha_{i-1}}{\partial \hat{b}_k}(\pi_i^{b_k} - \dot{\hat{b}}_k) + \sum_{k=1}^{i-1}(\hat{b}_k - b_k)\frac{\partial \alpha_{i-1}}{\partial x_k} g_k x_{k+1} - (\hat{b}_i - b_i) g_i z_{i+1} + (b_i \hat{\varrho}_i - 1)(g_i x_{r(i+1)} + \frac{g_i}{\hat{\varrho}_i}\alpha_i) \quad (3.19)$$

where $z_{i+1} = x_{i+1} - \hat{\varrho}_i x_{r(i+1)} - \alpha_i$, α_i is a fictitious control to be defined later, and

$$\begin{aligned} F_{is} &= F_i - \textstyle\sum_{k=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_k} F_k \\ f_{is} &= f_i + g_i x_{r(i+1)} - \textstyle\sum_{k=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_k} f_k - \sum_{k=1}^{i-1} \hat{b}_k \frac{\partial \alpha_{i-1}}{\partial x_k} g_k x_{k+1} \\ &\quad - \frac{\partial \alpha_{i-1}}{\partial \hat{\theta}}\tau_i - \textstyle\sum_{k=1}^{i-2} \frac{\partial \alpha_{i-1}}{\partial \hat{b}_k}\pi_i^{b_k} - \sum_{k=1}^{n} \frac{\partial \alpha_{i-1}}{\partial x_{rk}} f_{rk} \\ &\quad - \hat{\varrho}_{i-1} f_{ri} - \textstyle\sum_{k=1}^{i-2} \frac{\partial \alpha_{i-1}}{\partial \hat{\varrho}_k}\dot{\hat{\varrho}}_k - \dot{\hat{\varrho}}_{i-1}(x_{ri} + \frac{\partial \alpha_{i-1}}{\partial \hat{\varrho}_{i-1}}) - \frac{\partial \alpha_{i-1}}{\partial t} \end{aligned}$$

Using α_i as a control to stabilize the $(z_1, \cdots, z_i)$-subsystem, we choose the following Lyapunov function candidate

$$V_i = V_{i-1} + \frac{1}{2}z_i^2 + \frac{1}{2\gamma}(\hat{b}_i - b_i)^2 + \frac{|b_i|}{2\gamma}(\hat{\varrho}_i - \varrho_i)^2 \quad (3.20)$$

The derivative of V_i is

$$\begin{aligned} \dot{V}_i =& -\sum_{k=1}^{i-1} c_k z_k^2 + \hat{b}_i g_i z_i z_{i+1} + z_i(\hat{b}_{i-1} g_{i-1} z_1 + \frac{g_i}{\hat{\varrho}_i}\alpha_i + \hat{\theta}^T F_{is} \\ &+ f_{is}) + \sum_{k=1}^{i-2}\left(z_{k+1}\frac{\partial \alpha_k}{\partial \hat{\theta}}\right)(\tau_{i-1} - \dot{\hat{\theta}}) + \sum_{j=1}^{i-3}\sum_{k=1}^{i-j-2} z_{k+2} \\ &\frac{\partial \alpha_{k+1}}{\partial \hat{b}_j}(\pi_{i-1}^{b_j} - \dot{\hat{b}}_j) + (\theta - \hat{\theta})^T \Gamma^{-1}(\dot{\hat{\theta}} - \tau_{i-1} - \Gamma F_{is} z_i) \\ &+ \sum_{k=1}^{i-1}(\hat{b}_k - b_k)\gamma^{-1}(\dot{\hat{b}}_k - \pi_{i-1}^{b_k} + \gamma z_i \frac{\partial \alpha_{i-1}}{\partial x_k} g_k x_{k+1}) \\ &+ (\hat{b}_i - b_i)\gamma^{-1}(\dot{\hat{b}}_i - \gamma g_i z_i z_{i+1}) + z_i \frac{\partial \alpha_{i-1}}{\partial \hat{\theta}}(\tau_i - \dot{\hat{\theta}}) \\ &+ z_i \sum_{k=1}^{i-2}\frac{\partial \alpha_{i-1}}{\partial \hat{b}_k}(\pi_i^{b_k} - \dot{\hat{b}}_k) + (b_i \hat{\varrho}_i - 1) sgn(b_i)\gamma^{-1} \\ &(\dot{\hat{\varrho}}_i + sgn(b_i)\gamma z_i (g_i x_{r(i+1)} + \frac{g_i}{\hat{\varrho}_i}\alpha_i)) \end{aligned} \quad (3.21)$$

