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A Robust Tracking Control for Chaotic Chua's Circuits via Fuzzy Approach

Yeong-Chan Chang

Abstract—This paper addresses the problem of designing robust tracking controls for nonlinear chaotic Chua's circuits involving plant uncertainties and external disturbances. A hybrid adaptive-robust tracking control scheme which is based upon a combination of the H^∞ tracking theory, variable structure system (VSS) control algorithm, and adaptive fuzzy control is developed such that all the states and signals of the closed-loop system are bounded and an H^∞ tracking control from the tracking error to the external disturbance is guaranteed. In contrast to the previous investigations of controlling Chua's circuits the controller developed here can be extended to handle a broader class of uncertain nonlinear chaotic Chua's circuits.

Index Terms—Chaotic Chua's circuits, H^∞ tracking control, hybrid adaptive-robust design, Riccati-like matrix equation.

I. INTRODUCTION

The control of nonlinear chaotic Chua's circuits is an important topic for numerous practical applications since this circuit exhibits a wide variety of nonlinear dynamic phenomena such as bifurcations and chaos [5], [6], [18]. This chaotic circuit possesses the property of simplicity and universality, and has become a standard prototype for investigation of chaos. A significant research attention has been paid toward studying the control of such circuits in the past few years [2]–[6], [11], [14], [15], [17], [18]. In most of these previous investigations, the plant dynamics is assumed to be available for implementation or external disturbances are neglected. However, in practical applications, plant uncertainties and external disturbances which may affect the tracking performance are inevitable. Therefore, development of an alternative control approach to efficiently treat the robust tracking control of chaotic Chua's circuits involving a large class of uncertainties and variations is highly desirable.

Fuzzy control has recently found extensive applications for a wide variety of industrial systems and many adaptive fuzzy control

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Fig. 1. Diagram of nonlinear chaotic Chua's circuit.

schemes have been developed [3], [7], [12], [13], [16]. Specially, some fuzzy-based control schemes are also developed to treat the control of chaotic circuits [2], [5], [15]. In [2] a simple fuzzy logic based intelligent mechanism was developed for predicting and controlling an uncertain chaotic system to a desired target. Chen and Dong [5] applied fuzzy inference systems to identify and control chaotic systems. Tanaka *et al.* [15] developed a unified approach to controlling chaos via an linear matrix inequality (LMI)-based fuzzy control system design.

This paper addresses the robust tracking control for a large class of nonlinear chaotic Chua's circuits in the presence of plant uncertainties and external disturbances. Nonlinear H^∞ tracking theory, variable structure system (VSS) control algorithm and fuzzy control design are combined together to construct a VSS indirect adaptive fuzzy-based H^∞ tracking controller such that the resulting closed-loop circuit system guarantees a satisfactorily transient and asymptotic performance in the sense that the tracking error can be made as small as possible in terms of L_∞ boundedness and H^∞ tracking performance. In order to implement the developed controller, knowledge of the dynamic model of nonlinear Chua's circuits doesn't require for feedback and moreover only a linear Riccati-like equation must be solved. Consequently, the control scheme developed in this study can be easily implemented from the viewpoint of practical applications.

In Section II, the tracking control problem of nonlinear chaotic Chua's circuits is presented. Section III develops a hybrid adaptive-robust fuzzy-based controller. Section IV presents a simulation example. Finally, the conclusion is given in Section V.

II. PRELIMINARY

A. Problem Statement

The chaotic Chua's circuit, as shown in Fig. 1, is a simple electronic system that consists of one inductor L , two capacitors C_1 , C_2 , one linear resistor R , and one nonlinear resistor g . The dynamic equations of Chua's circuit with control inputs are described by [5], [18]

$$\dot{v}_{c1} = \frac{1}{C_1} \left(\frac{1}{R} (v_{c2} - v_{c1}) - g(v_{c1}) + u_1 \right) + d_1 \quad (1)$$

$$\dot{v}_{c2} = \frac{1}{C_2} \left(\frac{1}{R} (v_{c1} - v_{c2}) + i_L + u_2 \right) + d_2 \quad (2)$$

$$\dot{i}_L = \frac{1}{L} (-v_{c2} + u_3) + d_3 \quad (3)$$

where

i_L	current through the inductor L ;
v_{c1}, v_{c2}	voltages across C_1 and C_2 , respectively;
$g(\cdot)$	current through the nonlinear resistor;
u_1, u_2, u_3	control inputs;
d_1, d_2 and d_3	external disturbances.

