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Chua's Circuit: Ten Years Later

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SUMMARY More than 200 papers, two special issues (Journal of Circuits, Systems, and Computers, March, June, 1993, and IEEE Trans. on Circuits and Systems, vol. 40, no. 10, October 1993), an International workshop on "Chua's Circuit: chaotic phenomena and applications" at NOLTA'93, and a book (Edited by R. N. Madan, World Scientific, 1993) on Chua's circuit have been published since its inception a decade ago. This review paper attempts to present an overview of these timely publications, almost all within the last 6 months, and to identify four milestones of this very active research area. An important milestone is the recent fabrication of a *monolithic* Chua's circuit. The robustness of this IC chip demonstrates that an array of Chua's circuits can also be fabricated into a monolithic chip, thereby opening the floodgate to many unconventional applications in information technology, synergetics, and even music. The second milestone is the recent global unfolding of Chua's circuit, obtained by adding a linear resistor in series with the inductor to obtain a *canonical* Chua's circuit—now generally referred to as *Chua's oscillator*. This circuit is most significant because it is structurally the *simplest* (it contain only 6 circuit elements) but dynamically the *most complex* among all nonlinear circuits and systems described by a 21-parameter family of continuous odd-symmetric piecewise-linear vector fields. The third milestone is the recent discovery of several important new phenomena in Chua's Circuits, e.g., *stochastic resonance*, *chaos-chaos type intermittency*, *1/f noise spectrum*, etc. These new phenomena could have far-reaching theoretical and practical significance. The fourth milestone is the theoretical and experimental demonstration that Chua's circuit can be easily *controlled* from a chaotic regime to a prescribed periodic or constant orbit, or it can be *synchronized* with 2 or more identical Chua's circuits, operating in an oscillatory, or a chaotic regime. These recent breakthroughs have ushered in a new era where *chaos* is deliberately created and exploited for unconventional applications, e.g., secure communication.
key words: *Chua's circuit, Chua's oscillator*

1. Brief History of Evolution

Prior to 1983, no *autonomous* electronic circuit was known to be chaotic, in spite of numerous attempts by researchers to uncover such examples. In particular, Matsumoto and his students had struggled for years to build an electronic circuit analog of the Lorenz Equation. The history of how Matsumoto's disappointing failure had spurred the author to design a chaotic circuit from first principles was described vividly in [1]. Here, we only outline the chronological events, which began in the fall of 1983, where this chaotic circuit was designed by the author, using a systematic nonlinear circuit synthesis technique. After describing his design to Matsumoto and instructing him on how to choose the circuit parameters for a possible chaotic regime, the author's involvement in this circuit was abruptly interrupted for over a year due to illness.

Having no prior experimental background, Matsumoto uses computer simulation to verify that the author's circuit, which he had named *Chua's Circuit* [2], is indeed chaotic. Meanwhile, Matsumoto and his students had followed the author's suggestion to modify Rosenthal's circuit [3] in order to obtain an active 2-terminal *nonlinear resistor* with the desired piecewise-linear characteristic.* It took two years before his student Tokumasu finally succeeded in 1986 to adapt Rosenthal's circuit to obtain the desired nonlinearity [4].**

Meanwhile, using an op-amp circuit synthesis technique proposed by the author, Zhong and Ayrom [5] had succeeded to build a two-opamp Chua's circuit where chaos was first observed *experimentally* during the winter of 1984. Their experimental confirmation of chaos was published in January 1985, one month after Matsumoto's publication in December 1984 of his *computer observation* of chaos in Chua's Circuit [2].

The most robust and economical method to hook up an experimental Chua's circuit is given in [6], [7]. In this setup, the nonlinear resistor, called *Chua's diode* by Kennedy [7], is realized by a single IC package (containing two op amps) and 6 linear resistors. The entire setup can be easily hooked up in 30 minutes for less than $10. Because of its low cost and robustness, Chua's circuit has become the circuit of choice in applications where an inexpensive and robust source of chaotic signals is required.