Define tuning functions τ_i, $\pi_i^{b_1}, \cdots, \pi_i^{b_i}$ for $\hat{\theta}$, $\hat{b}_1, \cdots, \hat{b}_i$ respectively as

$$\begin{aligned} \tau_i &= \tau_{i-1} + \Gamma F_{is} z_i \\ \pi_i^{b_1} &= \pi_{i-1}^{b_1} - \gamma z_i \frac{\partial \alpha_{i-1}}{\partial x_1} g_1 x_2 \\ &\vdots \\ \pi_i^{b_{i-1}} &= \pi_{i-1}^{b_{i-1}} - \gamma z_i \frac{\partial \alpha_{i-1}}{\partial x_{i-1}} g_{i-1} x_i \\ \pi_i^{b_i} &= \gamma g_i z_i z_{i+1} \end{aligned}$$

To eliminate the $(b_i\hat{\varrho}_i - 1)$-term from equation (3.21), we choose the parameter update law for $\hat{\varrho}_i$ as $\dot{\hat{\varrho}}_i = -sgn(b_i)\gamma z_i(\frac{g_i}{\hat{\varrho}_i}\alpha_i + g_i x_{r(i+1)})$

Noting that $\tau_{i-1} - \dot{\hat{\theta}} = \tau_i - \dot{\hat{\theta}} + \tau_{i-1} - \tau_i = \tau_i - \dot{\hat{\theta}} - \Gamma F_{is} z_i$ and $\pi_{i-1}^{b_j} - \dot{\hat{b}}_j = \pi_i^{b_j} - \dot{\hat{b}}_j + \gamma z_i \frac{\partial \alpha_{i-1}}{\partial x_j} g_j x_{j+1}$, we rewrite $\dot{V}_i$ as

$$\begin{aligned} \dot{V}_i =& -\sum_{k=1}^{i-1} c_k z_k^2 + \hat{b}_i g_i z_i z_{i+1} + \sum_{k=1}^{i-1}(z_{k+1}\frac{\partial \alpha_k}{\partial \hat{\theta}})(\tau_i - \dot{\hat{\theta}}) \\ &+ z_i(\hat{b}_{i-1} g_{i-1} z_i + \frac{g_i}{\hat{\varrho}_i}\alpha_i + \hat{\theta}^T F_{is} + f_{is} - \sum_{k=1}^{i-2} z_{k+1} \\ &\frac{\partial \alpha_k}{\partial \hat{\theta}}\Gamma F_{is} + \sum_{j=1}^{i-3}\sum_{k=1}^{i-j-2} z_{k+2}\frac{\partial \alpha_{k+1}}{\partial \hat{b}_j}\gamma \frac{\partial \alpha_{i-1}}{\partial x_j} g_j x_{j+1}) \\ &+ (\theta - \hat{\theta})^T \Gamma^{-1}(\dot{\hat{\theta}} - \tau_i) + \sum_{j=1}^{i}(\hat{b}_j - b_j)\gamma^{-1}(\dot{\hat{b}}_j - \pi_i^{b_j}) \\ &+ \sum_{j=1}^{i-2}\sum_{k=1}^{i-j-1}\left(z_{k+2}\frac{\partial \alpha_{k+1}}{\partial \hat{b}_j}\right)(\pi_{i-1}^{b_j} - \dot{\hat{b}}_j) \end{aligned} \quad (3.22)$$