Let $x(t) \triangleq [v_{c1}(t), v_{c2}(t), i_L(t)]^T$ be the state variable.

Give a desired reference signal $x_d(t) \triangleq [x_{d1}(t), x_{d2}(t), x_{d3}(t)]^T$ which is assumed to be bounded and continuously differentiable, i.e., there is a compact set Ω_d such that $x_d(t) \in \Omega_d$ for all $t \geq 0$. The objective of this paper is to design an adaptive-robust tracking controller for nonlinear chaotic Chua's circuits (1)–(3) involving plant uncertainties and external disturbances such that the resulting closed-loop system guarantees that all the states and signals are bounded and the tracking error should be as small as possible.

Taking $e(t) \triangleq x(t) - x_d(t) = [v_{c1}(t) - x_{d1}(t), v_{c2}(t) - x_{d2}(t), i_L(t) - x_{d3}(t)]^T$, the tracking error dynamic equation can be expressed as

$$\dot{e} = F(x) + G(x)u + d - \dot{x}_d \quad (4)$$

where $u \triangleq [u_1, u_2, u_3]^T$, $d \triangleq [d_1, d_2, d_3]^T$, and

$$F(x) \triangleq \begin{bmatrix} f_1(x) \\ f_2(x) \\ f_3(x) \end{bmatrix} \triangleq \begin{bmatrix} \frac{1}{C_1} \left(\frac{1}{R} (v_{c2} - v_{c1}) - g(v_{c1}) \right) \\ \frac{1}{C_2} \left(\frac{1}{R} (v_{c1} - v_{c2}) + i_L \right) \\ \frac{1}{L} (-v_{c2}) \end{bmatrix}$$

$$G(x) \triangleq \begin{bmatrix} \frac{1}{C_1} & 0 & 0 \\ 0 & \frac{1}{C_2} & 0 \\ 0 & 0 & \frac{1}{L} \end{bmatrix}.$$

It is clear that if the function $F(x)$ and the matrix $G(x)$ are well-known, by employing the technique of feedback linearization [9] there exists a suitable control law such that the resulting closed-loop system can be shown to achieve a satisfactory tracking performance. However, in practical circuit systems, the inductor, capacitor, and resistor may have uncertain variations around their nominal values due to heating. The parameters and characteristics of electronic elements may vary while the circuit system has operated for a long time, and the circuit system may receive unpredictable interference from the environment where it resides, etc. Therefore, in order to efficiently control the chaotic Chua's circuit it is necessary to consider the effects due to plant uncertainties and external disturbances.

In this study, the values of the inductor L , capacitors C_1 and C_2 , and resistors R and $g(\cdot)$ are assumed to be unknown. Therefore, the nonlinear dynamics $F(x)$ and $G(x)$ are also unknown and cannot be available directly in the robust control design. The philosophy of our tracking design is expected that the fuzzy approximator equipped with an adaptive algorithm is introduced first to learn the uncertain dynamics. Next, two additional robust control algorithms, i.e., VSS algorithm and nonlinear H^∞ control algorithm, are employed to efficiently attenuate the effects on the tracking error due to the fuzzy approximation error and the external disturbance.

Two cases with respect to different knowledge of the input gain matrix will be proposed, sequentially. One is assumed that the values of C_1 , C_2 and L can be split into a nominal part plus an uncertain part and the other is assumed that these three values are nonlinear time-varying and unknown.