For mass applications, it would be desirable to

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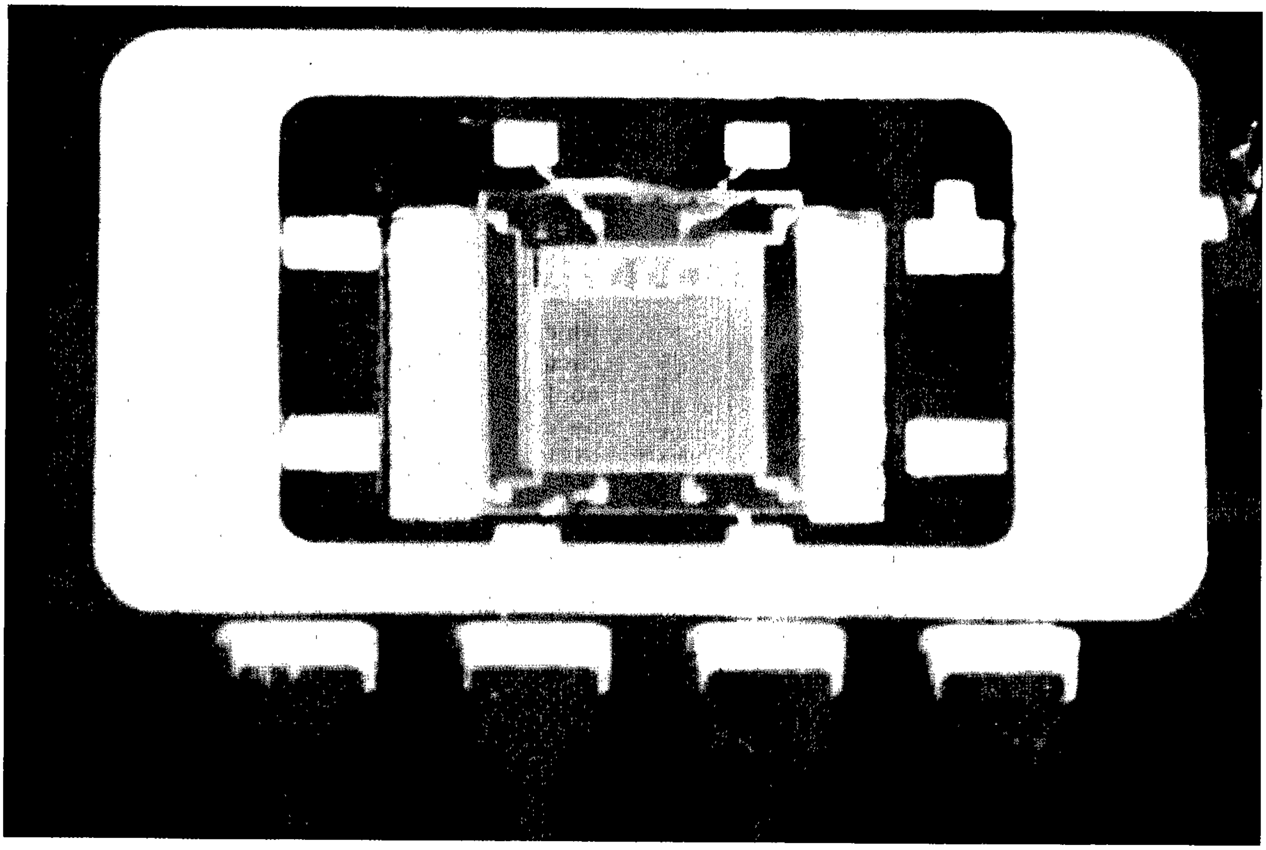
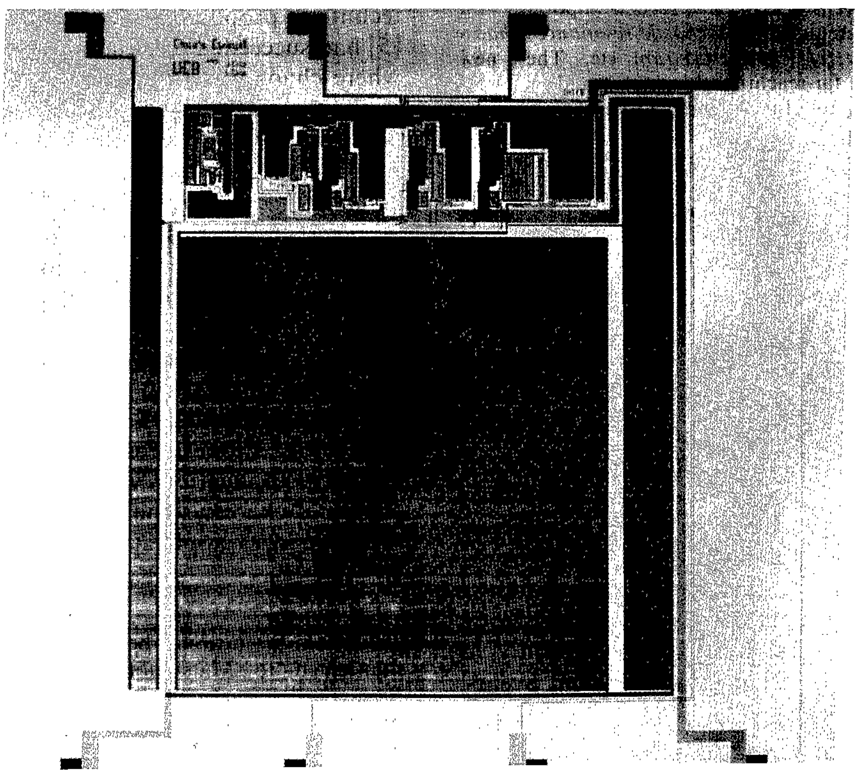
* From his hospital bed in Tokyo, the author had communicated to Matsumoto on the possibility of using Rosenthal's circuit as the basis for designing the desired nonlinearity.