To make the bracketed term multiplying z_i in equation (3.22) be equal to $-c_i z_i^2$, we choose

$$\begin{aligned} \alpha_i =& \frac{\hat{\varrho}_i}{g_i}(-c_i z_i - \hat{b}_{i-1} g_{i-1} z_{i-1} + \sum_{k=1}^{i-2} z_{k+1}\frac{\partial \alpha_k}{\partial \hat{\theta}}\Gamma F_{is} \\ &- \sum_{j=1}^{i-3}\sum_{k=1}^{i-j-2} z_{k+2}\frac{\partial \alpha_{k+1}}{\partial \hat{b}_j}\gamma \frac{\partial \alpha_{i-1}}{\partial x_j} g_j x_{j+1} - \hat{\theta}^T F_{is} - f_{is}) \end{aligned} \quad (3.23)$$

which yields

$$\begin{aligned} \dot{V}_i =& -\sum_{k=1}^{i-1} c_k z_k^2 + \hat{b}_i g_i z_i z_{i+1} + \sum_{k=1}^{i-1}(z_{k+1}\frac{\partial \alpha_k}{\partial \hat{\theta}})(\tau_i - \dot{\hat{\theta}}) \\ &+ (\theta - \hat{\theta})^T \Gamma^{-1}(\dot{\hat{\theta}} - \tau_i) + \sum_{j=1}^{i}(\hat{b}_j - b_j)\gamma^{-1}(\dot{\hat{b}}_j - \pi_i^{b_j}) \\ &+ \sum_{j=1}^{i-2}\sum_{k=1}^{i-j-1}\left(z_{k+2}\frac{\partial \alpha_{k+1}}{\partial \hat{b}_j}\right)(\pi_{i-1}^{b_j} - \dot{\hat{b}}_j) \end{aligned} \quad (3.24)$$

Step n. Since this is our last step, the derivative of z_n is expressed as

$$\begin{aligned} \dot{z}_n =& \frac{g_n}{\hat{\varrho}_n}u + \hat{\theta}^T F_{ns} + f_{ns} + \frac{\partial \alpha_{n-1}}{\partial \hat{\theta}}(\tau_n - \dot{\hat{\theta}}) + \sum_{k=1}^{n-1}\frac{\partial \alpha_{n-1}}{\partial \hat{b}_k} \\ &(\pi_n^{b_k} - \dot{\hat{b}}_k) - (\hat{\theta} - \theta)^T F_{ns} + \sum_{k=1}^{n-1}(\hat{b}_k - b_k)\frac{\partial \alpha_{n-1}}{\partial x_k} g_k x_{k+1} \\ &+ (b_n \hat{\varrho}_n - 1)(\frac{g_n}{\hat{\varrho}_n}u) \end{aligned} \quad (3.25)$$

where

$$\begin{aligned} F_{ns} &= F_n - \textstyle\sum_{k=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial x_k} F_k \\ f_{ns} &= f_n + g_n x_{rn} - \textstyle\sum_{k=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial x_k} f_k - \sum_{k=1}^{n-1} \hat{b}_k \frac{\partial \alpha_{n-1}}{\partial x_k} g_k x_{k+1} \\ &\quad - \textstyle\sum_{k=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial \hat{b}_k}\pi_n^{b_k} - \sum_{k=1}^{n} \frac{\partial \alpha_{n-1}}{\partial x_{rk}} f_{rk} - \sum_{k=1}^{n-2} \frac{\partial \alpha_{n-1}}{\partial \hat{\varrho}_k}\dot{\hat{\varrho}}_k \\ &\quad - \frac{\partial \alpha_{n-1}}{\partial \hat{\theta}}\tau_n - \hat{\varrho}_{n-1} f_{rn} - \dot{\hat{\varrho}}_{n-1}(x_{rn} + \frac{\partial \alpha_{n-1}}{\partial \hat{\varrho}_{n-1}}) - \frac{\partial \alpha_{n-1}}{\partial t} \end{aligned}$$