B. Description of Fuzzy Systems

The basic configuration of the fuzzy system shown in Fig. 2 is constructed from the fuzzy If-Then rules using some specific

Fig. 2. The basic configuration of a fuzzy logic system.

inference, fuzzification, and defuzzification strategies [13], [16]. The fuzzy system performs a mapping from $U \subset R^N$ to $V \subset R$. The fuzzifier maps a crisp point in U into a fuzzy set in U . The fuzzy rule base consists of a collection of fuzzy If-Then rules such as

$$R^{(l)}: \text{If } x_1 \text{ is } F_1^l, \dots, x_N \text{ is } F_N^l, \\ \text{Then } y \text{ is } G^l \quad (5)$$

in which $x = (x_1, \dots, x_N)^T \in U$ and $y \in V \subset R$ are the input and output of the fuzzy system, respectively, $F_i^l, i = 1, 2, \dots, N$ and G^l are fuzzy sets, and $l = 1, \dots, M$ where M denotes the number of fuzzy If-Then rules. The fuzzy inference engine performs a mapping from fuzzy sets in U to fuzzy sets in V , based upon the fuzzy If-Then rules and the compositional rule of inference. The defuzzifier maps a fuzzy set in V to a crisp point in V .

According to the universal approximation theorem [13], [16] the fuzzy system with center-average defuzzifier, product inference and singleton fuzzifier is of the following form:

$$y(x) = \frac{\sum_{l=1}^M \theta_l \left(\prod_{i=1}^N \mu_{F_i^l}(x_i) \right)}{\sum_{l=1}^M \left(\prod_{i=1}^N \mu_{F_i^l}(x_i) \right)} \quad (6)$$

where $\mu_{F_i^l}(\cdot)$ is the numbership function of the fuzzy set F_i^l and θ_l is the point at which μ_{G^l} achieves its maximum value. Moreover, for any given real continuous function $f(x)$ on a compact subset $U \subset R^N$ and arbitrary $\epsilon > 0$, there exists a fuzzy system $y(x)$ in the form of (6) such that $\max_{x \in U} \|f(x) - y(x)\| < \epsilon$.

III. CONTROLLER DESIGN AND STABILITY ANALYSIS

First, we shall focus on the case that L, C_1 and C_2 can be partitioned as $L = L_0 + \Delta L$, $C_1 = C_{10} + \Delta C_1$ and $C_2 = C_{20} + \Delta C_2$ where L_0, C_{10} and C_{20} denote the known nominal values and $\Delta L, \Delta C_1$ and ΔC_2 denote the unknown nonlinear time-varying perturbations. After some simple manipulations, the input matrix $G(t, x)$ can be expressed as $G(t, x) = G_0 + \Delta G(t, x)$ where $G_0 = \text{diag}[1/C_{10}, 1/C_{20}, 1/L_0]$ and $\Delta G = \text{diag}[-(\Delta C_1/C_{10}C_1), -(\Delta C_2/C_{20}C_2), -(\Delta L/L_0L)]$.

Take the universal approximation system $\hat{F}(x, \Theta)$ with $x \in U_x$ for some compact set $U_x \subset R^3$, to approximate the uncertain term $F(x)$ where Θ contains the tunable approximation parameters. Let $\hat{f}_i(x, \Theta_i), i = 1, 2, 3$ denote the i th component of $\hat{F}(x, \Theta)$ and use to approximate $f_i(x)$. From (6) $\hat{f}_i(x, \Theta_i)$ can be expressed as

$$\hat{f}_i(x, \Theta_i) = \zeta_i^T(x) \Theta_i, \quad i = 1, 2, 3 \quad (7)$$

where $\Theta_i \triangleq [\theta_{i1}, \dots, \theta_{im_i}]^T \in R^{m_i}$ for some $m_i > 0$, is a parameter vector and $\zeta_i(x) \triangleq [\zeta_{i1}(x), \dots, \zeta_{im_i}(x)]^T$ is a regressive vector with the regressor $\zeta_{il}(x)$ defined as

$$\zeta_{il}(x) = \frac{\prod_{j=1}^3 \mu_{F_j^l}(x_j)}{\sum_{l=1}^{m_i} \left(\prod_{j=1}^3 \mu_{F_j^l}(x_j) \right)}, \quad l = 1, \dots, m_i. \quad (8)$$