** Although neither acknowledged nor referenced in [4], the core of the resulting 2-transistor circuit is essentially Rosenthal's circuit. This author therefore wishes to take this opportunity to acknowledge Rosenthal's contribution to this 2-transistor Chua's circuit.



(a) 8-pin DIP package.



(b) The photomicrograph of the chip.

Fig. 1 Monolithic CMOS Chua's Circuit described in [10].

have the Chua's diode integrated into an IC chip. This has been achieved using a 2-μm CMOS process, with 39 CMOS transistors occupying a chip area of 0.5 mm^2 [8]. More recently, the *entire* Chua's circuit has been successfully integrated into a *monolithic* chip via two different designs [9], [10]. The Chua's Circuit IC Chip in [10] is housed in a standard 8-pin DIP package, as shown in Fig. 1(a). It is fabricated using the standard 2-micron CMOS technology, whose photomicrograph is shown in Fig. 1(b), and occupies a silicon area of 2.5 mm$\times$2.8 mm. This chip is powered by a single 9-volt battery and dissipates 0.001 Watts of power. This latest evolution represents a milestone in the ever-widening studies and exploitation of Chua's circuits because it demonstrates that an entire array of closely matched Chua's circuits can be fabricated in monolithic form. Indeed, we have estimated that using a 0.5 micron technology, a locally-connected array of 10,000 identical Chua's Circuits can be fabricated in a single chip.

2. Generalizations of Chua's Diode

The original piecewise-linear characteristic of Chua's diode has been generalized by many researchers to assume various different forms. For example, the original piecewise-linear function has been replaced by a *discontinuous* function in [11], a C^∞ "sigmoid function" in [12] and a cubic polynomial $f(x)=c_0x+c_1x^3$ in [13], [14] and [15]. The main motivation for choosing a "smooth" rather than "piecewise-linear" function for Chua's diode is to obtain a smooth state equation so that analytical tools from nonlinear dynamics can be brought to bear. Indeed, it is obvious from the original principles used to design this circuit, as described in [1], that the smoothness of the *v-i* characteristic is quite irrelevant to obtaining a strange attractor. A piecewise-linear characteristic was chosen in [1] only to facilitate a rigorous analysis.

In some applications, the symmetry in the Chua's diode *v-i* characteristic is deliberately broken in order to allow nonlinear wave propagations in a chain of Chua's circuits [16], or in a Chua's circuit terminated by a transmission line [17]. The symmetry can be broken either by shifting the origin, or changing the shape, of the characteristic.

In yet another application, the number of segments of the original piecewise-linear characteristic of Chua's diode has been increased in order to synthesize strange attractors with multiple scrolls [18].

3. Generalizations to Higher Dimensions

In order to investigate chaotic dynamics and new phenomena in higher-dimensional spaces, Chua's circuit has been generalized by replacing the resonant tank circuit by an RLC ladder circuit in [19], by a coaxial cable in [20], and by a lossless transmission line terminated in a short circuit in [17]. The dynamics in the latter case is described by 2 *linear* partial differential equations with a *nonlinear* boundary condition. However, in the limiting case where the capacitance C_1 across Chua's diode tends to zero, the partial differential equations reduces to a 1-dimensional map with a time delay, which makes it analytically tractable. A further generalization of Chua's circuit to a Banach Space has been given in [21].

Another direction of generalization consists of a *chain* [16], [22], or an *array* [23], [24] of Chua's circuits. This generalization leads to a system of nonlinear *ordinary* differential equations which has the same form on the right hand side as that of a system of nonlinear *reaction-diffusion partial* differential equations. Consequently, several currently very active research areas in nonlinear lattice dynamics, including "spiral waves", "Turing patterns", and "spatio-temporal chaos", can all be efficiently studied by an array of Chua's circuits. Moreover, the signals extracted from such an array have many potential applications.

4. Generalizations to Chua's Circuit Family

The theory† developed in [25] for investigating the dynamics of Chua's circuit is actually applicable to any family $\mathscr{C}$ of *continuous, odd-symmetric piecewise-linear vector fields* in $\mathscr{R}^3$, partitioned by two parallel planes U_1 and U_{-1} (of arbitrary orientation) into an inner region D_0 and two outer regions D_1 and D_{-1}, respectively. Any member of $\mathscr{C}$ is described by††

$$\begin{aligned}\dot{\boldsymbol{x}} &= \boldsymbol{A}\boldsymbol{x}+\boldsymbol{b}, & x_1 &\geqq 1\\ &= \boldsymbol{A}_0\boldsymbol{x}, & -1 \leqq x_1 &\leqq 1\\ &= \boldsymbol{A}\boldsymbol{x}-\boldsymbol{b}, & x_1 &\leqq -1\end{aligned} \tag{1}$$

where

$$\boldsymbol{A}=\begin{bmatrix} a_{11} & a_{12} & a_{13}\\ a_{21} & a_{22} & a_{23}\\ a_{31} & a_{32} & a_{33}\end{bmatrix},\quad \boldsymbol{b}=\begin{bmatrix} b_1\\ b_2\\ b_3\end{bmatrix} \tag{2}$$

defines an *affine* vector field in the outer regions D_1 and D_{-1}, and

† As a compromise for departing from Matsumoto's previous tradition of placing his name first in all publications involving Chua's circuit, the authors in this paper have been reordered *alphabetically*. Indeed, most of the results in this paper was developed by Komuro.