Using control u to stabilize the $(z_1, \cdots, z_n)$-system, we choose the following Lyapunov function candidate

$$V_n = V_{n-1} + \frac{1}{2}z_n^2 + \frac{|b_n|}{2\gamma}(\hat{\varrho}_n - \varrho_n)^2 \quad (3.26)$$

The derivative of V_n is

$$\dot{V}_n = -\sum_{k=1}^{n-1} c_k z_k^2 + \sum_{k=1}^{n-1} z_{k+1}\frac{\partial \alpha_k}{\partial \hat{\theta}}(\tau_{n-1} - \dot{\hat{\theta}}) + z_n(\hat{b}_{n-1}g_{n-1}z_n + \frac{g_n}{\hat{\varrho}_n}u + \hat{\theta}^T F_{ns} + f_{ns}) + z_n\frac{\partial \alpha_{n-1}}{\partial \hat{\theta}}(\tau_n - \dot{\hat{\theta}}) + z_n\sum_{k=1}^{n-1}\frac{\partial \alpha_{n-1}}{\partial \hat{b}_k}(\pi_n^{b_k} - \dot{\hat{b}}_k) + (\theta - \hat{\theta})^T\Gamma^{-1}(\dot{\hat{\theta}} - \tau_{n-1} - \Gamma F_{ns}z_n)$$
$$+\sum_{k=1}^{n-1}(\hat{b}_k - b_k)\gamma^{-1}(\dot{\hat{b}}_k - \pi_{n-1}^{b_k} + \gamma z_n\frac{\partial \alpha_{n-1}}{\partial x_k}g_k x_{k+1})$$
$$+\sum_{j=1}^{n-3}\sum_{k=1}^{n-j-2}\left(z_{k+2}\frac{\partial \alpha_{k+1}}{\partial \hat{b}_j}\right)(\pi_{n-1}^{b_j} - \dot{\hat{b}}_j)$$
$$+ (b_n\hat{\varrho}_n - 1)sgn(b_n)\gamma^{-1}(\dot{\hat{\varrho}}_n + sgn(b_n)\gamma z_n(\frac{g_n}{\hat{\varrho}_n}u)) \quad (3.27)$$

To eliminate the $(b_n\hat{\varrho}_n - 1)$-term from equation (3.27), we choose the parameter update law for $\hat{\varrho}_n$ as $\dot{\hat{\varrho}}_n = -sgn(b_n)\gamma\frac{g_n}{\hat{\varrho}_n}u$.

To eliminate the $(\hat{\theta} - \theta)$, $(\hat{b}_1 - b_1), \cdots, (\hat{b}_n - b_n)$-terms in $\dot{V}_n$ from equation (3.27), we choose the parameter update law for $\hat{\theta}$, $\hat{b}_1, \cdots, \hat{b}_{n-1}$ respectively as

$$\begin{aligned}
\dot{\hat{\theta}} &= \tau_n = \tau_{n-1} + \Gamma F_{ns}z_n \\
\dot{\hat{b}}_1 &= \pi_n^{b_1} = \pi_{n-1}^{b_1} - \gamma z_n\tfrac{\partial \alpha_{n-1}}{\partial x_1}g_1 x_2 \\
&\vdots \\
\dot{\hat{b}}_{n-1} &= \pi_n^{b_{n-1}} = \pi_{n-1}^{b_{n-1}} - \gamma z_n\tfrac{\partial \alpha_{n-1}}{\partial x_{n-1}}g_{n-1}x_n
\end{aligned}$$

