Dynamic Phenomena in Chain Interconnections of Chua's Circuits

Andrzej M. Dąbrowski, *Member, IEEE,* Władysław R. Dąbrowski, and Maciej J. Ogorzałek, *Senior Member, IEEE*

***Abstract*—Dynamic phenomena in long chain interconnections of Chua's chaotic circuits are studied. We show via numerical simulations that a large variety of behavior is typically encountered in such systems. Most interesting observations include transmission delay of chaotic "waves," disorganized propagation, synchronized states, and "chaotic interference." Results of first laboratory tests are also included.**

I. Introduction

There is a considerable interest in the studies of networks of chaotic elements in recent years [1], [2], [4]–[7], [9]–[13]. This is due to two facts: firstly, such networks are important as a model for physical systems with many degrees of freedom; secondly, such networks have possible interesting engineering applications, e.g., in information processing [13]. Due to the high dimensionality of these systems, most of the studies are based on simulation experiments alone. One of the very few analytical results concerning possible synchronization in chaotic arrays is found in [1] and [2].

The variety of phenomena in chaotic oscillator arrays is astonishing. Kaneko [6], [7] introduced a classification of these phenomena. The simplest type of "organized" behavior is the so-called coherent attractors when the variables in the ith and jth cell are identical in time. Another interesting phenomenon is "clustering," when groups of neighboring cells are coherent [5], [7]. Oscillator arrays often show a universal behavior [10] characterized by Feigenbaum constants. The most interesting type of behavior encountered in large interconnections of chaotic oscillators is the spatio-temporal chaos (sometimes referred to as turbulence [6], [7]) when the observed trajectories exhibit chaotic properties both in time and space.

In our paper we study the dynamic behaviors encountered in a chain interconnection of simple electronic oscillators (Chua's circuits). Apart from an abundance of different attractors, we discovered in our simulation studies some phenomena that are typical in transmission lines. These include propagation delay and interference. One should bear in mind, however, that the "wave" described in all cases is chaotic.

II. Model Description

The model we investigate in this paper is a chain interconnection of identical chaotic oscillators (cells). We employ as a standard chaotic cell the canonical Chua's circuit described in [3]. Its circuit diagram and typical characteristic of the nonlinear resistor are shown in Fig. 1(a) and (b), respectively. The state equations of the canonical Chua's circuit have the following form:

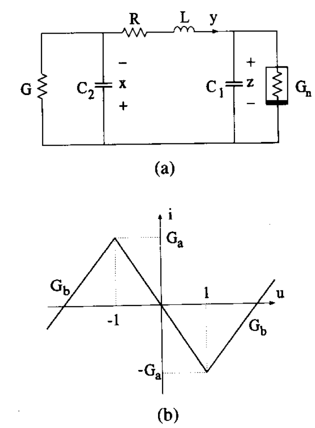

Fig. 1. (a) Circuit diagram of canonical Chua's circuit and (b) the characteristic of the nonlinear resistor.

$$\begin{aligned}\dot{x} &= \frac{1}{C_2}y - \frac{G}{C_2}z\\ \dot{y} &= -\frac{1}{L}x - \frac{R}{L}y - \frac{1}{L}z\\ \dot{z} &= \frac{1}{C_1}y - \frac{1}{C_1}f(z).\end{aligned} \tag{1}$$

In our study, we use the following notation: $a_1 = -(G/C_2)$, $a_2 = 1/C_2$, $a_3 = -(1/L)$, $a_4 = -(R/L)$, $a_5 = 1/C_1$, $a_6 = -(G_1/C_1)$, $a_7 = -(G_1/C_2)$, $a_8 = a_1 + a_7$.

By interconnecting the Chua's circuit unit as shown in Fig. 2, we obtain a chain structure of chaotic oscillators (this is similar to the n-dimensional canonical Chua's circuit [9]).

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The authors are with the Department of Electrical Engineering, University of Mining and Metallurgy, Kraków, Poland.