†† We would like to take this opportunity to correct the following equations in [26].

$$A_0=\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \quad (3)$$

defines a *linear* vector field in the inner region D_0.

Using the nonlinear state equation synthesis technique developed in [27], [28], any member of this 21-parameter family of vector fields can be synthesized by a circuit using one Chua's diode and linear resistive elements, including controlled sources. The basic structure of this circuit family is summarized in Fig. 14, page 1031, of Wu's paper [29]. In Wu's Fig. 14, the Chua's diode is fixed to be a *passive* element, made of two ideal back-to-back Zener diodes connected in parallel across a linear passive resistor. In this case, the linear resistive 4-port N must contain at least one *negative* resistor or controlled source. By using standard circuit transformation techniques, one can choose N to be passive and transfer the activity requirement into Chua's diode. Indeed, once the 21 parameters defining $\boldsymbol{A}$, $\boldsymbol{A}_0$, and $\boldsymbol{b}$ are given, much simpler circuits equivalent to that of Fig. 14 can be synthesized. In the *simplest* case, only 4 circuit elements are needed; namely, 2 capacitors, 1 inductor, and a Chua's diode with an appropriate *v-i* characteristic. One such circuit,† synthesized, built, and analyzed by Tokunaga [30], has a strange attractor which is spawned by the breakdown of a torus. A detailed analysis of this member of Chua's circuit family shows that the complicated bifurcation phenomena follows the scenarios predicted by the Afraimovich-Shil'nikov torus breakdown theorem [31].

Many other members of Chua's circuit family have been synthesized and built [32]-[35]. Except for the circuit reported in [35], the dynamics of the above cited circuits, including the original Chua's circuit, are not sufficiently general in the sense that certain phenomenon observed from one such member of Chua's circuit family can *not* be observed from another member, regardless of the choice of circuit parameters. From the nonlinear circuit foundation point of view, it is highly desirable to synthesize the *simplest* circuit topology which is capable of reproducing the *qualitative phenomena exhibited by every* member of Chua's circuit family. The circuit structure provided by Fig. 14 of Wu [29] has this property but it is not the *simplest* in the sense that there exist circuits with fewer number of circuit elements that are also endowed with this property. In fact, the circuit reported in [35] is one (among several others) such circuit and is therefore said to be *canonical* because no circuit having fewer number of elements has this property.

Among several equivalent *canonical* Chua's circuits, Dr. Madan, guest editor of two recent special issues on Chua's circuit [36], [37] has chosen the globally unfolded Chua's circuit [26]—obtained by adding a *linear resistor* R_0 in series with the inductor —as the standard bearer of the name *canonical*, and had named it *Chua's Oscillator* to distinguish it from the original Chua's circuit. Dr. Madan chose this augmented circuit as canonical not only because Chua's oscillator reduces to Chua's Circuit upon setting the linear resistor R_0 to zero, but also because all recent publications on canonical Chua's circuit are already based on the Chua's oscillator [24], [26], [38]-[45]. From the theoretical point-of-view, the significance of Chua's oscillator is fully analyzed in [26], [44]. Here, we simply paraphrase the main result succintly as follows:

Significance of Chua's Oscillator

Chua's oscillator is structurely the simplest and dynamically the most complex member of the Chua's circuit family.

It is the simplest circuit because no circuit with fewer number of circuit elements is as general. It is the most complex because no circuit belonging to the Chua's circuit family can exhibit more complex dynamics.

The significance of the Chua's oscillator transcends beyond nonlinear circuit theory. Indeed, there are many publications involving *systems* which are *not* circuits but which are also described by Eqs. (1)-(3), e.g., [46]-[50].

No longer is it necessary for beginners in nonlinear dynamics to study all of these seemingly unrelated papers, along with their diverse notations and jargons. Instead, since *nothing* new can be learned that is not already included as special cases of the dynamics endowed upon Chua's oscillator, future researchers on chaos need only obtain an in-depth understanding of the nonlinear dynamics and bifurcation phenomena of this single circuit. In short, Chua's oscillator has unified the nonlinear dynamics of the entire 21-parameter family of piecewise-linear vector fields into a single system defined by (1)-(3). Moreover, the dynamics of many non-piecewise-linear systems can also be understood and explained by the dynamics of Chua's oscillator, as illustrated in [51]-[53].

5. Some Recent Phenomena Observed from Chua's Circuit

In addition to the various standard routes to chaos (e.g., period doubling, torus breakdown) that are now well known for Chua's circuit, several interesting new phenomena have been discovered recently which we now briefly summarized.