Noting that $\tau_{n-1} - \dot{\hat{\theta}} = \tau_{n-1} - \tau_n = -\Gamma F_{ns}z_n$ and $\pi_{n-1}^{b_j} - \dot{\hat{b}}_j = \gamma z_n\frac{\partial \alpha_{n-1}}{\partial x_j}g_j x_{j+1}$, equation (3.27) can be written as

$$\dot{V}_n = z_n\left[\hat{b}_{n-1}g_{n-1}z_n + \frac{g_n}{\hat{\varrho}_n}u + \hat{\theta}^T F_{ns} + f_{ns} - \sum_{k=1}^{n-2} z_{k+1}\frac{\partial \alpha_k}{\partial \hat{\theta}}\Gamma F_{ns} + \sum_{j=1}^{n-3}\sum_{k=1}^{n-j-2}\left(z_{k+2}\frac{\partial \alpha_{k+1}}{\partial \hat{b}_j}\right)\gamma\frac{\partial \alpha_{n-3}}{\partial x_j}g_j x_{j+1}\right] - \sum_{k=1}^{n-1} c_k z_k^2 \quad (3.28)$$

Finally, we choose the control u such that the bracketed term multiplying z_n in equation (3.28) equals $-c_n z_n^2$

$$u = \frac{\hat{\varrho}_n}{g_n}(-c_n z_n - \hat{b}_{n-1}g_{n-1}z_{n-1} + \sum_{k=1}^{n-2} z_{k+1}\frac{\partial \alpha_k}{\partial \hat{\theta}}\Gamma F_{ns} - \sum_{j=1}^{n-3}\sum_{k=1}^{n-j-2}(z_{k+2}\frac{\partial \alpha_{k+1}}{\partial \hat{b}_j})\gamma\frac{\partial \alpha_{n-3}}{\partial x_j}g_j x_{j+1} - \hat{\theta}^T F_{ns} - f_{ns}) \quad (3.29)$$

Thus, we have

$$\dot{V}_n = -\sum_{k=1}^{n} c_k z_k^2 \quad (3.30)$$

Theorem 1 The closed-loop adaptive system consisting of the plant (1.1), the reference model (3.1), the controller (3.29) and the parameter update law (3.28) has a globally uniformly stable equilibrium at $z = [z_1, z_2, \cdots, z_n]^T = 0$. This guarantees the global boundedness of all the signals in the closed-loop system, including the states $x = [x_1, x_2, \cdots, x_n]^T$, the control u and parameter estimates $\hat{\theta}$, $\hat{b}_1, \cdots, \hat{b}_{n-1}$ and $\hat{\varrho}_1, \cdots, \hat{\varrho}_n$, and $\lim_{t\to\infty} z(t) = 0$, i.e., subsequently,

$$\lim_{t\to\infty}[y(t) - y_r(t)] = 0 \quad (3.31)$$

Proof: The $(z_1, \cdots, z_n)$-system corresponds to the closed-loop adaptive system, which consists of the plant (1.1), the reference model (3.1), the controller (3.29) and the parameter update law (3.28). The derivative of the Lyapunov function (3.13) along $(z_1, \cdots, z_n)$-system is (3.30), which proves that equilibrium $z = 0$ is globally uniformly stable.

Combining (3.26) with (3.30), we conclude that $\hat{\theta}$, $\hat{b}_1, \cdots, \hat{b}_{n-1}$ and $\hat{\varrho}_1, \cdots, \hat{\varrho}_n$ are bounded. Since $z_1 = x_1 - x_{r1}$ and x_{r1} is bounded, we see that x_1 is also bounded. The boundedness of x_i, $i = 2, \cdots, n$ follows from the boundedness of α_{i-1} and $\hat{\varrho}_{i-1}$, $i = 2, \cdots, n$ and x_{ri}, and the fact that $x_i = z_i + \hat{\varrho}_{i-1}x_{ri} + \alpha_{i-1}$, $i = 2, \cdots, n$. Using (3.29), we conclude that the control u is also bounded.

From the LaSalle-Yoshizawa theorem [11], it further follows that, all the solutions of the $(z_1, \cdots, z_n)$-system converge to the manifold $z = 0$ as $t \to \infty$. From the definition $z_1 = x_1 - x_{r1}$, we conclude that $|y(t) - y_r(t)| \to 0$ as $t \to \infty$.