Here, the linearly parametrized fuzzy model [3], [12], [13], [16] is employed in the approximation procedure. That is, the membership functions $\mu_{F_j^l}(x_j)$ for $1 \leq l \leq m_i$ and $1 \leq j \leq 3$ are specified beforehand and the number m_i is given so that $\hat{f}_i(x, \Theta_i)$ can approximate $f_i(x)$ as best as possible. Consequently, the fuzzy system $\hat{F}(x, \Theta)$ can be expressed as

$$\hat{F}(x, \Theta) = \begin{bmatrix} \zeta_1^T(x) & 0 & 0 \\ 0 & \zeta_2^T(x) & 0 \\ 0 & 0 & \zeta_3^T(x) \end{bmatrix} \begin{bmatrix} \Theta_1 \\ \Theta_2 \\ \Theta_3 \end{bmatrix} \triangleq Y(x)\Theta \quad (9)$$

where $Y(x) \in R^{3 \times m}$ is a basis matrix and $\Theta \triangleq [\Theta_1^T, \Theta_2^T, \Theta_3^T]^T \in R^m$ with $m = m_1 + m_2 + m_3$.

According to the universal approximation theorem, there exists an optimal approximation parameter Θ^* [3], [13], [16] such that $\hat{F}(x, \Theta^*)$ can approximate $F(x)$ as best as possible. As in many previous adaptive fuzzy designs, Θ^* will be learned by using an adaptive algorithm. Suppose the constrained region of Θ is chosen to be a convex hypercube. That is, consider $\Omega_0 \triangleq \{\Theta | b_{ij} \leq \theta_{ij} \leq c_{ij}, 1 \leq j \leq m_i, 1 \leq i \leq 3\}$ and $\Omega_\theta \triangleq \{\Theta | b_{ij} - \delta \leq \theta_{ij} \leq c_{ij} + \delta, 1 \leq j \leq m_i, 1 \leq i \leq 3\}$ where the values of b_{ij} , c_{ij} and $\delta > 0$ can be arbitrarily specified by the designer. Give a smooth function $\Phi \triangleq [\Phi_1^T, \Phi_2^T, \Phi_3^T]^T$ where $\Phi_1 \in R^{m_1 \times 1}$, $\Phi_2 \in R^{m_2 \times 1}$ and $\Phi_3 \in R^{m_3 \times 1}$ should be specified later. Therefore, the smooth projection algorithm with respect to Ω_θ can be expressed as [10]

$$\text{Proj}[\Phi, \Theta] \triangleq \begin{cases} \bar{\phi}_{ij}, & \text{if } (\theta_{ij} > c_{ij} \text{ and } \phi_{ij} > 0) \\ \check{\phi}_{ij}, & \text{if } (\theta_{ij} < b_{ij} \text{ and } \phi_{ij} < 0) \\ \phi_{ij}, & \text{otherwise, } 1 \leq j \leq m_i, 1 \leq i \leq 3 \end{cases} \quad (10)$$

where $\bar{\phi}_{ij} \triangleq (1 + (c_{ij} - \theta_{ij}/\delta))\phi_{ij}$, $\check{\phi}_{ij} \triangleq (1 + (\theta_{ij} - b_{ij}/\delta))\phi_{ij}$ and ϕ_{ij} denotes the j element of Φ_i .

Let $\Delta F(x) = F(x) - \hat{F}(x, \Theta^*)$ be the minimum approximation error and Δf_i be the i element of ΔF . Throughout this section we need the following assumptions.

- A1)** There exists a constant $0 < \kappa < 1$ such that $|\lambda_i(\Delta G G_0^{-1})| \leq \kappa, \forall 1 \leq i \leq 3$.
- A2)** There exists a constant $\epsilon_f > 0$ such that $|\Delta f_i| \leq \epsilon_f, \forall 1 \leq i \leq 3$.
- A3)** There exists a constant $M_d > 0$ such that $\int_0^\infty \|d(t)\|^2 dt \leq M_d$, i.e., $d \in L_2[0, \infty)$. $\square$

The assumption **A1**) is equivalent to $|\Delta C_1/C_1| \leq \kappa, |\Delta C_2/C_2| \leq \kappa$ and $|\Delta L/L| \leq \kappa$, and so it always holds for small parameter perturbations. According to the universal approximation theorem the assumption **A2**) also holds. The assumption **A3**) implies the external disturbance is of finite-energy. This kind of disturbance usually appears in a physically operating circuit.