† The content of this paper is due almost exclusively to Tokunaga.

5. 1 Stochastic Resonance

It is well known that when two spiral Chua's attractors (similar in structure to the Rossler attractor) collide in a *crisis* bifurcation, they merge into a single attractor; namely, the double-scroll Chua's attractor. Within a narrow "band" along this bifurcation boundary in the α-β parameter space, a *chaos-chaos type* of intermittency phenomena is observed. If a small sinusoidal signal with the appropriate frequency close to some "natural frequency" of the circuit is applied, a significantly amplified version of this signal is observed. The power gain seems to come at the expense of the energy previously distributed over the entire chaotic power spectrum. Moreover, under certain conditions, the *signal-to-noise ratio* of the amplified output signal is observed to be greater than the signal-to-noise ratio of the input signal—a novel phenomenon which can not be achieved with a *linear* amplifier. This phenomenon is called *stochastic resonance* [54], and is currently a very active research area being pursued by many scientists, specially physicists and biologists.

5. 2 Signal Amplification via Chaos

Apart from the stochastic resonance phenomenon described above, another mechanism for achieving voltage gain (up to 50 dB has been demonstrated experimentally) from Chua's circuit has been discovered recently [55]. The mechanism of this voltage gain is different from that of stochastic resonance because the effect is observed even when Chua's circuit is operating in a spiral Chua's attractor regime far from the bifurcation boundary where stochastic resonance takes place.

5. 3 $1/f$ Noise Phenomenon

In addition to the chaos-chaos type intermittency one observes a $1/f$ power spectrum near the bifurcation boundary between the spiral Chua's attractor regime and the double-scroll Chua's attractor regime [36]. Extensive numerical simulations of Chua's circuit have shown that the associated power spectrum is characterized by a $1/f$ divergence in the low-frequency region. This phenomenon can be used as a $1/f$ noise generator, and can lead to a better understanding of the ubiquitous yet still poorly understood $1/f$ phenomenon.

5. 4 Antimonotonicity Phenomenon

Yorke and his co-workers have predicted that antimonotonicity—i.e., inevitable reversal of period—doubling cascades, is a fundamental phenomenon for a large class of nonlinear systems [56]. Recent experiments on Chua's circuit have provided the first *experimental* confirmation of this phenomenon [57].

5. 5 Period-Adding Phenomenon

Most readers of chaos are familiar with the *period-doubling* phenomenon, where the oscillation period doubles at a geometric rate in accordance with the Feigenbaum number. Another phenomenon rarely observed in autonomous system is that of *period adding*, where the oscillation period increases by consecutive integers, while interspersed between chaotic regimes. Such a phenomenon has been observed in Chua's oscillator [58] and is in fact the basis for designing a bassoon-like musical instrument [45]. The same phenomenon has been observed and rigorously proved by Sharkovsky et al. for the Chua's circuit characterized by a 1-D map with a time delay [17].

5. 6 Autowave Phenomenon

A 1-dimensional chain, or a 2-dimensional array, of identical Chua's circuits with resistive couplings has been shown to support stable *autowave* solutions for a wide range of coupling resistances [16], [22], [23], [59]. Below a certain diffusion coefficient, however, the autowave suddenly ceases to propagate. This propagation failure mechanism is similar to that observed in diseased nerve fibers, such as multiple sclerosis. Such a phenomenon has baffled biologist for many years because no such phenomenon has been observed from simulations of the various associated nonlinear *partial differential* equation models. Indeed, it has only recently been proved mathematically that no such propagation phenomenon can occur in any 1-dimensional active medium modeled by a partial differential equation. Consequently, our observation of this phenomenon from a chain of Chua's circuits implies that a "discrete" chain of Chua's circuits (described by *ordinary* differential equations) is richer in dynamics,

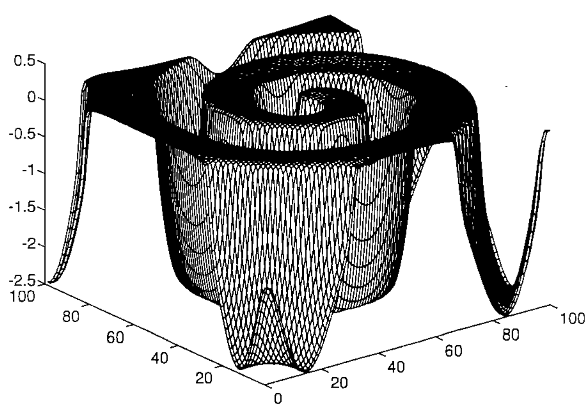


Fig. 2 Example of a *spiral wave* generated from a 100×100 array of identical Chua's oscillator.

than its limiting *continuum* version, and that it can predict certain phenomenon which its limiting partial differential equation model can not [23].