4 Example: Tracking Control of Chua's Circuit

We assume that the controlled Chua's circuit is originally ($u = 0$) in the periodic state, period-1 attractor [9], with parameters $C_1 = 0.11364, C_2 = 1, L = 0.0625, R = 1, G_a = -1.143$ and $G_b = -0.714$, i.e., $b_1 = 16$, $b_2 = 1$ and $\theta = [1.0000, 1.0000, 8.7997, 2.5167, -1.8875]^T$. The objective is to force the output $y = x_1(t)$ of the controlled Chua's circuit (2.4) to asymptotically track the chaotic reference signal $y_r = x_{r1}(t)$ generated from another uncontrolled Chua's circuit (2.4) ($u = 0$) in chaotic state, double-scroll attractor [9], with parameters $C_1 = 0.10204, C_2 = 1, L = 0.0625, R = 1, G_a = -1.143$ and $G_b = -0.714$.

The design parameters of controller (3.29) and parameter update law (3.28) are chosen as $c_1 = 10, c_2 = 20, c_3 = 50, \gamma = 0.1$ and $\Gamma = diag\{0.03, 0.1, 0.1, 0.02, 0.07\}$. These gains are chosen by trial and error for better performance. The initial conditions are chosen that $x_1(0) = 2$, $x_2(0) = 0.3$, $x_3(0) = 0.4$, $x_{r1}(0) = 0.2$, $x_{r2}(0) = 0.5$ and $x_{r3}(0) = 0.3$.

Numerical simulation results are shown in Figures 1-3. As shown in Figure 1, the output $y = x_1(t)$ of the controlled Chua's circuit (2.4) asymptotically track the chaotic reference signal $y_r = x_{r1}(t)$. It can be shown that at the same time the states $x_2(t)$ and $x_3(t)$ of the controlled Chua's oscillator (2.4), the parameter estimates $\hat{\theta}$, $\hat{b}_1$, $\hat{\varrho}_1$, $\hat{b}_2$, $\hat{\varrho}_2$ and the control u remain bounded. The boundedness of parameter estimates and control signal u is shown in Figures 2 and 3 respectively.

5 Conclusion

In this paper, firstly we showed that several Chua's circuits of different types, including Chua's oscillator, Chua's circuit with cubic nonlinearity, and the non-autonomous Chua's circuit, can all be transformed into the class of nonlinear system in the so-called non-autonomous "strict-feedback" form. Then, an adaptive backstepping with tuning functions method has been extended to the non-autonomous "strict-feedback" system, and it is used to control the output of the Chua's circuit to asymptotically track arbitrarily given reference signal generated from known, bounded and smooth nonlinear reference model.

References

[1] L. O. Chua, M. Komuro and T. Matsumoto "The double scroll family parts I and II," *IEEE Transactions on Circuits and Systems, Part I*, **33**, 1996, pp. 1073-1118.

[2] L.O. Chua "Global unfolding of Chua's circuit," *IEICE Trans. Fundamentals*, **E76-A**, 1993, pp. 704-734.

[3] L. O. Chua, C. W. Wu, A.-S. Huang, and G.-Q. Zhong "Universal circuit for studying and generating chaos - Part I: routes to chaos ," *IEEE Transactions on Circuits and Systems, Part I*, **40**, 1993, pp. 732-744.

[4] L. O. Chua, C. W. Wu, A.-S. Huang, and G.-Q. Zhong, "Universal circuit for studying and generating chaos - Part II: strange attractors," *IEEE Transactions on Circuits and Systems, Part I*, **40**, 1993, pp. 745-761.

[5] L. O. Chua, T. Yang, G.-Q. Zhong and C. W. Wu, "Adaptive synchronization of Chua's oscillators," *International Journal of Bifurcation and Chaos*, **6**, 1996, pp. 189-201.

[6] G. Chen and X. Dong, *From Chaos To Order*, World Scientific, Singapore, 1998.