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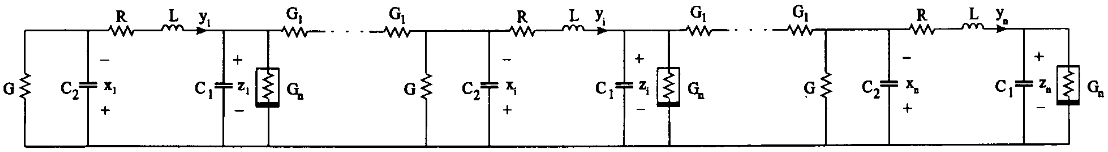

Fig. 2. Circuit diagram of the considered chain interconnection of canonical Chua's circuits.

The dynamics of this chain structure are described by

$$\left.\begin{array}{l}\dot{x}_1 = a_1x_1 + a_2y_1\\ \dot{y}_1 = a_3x_1 + a_4y_1 + a_3z_1\\ \dot{z}_1 = a_5y_1 + a_6z_1 + a_6x_2 - a_5f(z_1)\end{array}\right\}\text{1st section}$$
$$\vdots$$
$$\left.\begin{array}{l}\dot{x}_i = a_7z_{i-1} + a_8x_i + a_2y_i\\ \dot{y}_i = a_3x_i + a_4y_i + a_3z_i\\ \dot{z}_i = a_5y_i + a_6z_i + a_6x_{i+1} - a_5f(z_i)\end{array}\right\}i\text{th section} \quad (2)$$
$$\vdots$$
$$\left.\begin{array}{l}\dot{x}_n = a_7z_{n-1} + a_8x_n + a_2y_n\\ \dot{y}_n = a_3x_n + a_4y_n + a_3z_n\\ \dot{z}_n = a_5y_n + a_6z_n - a_5f(z_n)\end{array}\right\}n\text{th (last) section.}$$

III. Simulation Results

In our simulation experiments we used typical parameter values for the canonical Chua's circuit given in table III in [3] for which an isolated Chua's oscillator generates chaotic oscillations. Depending on the actual coupling resistor values, a variety of dynamic behaviors could be observed. In all simulation experiments we used a chain of ten canonical Chua's circuits.

One common type of behavior is shown in Fig. 3. Successive graphs show time plots of voltages across the capacitors C_2 in successive cells. Introducing a nonzero initial condition on the capacitor C_2 in the first cell only, one can observe how the chaotic "wave" develops in the chain—we observe a delay in the onset of chaotic oscillations in succesive cells and further a chaotic, spatially disorganized behavior. We call this kind of dynamic behavior "chaotic wave propagation."

If the coupling resistance is less then some threshold level one can observe a destabilization of chaotic oscillations and locking into large amplitude pariodic oscillation (Fig. 4). Succesive graphs show the time plots of C_2 voltages in every second cell in the chain. Eventually, all cells oscillate at the same frequency but the phase shift depends on the position of the cell in the chain.