5. 7 Spiral Wave Phenomenon

Spiral waves are special cases of autowaves that are widely observed in active chemical media—e.g., the classic Belousov-Zhabotinski reaction. Such media are modeled by nonlinear reaction—diffusion partial differential equations. It has been shown recently that an array of Chua's oscillators can also support a stable spiral wave solution [24], an example of which is shown in Fig. 2. This is the *first* spiral wave phenomenon that has been observed in Electrical Engineering, and could lead to novel applications.

5. 8 Universality and Self-Similarity

The observation that the α-β plane bifurcation patterns contain *self-similar* features resembling that of "swallow tails" has been pointed out in [25] and [60]. A recent in-depth analysis of this phenomenon using renormalization group analysis [61] has resulted in a definitive characterization of the geometry of this self-similarity phenomenon. In particular, the complex fine structure in the topography of regions of different dynamical behavior near the onset of chaos has been investigated in a 2-parameter 1-D map which describes approximately the dynamics of Chua's circuit. Besides the typical piecewise-smooth Feigenbaum critical lines, the boundary of chaos contains an infinite set of codimension-two critical points, which may be coded by itineraries on a binary tree. In regions nearby critical points having periodic codes, the infinite topography of the parameter plane reveals a property of self-similarity. Moreover, the well-known "Feigenbaum number" for 1-parameter 1-D maps has been generalized to *two universal numbers* for 2-parameter 1-D maps [61].

6. Recent Analytical Investigations of Chua's Circuit

Many deep mathematical analysis, and their generalizations, of Chua's circuit have been published during the past two years. We now summarized some of these *analytical* results:

6. 1 The Double-Hook Attractor

For certain regions in the α-β parameter plane, all eigenvalues associated with the equilibrium point O of Chua's equation are *real* numbers. The strange attractor associated with this chaotic regime is called a *double-hook* attractor in [62]. An in-depth analytical study of this regime has been made by Silva [62]–[64].

6. 2 One-Dimensional Chua's Map

The original Chua's 1-D map presented in [25] has been extensively investigated numerically [65], [66] and analytically [12], [17], [67], [68]. Using the generalized framework developed by Brown [12], Misiurewicz has investigated maps of the real line into itself obtained from the modified Chua's equation [67]. For a large range of parameters, Misiurewicz found the existence of *invariant* intervals as well as invariant sub-intervals on which the associated Chua's circuit is *unimodal* and resembles the well-known *logistic* map. Moreover, this map is found to have a negative Schwarzian derivative, implying the existence of at most one attracting periodic orbit. Moreover, Misiurewicz has proved that there is a set of parameters of positive measure for which chaos occurs.

6. 3 Universality in Cycles of Chaotic Intervals

The order of the bifurcation sequence in piecewise-linear maps is different from that of smooth maps. In the case of the piecewise-linear map associated with the Chua's circuit with time-delay [17], Maistrenko et al. have found that when a period-*n point cycle* loses its stability, a "rigid" period-doubling bifurcation occurs which leads to the emergence of *not* point cycles but *interval cycles* of double period having chaotic trajectories [69]. This is followed by an inverse period-doubling bifurcation, i.e., interval cycles of period $2n$ are merged pairwise, giving birth to a period—n interval cycle. Finally, in the next bifurcation all intervals of interval cycles will merge into the full *interval* cycle $I=[0, 1]$. In this case, there are no subintervals of I which recur periodically under the map of f. Among many elegant mathematical properties concerning interval cycles, Maistrenko et al. has derived two *universal* constants analytically, and in explicit form [69]. This result is most surprising since the well-known Feigenbaum universal constant can be calculated only numerically.

6. 4 Global Stability and Instability of Chua's Oscillator

Recently, Leonov et al. has investigated Chua's oscillators as a feedback control system and derived a frequency-domain criterion for global stability and instability [70]. This analytical study has led to a new version of the generalized Kalman's conjecture.

6. 5 The Double-Horseshoe Theorem

Using a new geometric model of Chua's circuit, Belykh and Chua have presented an analytical study of a new type of strange attractors generated by an odd-symmetric three-dimensional orbit at the origin. This type of attractor is intimately related to the double-scroll Chua's attractor. They have proved rigorously that the chaotic nature of this attractor is different from that of a Lorenz-type attractor, or a quasi-attractor. In particular, this attractor has the geometry of a *double horseshoe*. For certain nonempty intervals of parameters, this strange attractor has no stable orbits. Unlike other known attractors, the double horseshoe attractor contains not only a Cantor set structure of hyperbolic points typical of horseshoe maps, but unstable points (i.e., stable in reverse time) as well. This inplies that the points from the stable manifolds of the hyperbolic points must necessarily attract the unstable points.