Theorem 1: Consider the nonlinear chaotic Chua's circuit (1)–(3). Suppose that Assumptions **A1**)–**A3**) are satisfied. If there exists a sym-

metric positive matrix $P = P^T > 0$ satisfying the Riccati-like equation

$$-K^T P - PK + Q + P \left(\frac{1}{\rho^2} I_{3 \times 3} - (1 - \kappa) R^{-1} \right) P = 0 \quad (11)$$

where $\rho > 0$ is a prescribed attenuation level, $Q = Q^T > 0$ is a prescribed weighting matrix, $K \triangleq \text{diag}[k_1, k_2, k_3]$ for some constants $k_i > 0$ is a state feedback control gain, and $R \triangleq \text{diag}[r_1, r_2, r_3]$ for some constants $r_i > 0$ is an H^∞ control gain, then the following VSS adaptive fuzzy-based control law

$$u = G_0^{-1}(-Y(x)\Theta + \dot{x}_d - Ke + u_h + u_s) \quad (12)$$

with

$$u_h = -\frac{1}{2} R^{-1} P e \quad (13)$$

$$u_s = -\frac{\epsilon_f + \kappa M_e}{1 - \kappa} \text{sgn}(Pe) \quad (14)$$

$$\dot{\Theta} = \gamma \text{Proj}[\Phi, \Theta] \quad (15)$$

where $M_e \triangleq \| -Y\Theta + \dot{x}_d - Ke \|$, $\text{sgn}(Pe) \triangleq [\text{sgn}((Pe)_1), \text{sgn}((Pe)_2), \text{sgn}((Pe)_3)]^T$, $\Phi \triangleq Y^T P e$, and γ denotes the adaptive gain, guarantees that i) all the variables of the closed-loop system (1)–(3), (12)–(15) are bounded and ii) an H^∞ tracking performance from $d(t)$ to $e(t)$ is achieved, i.e.,

$$\int_0^T \|e(t)\|_Q^2 dt \leq 2V(0) + \rho^2 \int_0^T \|d(t)\|^2 dt \quad \forall 0 \leq T < \infty \quad (16)$$

where $V(0)$ denotes the initial conditions. $\square$

Proof: Taking into account the minimum approximation error and the control law (12), the error dynamic equation (4) can be rewritten as

$$\begin{aligned} \dot{e} &= \hat{F}(x, \Theta^*) + \Delta F(x) + G_0 u + \Delta G(t, x)u + d - \dot{x}_d \\ &= -Ke - Y(x)\tilde{\Theta} + \Delta F(x) + \Delta G(t, x)u + u_h + u_s + d \end{aligned} \quad (17)$$

where $\tilde{\Theta} = \Theta - \Theta^*$. Choose a Lyapunov function as

$$V = \frac{1}{2} e^T P e + \frac{1}{2\gamma} \tilde{\Theta}^T \tilde{\Theta}. \quad (18)$$

By completing the squares and using the control input u_h in (13), the time derivative $\dot{V}$ along the error trajectory (17) is