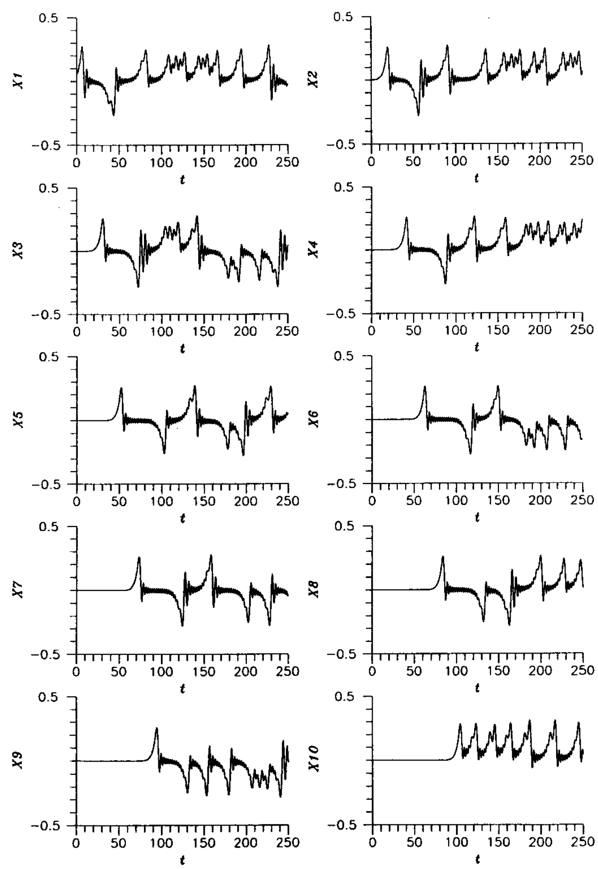

Fig. 3. Results showing propagation delay of a chaotic wave developing in a ten-element chain structure of Chua's circuits.

IV. Results of Laboratory Experiments

We have also carried out several laboratory tests confirming the existence of the phenomena discovered in simulations in real circuits. In all laboratory tests we used the classical Chua's circuits (instead of the canonical ones) for unit cells. Kennedy's [8] op-amp implementations have been used for nonlinear resistors. The cells were interconnected into a ladder in the same manner as in the simulation experiments (coupling resistors between capacitors C_2 and C_1 of the successive cells). Fig. 5 shows the photograph of the laboratory test circuit consisting of 30 Chua's circuits. During our experiments we confirmed the existence of different types of attractors. Some of these attractors are shown in Fig. 6. When changing the coupling resistance towards smaller values, we observed more and more "organized" states in the circuit form chaotic through quasi-periodic to periodic states (Fig. 6(a)–(c)).

In real circuit experiments we observed also "wave" phenomena similar to those encountered in simulation experiments. The visualization of these results is, however, extremely difficult using standard equipment (oscilloscope).

V. Conclusions

Our study reveals extremely interesting phenomena not described so far in the literature. These include "chaotic wave propagation" and destabilization of chaotic oscillations

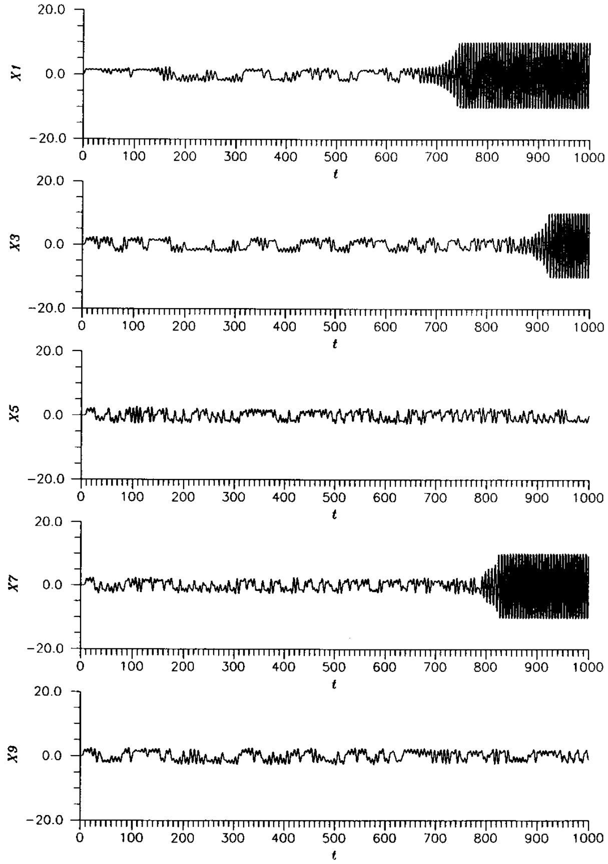

Fig. 4. Simulation results showing a destabilization of the chaotic wave. This phenomenon is interpreted as a "chaotic interference" between the forward and backward (reflected at the end of the chain structure) waves.