6.6 Synchronization, Trigger Wave, and Spatial Chaos

Several criteria for synchronizing two *mutually-coupled* Chua's circuits operating under chaotic regimes are derived in [71]. For a chain of Chua's oscillators, analytical results couched in terms of a moving coordinate system have been derived which guarantee the existence of *heteroclinic* orbits [72]. This analytical study is highly significant because it proves, among other things, the presence of a trigger wave along the chain. The proof of the existence of heteroclinic orbits represents a major breakthrough since it is generally extremely difficult if not impossible to derive such analytical results. In addition to trigger waves, this investigation also proves the existence of *Spatial Chaos* along a finite chain of Chua's oscillators.

6.7 Fine Structure of the Double-scroll Chua's Attractors

Using the theory of *confinors* [73]-[75], Lozi and Ushiki have developed an *analytical* approach, in sharp contrast to numerical integration methods, for examining the fine features of various Chua's attractors. The keystone of the original definition of confinors is that very often, changes in the shape of experimentally observed signals are more significant in characterizing the phase portrait, than any topological change between chaotic attractors. The theory of confinor takes into account the "shape" of the signals, and is capable of modeling both transient and asymptotic regimes. Applying this unique approach to Chua's equation, Lozi and Ushiki have discovered the co-existence of 3 *distinct* double-scroll Chua's attractors in close proximity of each other for *the same value* of parameters [75]. Without a precise knowledge of initial conditions, which only the confinor theory can supply, it would be virtually impossible to pick these 3 attractors apart. This explains why in spite of the rather extensive numerical and experimental works of many researchers on Chua's circuit over the last 10 years, no one has ever observed the simultaneous existence of 3 chaotic attractors.

In addition to this discovery, Lozi and Ushiki have also provided the most precise characterizations of the structure of the double-scroll Chua's attractors via an exact 2-dimensional Poincare map. Moreover, they have discovered some very unusual bifurcation phenomena which are distinct from the usual period-doubling cascades [74]. Since these results are all highly original and robust, they can be used as a guide for characterizing strange attractors of other chaotic systems, thereby demonstrating yet another application of Chua's Circuit as a universal paradigm for chaos.

7. Some Recent Applications of Chaos from Chua's Circuit

During the last 20 years, many researchers have begun to control and exploit chaos from various dynamical systems for novel applications. In this final section, we summarize some recent results in this area which have been applied to Chua's circuit.

7.1 Controlling Chaos in Chua's Circuit

To control chaos in Chua's circuit means to influence its normal chaotic regime and transform it into some "desired" dynamic operation, such as a fixed point, a periodic orbit, or a particular strange attractor. Many different techniques have been developed successfully to control chaos in Chua's circuit [14], [76]-[85]. Some of the control techniques involve varying the circuit parameters, stabilizing some unstable orbits embedded in a strange attractor, absorbing the chaotic dynamics by a controlling circuit, etc. Since different controlling techniques have their advantages and drawbacks, the best approach will depend on specific applications.

7.2 Secure Communication via Chua's circuit

One of the most intriguing applications of chaos is to "hide" the small information—bearing signal within a much larger chaotic signal. Such a signal cannot be recovered unless the receiver is tuned to the exact circuit parameters—the *decoding* key—of the transmitter (Chua's circuit). Several secure communication systems based on Chua's circuit have been proposed [86]-[90]. So far, the approaches proposed in [89], [90] appear to be the most secure and accurate.

7.3 Trajectory Recognition via Array of Chua's Circuits

Recently, Altman uses the center manifold and normal form theory to relate the local behavior of Chua's circuit to some input trajectory to be recognized [91].

This mathematical problem arises in the recognition of hand gestures in the design of artificial intelligence, where the hand position as a function of time is used to drive Chua's circuit to an attracting surface. Since Chua's circuit is known to undergo a series of bifurcations from fixed points, to limit cycles, to a cascade of period-doubling oscillations leading to chaotic oscillations in the vicinity of the center manifold surface, the rapid entrainment of the chaotic system to an external signal having a trajectory near the center manifold surface provides the basic mechanism for trajectory recognition. The recognition of many trajectories can be achieved by using a 2-dimensional array of Chua's circuits. In this case, the variation of responses to the common input trajectory creates a *spatial pattern* which can be used to recognize the input trajectory. The above approach to trajectory recognition is both novel and fascinating.

7.4 Handwritten Character Recognition Using Chua's Oscillator

A neural network architecture and learning algorithm for associative memory storage of analog patterns, continuous sequences, and chaotic attractors via a network of Chua's oscillators has recently been designed by Baird and Hirsch [92]. Their design is used in the application to the problem of real-time handwritten digit recognition. They have demonstrated that several of the attractors from Chua's oscillator have out-performed the previously studied Lorenz attractor system in terms of both accuracy and speed of convergence.