[7] X. Dong, G. Chen and L. Chen, "Adaptive control of the uncertain Duffing oscillator," *International Journal of Bifurcation and Chaos*, **7**, 1997, pp. 1651-1658.

[8] I. Kanellakopoulos, P. Kokotovič and A. Morse, "Systematic design of adaptive controllers for feedback linearizable systems", *IEEE Transactions on Automatic Control* , **36**, 1991, pp. 1241-1253.

[9] M. P. Kennedy and C. W. Wu, "ABC: Adventures in Bifurcation and Chaos", software accompanying Chua's circuit: A paradigm for Chaos (R. N. Madan, ed.), Singapore: World Scientific, 1993.

[10] M. Krstič, I. Kanellakopoulos and P. Kokotovič, "Adaptive nonlinear control without overparametrization", *Systems & Control Letters*, **19**, 1992, pp. 177-185.

[11] M. Krstič, I. Kanellakopoulos and P. Kokotovič, *Nonlinear and Adaptive Control Design*, John Wiley, New York, 1995.

[12] M. Lakshmanan and K. Murali, *Chaos in Nonlinear Oscillations*, World Scientific, Singapore, 1996.

[13] K. Murali, M. Lakshmanan and L. O. Chua, "Simplest dissipative nonautonomous chaotic circuit," *IEEE Transactions on Circuits and Systems, Part I*, **41**, 1994, pp. 462-463.

[14] C.W. Wu, T. Yang, and L. O. Chua, "On adaptive synchronization and control of nonlinear dynamical systems," *International Journal of Bifurcation and Chaos*, **6**, 1996, pp. 455-472.

[15] G. Q. Zhong, "Implementation of Chua's circuit with a cubic nonlinearity," *IEEE Transactions on Circuits and Systems, Part I*, **41**, 1994, pp. 934-941.

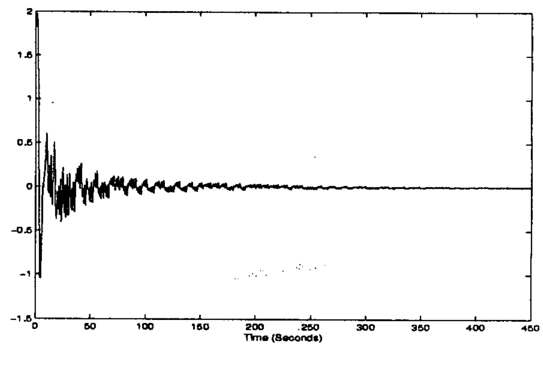

Figure 1: Tracking error $x_1(t) - x_{r1}(t)$

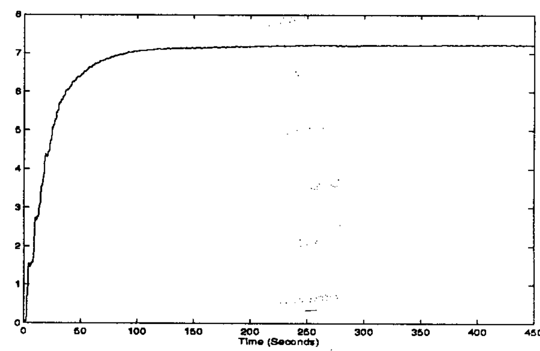

Figure 2: Boundedness of parameter estimates $||\hat{\theta}||$.

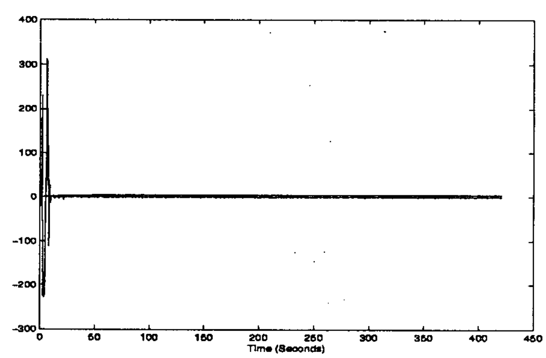

Figure 3: Boundedness of control signal u.