$$\begin{aligned} \dot{V} &= \frac{1}{2} e^T (-K^T P - PK) e + u_h^T (I + \Delta G G_0^{-1})^T P e \\ &\quad + d^T P e + u_s^T (I + \Delta G G_0^{-1})^T P e \\ &\quad + (\Delta F + \Delta G G_0^{-1}(-Y\Theta + \dot{x}_d - Ke))^T P e \\ &\quad - \tilde{\Theta}^T Y^T P e + \frac{1}{\gamma} \dot{\tilde{\Theta}}^T \tilde{\Theta} \\ &= \frac{1}{2} e^T \left(-K^T P - PK \right. \\ &\quad \left. + P \left(\frac{1}{\rho^2} I - R^{-1}(I + \Delta G G_0^{-1})^T \right) P \right) e \\ &\quad - \frac{1}{2} \left(\frac{1}{\rho} P e - \rho d \right)^T \left(\frac{1}{\rho} P e - \rho d \right) + \frac{1}{2} \rho^2 d^T d \\ &\quad + u_s^T (I + \Delta G G_0^{-1})^T P e + (\Delta F + \Delta G G_0^{-1} \\ &\quad \cdot (-Y\Theta + \dot{x}_d - Ke))^T P e - \tilde{\Theta}^T Y^T P e + \frac{1}{\gamma} \dot{\tilde{\Theta}}^T \tilde{\Theta}. \end{aligned} \quad (19)$$

From the update law (15) which is a standard smooth projection algorithm [10] we can guarantee that $1/\gamma \dot{\Theta}^T \tilde{\Theta} - \tilde{\Theta}^T Y^T P e \leq 0$ and $\Theta(t) \in \Omega_\theta$ for all $t \geq 0$ if $\Theta(0) \in \Omega_\theta$. Moreover, by u_s in (14) and the assumptions **A1**–**A2**, we get

$$\begin{aligned} u_s^T (I + \Delta G G_0^{-1})^T P e + (\Delta F + \Delta G G_0^{-1}(-Y\Theta + \dot{x}_d - K e))^T P e \\ \leq -(\epsilon_f + \kappa M_e) \sum_{i=1}^3 |(P e)_i| + \sum_{i=1}^3 \\ \cdot |(\Delta F + \Delta G G_0^{-1}(-Y\Theta + \dot{x}_d - K e))_i| |(P e)_i| \leq 0. \end{aligned} \quad (20)$$

Summarily, using the Riccati-like equation (11) the derivative $\dot{V}$ can be bounded as

$$\begin{aligned} \dot{V} &\leq \frac{1}{2} e^T \left(-K^T P - P K + P \right. \\ &\quad \cdot \left(\frac{1}{\rho^2} I - R^{-1} (I + \Delta G G_0^{-1})^T P \right) e + \frac{1}{2} \rho^2 d^T d \\ &\leq \frac{1}{2} e^T \left(-K^T P - P K + P \right. \\ &\quad \cdot \left(\frac{1}{\rho^2} I - (1 - \kappa) R^{-1} \right) P \left. \right) e + \frac{1}{2} \rho^2 d^T d \\ &\leq -\frac{1}{2} e^T Q e + \frac{1}{2} \rho^2 d^T d. \end{aligned} \quad (21)$$

Integrating the above inequality from $t = 0$ to $t = T$ yields

$$2V(T) + \int_0^T \|e(t)\|_Q^2 dt \leq 2V(0) + \rho^2 \int_0^T \|d(t)\|^2 dt \quad (22)$$

for all $0 \leq T < \infty$, i.e., the H^∞ performance in (16) is achieved [1]. Moreover, from $V(t)$ in (18) and **A3** it is clear that $e(t) \in \Omega_e \triangleq \{e | e^T P e \leq 2V(0) + \rho^2 M_d\}$ and so $x(t) \in U_x \triangleq \{x | e(t) \in \Omega_e, x_d(t) \in \Omega_d\} \forall t \geq 0$. This implies all the variables are bounded. $\square$

Remark 1: The proposed controller (12) consists of three parts: the adaptive fuzzy system, $\hat{F}(x, \Theta) = Y(x)\Theta$ equipped with (15), which is used to learn the unknown dynamics $F(x)$, the VSS controller, u_s , to eliminate the effect of the approximation error, and the robust H^∞ controller, u_h , to achieve the desired H^∞ tracking performance. Hence, in practice this controller is a hybrid adaptive-robust controller. $\square$