7.5 Applications of Chua's Circuit to Music

Perhaps the most fascinating application to date of Chua's circuit and its generalizations is in music. Recently, Mayer-Kress et al. [45] have discovered that in the α-β-γ parameter space of Chua's Oscillator, there is a manifold which gives rise to novel musical sounds. For example, a point on this manifold gives rise to a consecutive sequence of bassoon-like musical tones. This research project is presently conducted by a multidisciplinary team at the university of Illinois, Urbana, and consists of Professors G. Mayer-Kress and A. Hubler from the Physics department, Robin Bargar, project leader of sonification research and development at the National Center for supercomputing Applications, and Insook Choi, a doctoral candidate in musical arts. Their music-making method may herald an attractive alternative to the time-consuming pre-mixing of the audio frequencies, an essential step for electronic synthesizers of musical sounds. By varying the circuit element values from Chua's Oscillator, frequencies and overtones characteristic of musical instruments can be easily generated without the necessity of separately programming each frequency. Already these researchers have generated unharmonious sound and created music in overtones never heard before because no instrument exists that can make them. In fact by exploiting these unusual musical tones, choi has composed some truly *avant-garde* music via Chua's oscillator for a recent concert at Expo'93 in Seoul and Taejeoun, Korea in October 1993.

Independent of the research group from the university of Illinois, Professor Xavier Rodet from the Institute de Recherche et de coordination Acoustique/Musique (IRCAM) in Paris, and the University of Paris has used a time-delay version of Chua's circuit, not only to generate musical sounds, but also as a unified model of an interesting class of musical instruments, including those (e.g., clarinet) consisting of a massless reed coupled to a passive linear system. The surprisingly rich and novel family of periodic and chaotic musical sounds generated by Rodet has already enriched the sound synthesis repertoire of tools for researchers in computer music [93], [94].

8. Concluding Remarks

Although first conceived only 10 years ago, more than 200 papers, two special issues (Journal of Circuits, Systems, and Computers, vol. 3, no. 1 and no. 2, 1993), an international Workshop on Chua's Circuit: chaotic phenomena and applications at NOLTA'93, and a book (Edited by R. N. Madan, World Scientific, 1088 pages, 1993) have been published on all aspects of bifurcation and chaos of Chua's Circuit and its recent global unfolding, the Chua's oscillator. Yet, our understanding of this simplest among all chaotic circuits is still far from complete. Indeed, what has been published on Chua's circuit represents only the tip of an iceberg [95], [96], specially when viewed from the broader perspective of an array of driven Chua's oscillators, as well as other higher-dimensional generalizations of Chua's circuit. This yet uncharted territory will no doubt be systematically explored and exploited for novel applications in the next decade, and beyond.

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Leon O. Chua received the S.M. degree from the Massachusetts Institute of Technology in 1961 and the Ph.D. degree from the University of Illinois, Urbana, in 1964. He was also awarded a Doctor Honoris Causa from the Ecole Polytechnique Federale-Lausanne, Switzerland, in 1983, an Honorary Doctorate from the University of Tokushima. Japan, in 1984 an Honorary Doctorate (DR.-Ing. E.h.) from the Techniche Universiät Dresden in 1992, and an Honorary Doctor's degree from the Technical University of Budapest in 1994. He is presently a professor of Electrical Engineering and Computer Sciences at the University of California, Berkeley. Professor Chua's research interests are in the areas of general nonlinear network and system theory. He has been a consultant to various electronic industries in the areas of nonlinear network analysis, modeling, and computer-aided design. He is the author Introduction to Nonlinear Network Theory (New York: McGraw-Hill, 1969), and a coauthor of the books Computer-Aided Analysis of Electronic Circuits: Algorithms and Computational Techniques (Englewood Cliffs, NJ: Prentice-Hall, 1975), Linear and Nonlinear Circuits (New York: McGraw-Hill, 1987), and Practical Numerical Algorithms for Chaotic Systems (New York: Springer-Verlag, 1989). He has published many research papers in the area of nonlinear networks and systems. Professor Chua was elected Fellow of the IEEE in 1974. He served as Editor of the IEEE Transactions on Circuits and Systems from 1973 to 1975 and as the President of the IEEE Society on Circuits and Systems in 1976. He is presently the editor of the International Journal of Bifurcation and Chaos, a deputy editor of the International Journal of Circuit Theory and Applications, and the Editor of the World Scientific Book Series on Nonlinear Science. Professor Chua has been awarded six U.S. patents. He is also the recipient of several awards and prizes, including the 1967 IEEE Browder J. Thompson Memorial Prize Award, the 1973 IEEE W. R. G. Baker Prize Award, the 1974 Frederick Emmons Terman Award, the 1976 Miller Research Professorship from the Miller Institute, the 1982 Senior Visiting Fellowship at Cambridge University, England, the 1982/83 Alexander von Humboldt Senior U.S. Scientist Award at the Technical University of Munich, W. Germany, the 1983/84 Visiting U.S. Scientist Award at Waseda University, Tokyo, from the Japan Society for Promotion of Science, the IEEE Contennial Medal in 1985, the 1985 Myril B. Reed Best Paper Prize, and both the 1985 and 1989 IEEE Cuillemin-Cauer Prize. In the fall of 1986, Professor Chua was awarded a Professor Invite International Award at the University of Paris-Sud from the Freneh Minstry of Education.