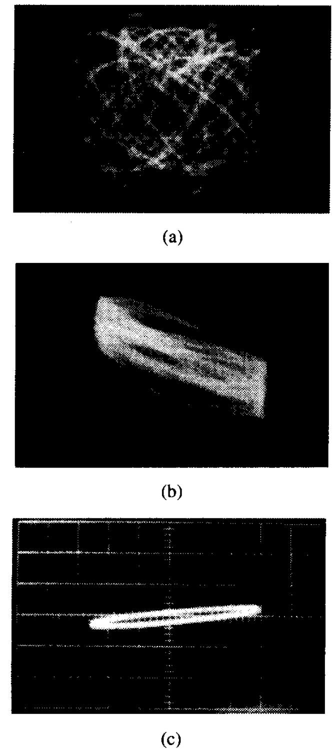

Fig. 6. Phase plots observed in the ten-cell ladder circuit showing: (a) Chaotic, disorganized state observed in the case of the coupling resistance equal to 10 kΩ (horizontal axis: x_1 100 mV/div, vertical axis: x_{10}–100 mV/div); (b) torus-like attractor observed for the copling resistance equal to 8.64 kΩ (horizontal axis: x_1 100 mV/div, vertical axis: x_{10}–200 mV/div); (c) periodic attractor observed for a coupling resistence equal to 7.24 kΩ (horizontal axis: x_1 100 mV/div, vertical axis: x_{10}–200 mV/div). Every single Chua's circuit was built using component values such that a double scroll attractor exists in an uncoupled circuit (compare with [8]).

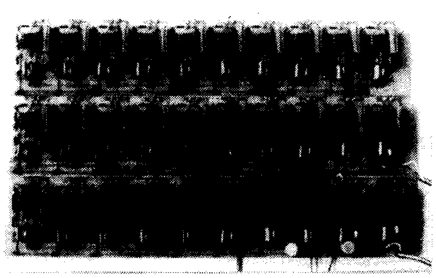

Fig. 5. Photograph of the laboratory test circuit consisting of 30 standard Chua's circuits (three subcircuits of ten cells each).

resulting in locking into large amplitude periodic oscillation. During our studies we also encountered a large number of different attractors. Currently we are introducing a classification of these attractors. Apart from the introductory test with ten Chua's circuits reported above, recently experiments in a 30-cell ladder network were carried out in our laboratory. Our test circuit shown in Fig. 5 enables tests in various interconnections of chaotic unit circuits (ladder, ring, array, etc.). The results of these experiments will be reported in a successive paper.

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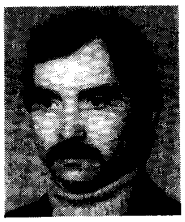
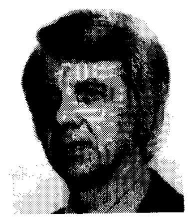


Andrzej M. Dąbrowski (M'92) was born in Tarnow, Poland. He received the M. S. degree in in electronic engineering from the University of Mining and Metallurgy, Cracow. He is currently working toward the Ph.D. degree at the Department of Electrical Engineering of the University of Mining and Metallurgy. His research interests are in nonlinear circuit theory, signal processing , and neural networks.



Władysław R. Dąbrowski received the Dr. Eng. degree in electrical engineering from the University of Mining and Metallurgy, Cracow, Poland, in 1973.

He is currently an Adjunct Professor in the Department of Electrical Engineering of the University of Mining and Metallurgy. His research interests include network synthesis and nonlinear cirucit analysis.

Maciej J. Ogorzałek (M'88–SM'93), for photograph and biography please see page 699 of the October 1993 issue of IEEE TRANSACTIONS ON CIRCUITS AND SYSTEMS—I: FUNDAMENTAL THEORY AND APPLICATIONS.