Remark 2: i) Result in Theorem 1 indicates that the control design relies only on the solution of an algebraic Riccati-like equation (11). By setting $R^{-1} = (1/(1-\kappa)\rho^2)I_{3 \times 3}$ and Q be a diagonal matrix, a simple solution is obtained as $P = (1/2)K^{-1}Q$. Since a small perturbation $\Delta G(t)$ implies the bounded value κ is smaller, there is a trade-off between the magnitude of $\Delta G(t)$ and the control gain in u_h . ii) Since the value ϵ_f may be made arbitrarily small by increasing the number of fuzzy If–Then rules, from (14) there is a trade-off between the number of fuzzy rules as well as the control gain in u_s . $\square$

Remark 3: From (15) it is shown that $b_{ij} - \delta \leq \theta_{ij} \leq c_{ij} + \delta$ for $1 \leq j \leq m_i$ and $1 \leq i \leq 3$. So, it can be concluded that $\underline{b}_i \leq \hat{f}_i(x, \Theta_i) \leq \bar{c}_i$ where $\underline{b}_i \triangleq \min_{1 \leq j \leq m_i} (b_{ij} - \delta)$ and $\bar{c}_i \triangleq \max_{1 \leq j \leq m_i} (c_{ij} + \delta)$. Actually, since each $\zeta_{ij}(x) \in (0, 1]$ and $\sum_{j=1}^{m_i} \zeta_{ij}(x) = 1$, we get $\underline{b}_i \leq \sum_{j=1}^{m_i} (b_{ij} - \delta)\zeta_{ij} \leq \hat{f}_i(x, \Theta_i) \leq \sum_{j=1}^{m_i} (c_{ij} + \delta)\zeta_{ij} \leq \bar{c}_i$. Based on this property if the upper and lower bounds of $f_i(x)$ (e.g., $\underline{f}_i \leq f_i(x) \leq \bar{f}_i$) are well-known, then we can simply set $b_{ij} = \underline{f}_i$, $c_{ij} = \bar{f}_i$ for all $j = 1, \dots, m_i$. $\square$

Since the nonsmoothness of $\text{sgn}(P e)$ results in the VSS control algorithm (14) with discontinuities between the sliding surfaces, the VSS adaptive fuzzy-based controller developed above may lead to control chattering and could excite high-frequency unmodeled modes [8]. To avoid this discontinuity we use the exponential modification to replace u_s in (14).

Theorem 2: Consider the nonlinear chaotic Chua's circuit (1)–(3). Under the same conditions as in Theorem 1, if the VSS type control u_s in (14) is modified to the continuous control

$$u_s = -\frac{M_E(x)}{1 - \kappa} \frac{M_E(x) P e}{\|M_E(x) P e\| + \epsilon e^{-\nu t}} \quad (23)$$

where $M_E(x) = \sqrt{3}\epsilon_f + \kappa M_e(x)$ and $\epsilon > 0$, $\nu > 0$ are positive constants, then i) if $d \in L_2[0, \infty)$, an H^∞ tracking performance is achieved; ii) if $d \in L_\infty[0, \infty)$, the tracking error is uniformly ultimately bounded; iii) if $d \in L_2[0, \infty) \cap L_\infty[0, \infty)$, then $\lim_{t \rightarrow \infty} e(t) = 0$; and iv) all the variables are bounded. $\square$

Proof: Following the proof in Theorem 1 and using (11)–(13), (15) it can be shown that

$$\begin{aligned} \dot{V} &\leq -\frac{1}{2} e^T Q e + \frac{1}{2} \rho^2 d^T d + u_s^T (I + \Delta G G_0^{-1})^T P e \\ &\quad + (\Delta F + \Delta G G_0^{-1}(-Y\Theta + \dot{x}_d - K e))^T P e. \end{aligned} \quad (24)$$

By u_s in (23) and the assumptions **A1**–**A2** we get

$$\begin{aligned} u_s^T (I + \Delta G G_0^{-1})^T P e \\ + (\Delta F + \Delta G G_0^{-1}(-Y\Theta + \dot{x}_d - K e))^T P e \\ \leq -\frac{(M_E P e)^T (I + \Delta G G_0^{-1})^T (M_E P e)}{(1 - \kappa)(\|M_E P e\| + \epsilon e^{-\nu t})} + \|M_E P e\| \\ \leq \epsilon e^{-\nu t}. \end{aligned} \quad (25)$$

Consequently, the derivative $\dot{V}$ can be bounded as

$$\dot{V} \leq -\frac{1}{2} e^T Q e + \frac{1}{2} \rho^2 d^T d + \epsilon e^{-\nu t}. \quad (26)$$

Integrating the above inequality yields

$$\int_0^T \|e(t)\|_Q^2 dt \leq 2V(0) + \frac{2\epsilon}{\nu}(1 - e^{-\nu T}) + \rho^2 \int_0^T \|d(t)\|^2 dt. \quad (27)$$

This implies an H^∞ tracking performance is achieved.

Next, suppose d is bounded, i.e., there is an $\epsilon_d > 0$ such that $\|d\| \leq \epsilon_d$. The inequality (26) can be rewritten as $\dot{V} \leq -(1/2)\lambda_q \|e\|^2 + (1/2)\rho^2 \epsilon_d^2 + \epsilon$ where λ_q denotes the minimum eigenvalue of Q . Then, for any given $\eta > 0$ there is a choice of λ_q such that $\dot{V} \leq -\xi \|e\|^2 < 0 \forall \|e(t)\| > \eta$ for some $\xi > 0$. This implies that there is a $T > 0$ such that $\|e(t)\| \leq \eta$ for all $t \geq T$ [10].

Moreover, if $d(\cdot) \in L_2 \cap L_\infty[0, \infty)$, from the closed-loop error system it is clear that $\dot{e}(t)$ is uniformly bounded. This implies that $e(t)$ is uniformly continuous. Based on the Barbalat's lemma [10] and the inequality (26), it can be concluded that $\lim_{t \rightarrow \infty} e(t) = 0$.

Finally, as in the proof of Theorem 1 we can conclude that all the states and signals of the closed-loop system are bounded. $\square$

Remark 4: The term $(2\epsilon/\nu)(1 - e^{-\nu T})$ in (27) is yielded owing to the smooth modification of u_s and can be viewed as an external disturbance. The smaller the value ϵ and the larger the value ν , the less smooth is the robust controller u_s and the smaller is the term $(2\epsilon/\nu)(1 - e^{-\nu T})$. As $\epsilon = 0$, this term is equal to zero and this H^∞ tracking performance is reduced to (16). $\square$

In the above analyses, the input gain matrix $G(t, x)$ is assumed to be expressed as a nominal part plus a small perturbation. However, in practical applications $G(t, x)$ may be completely unknown and cannot be available directly in the control design. Without loss of generality, assume the values of C_1 , C_2 and L are nonlinear time-varying, dependent on the state variable, and unknown. For simplicity of notation, let $g_1(t, x) \triangleq 1/C_1(t, x)$, $g_2(t, x) \triangleq 1/C_2(t, x)$ and $g_3(t, x) \triangleq 1/L(t, x)$.

Fig. 3. The responses for the nonlinear chaotic Chua's circuit using the developed controller. (a) Capacitor voltage $v_{c1}(t)$. (b) Capacitor voltage $v_{c2}(t)$. (c) Inductor current $i_L(t)$. (d) Control input $u_1(t)$.

Take the fuzzy system $\hat{g}_i(x, \Theta_{gi})$ to approximate the uncertain term $g_i(t, x)$. From (6) $\hat{g}_i(x, \Theta_{gi})$ can be expressed as

$$\hat{g}_i(x, \Theta_{gi}) = \zeta_{gi}^T(x) \Theta_{gi}, \quad i = 1, 2, 3 \quad (28)$$

where $\Theta_{gi} \in R^{m_{gi}}$ for some $m_{gi} > 0$ is a parameter vector and $\zeta_{gi}(x) \in R^{m_{gi}}$ is a regressive vector. Suppose the constrained regions of Θ_{gi} , $i = 1, 2, 3$ are convex hypercubes. That is, consider $\Omega_{gi0} \triangleq \{\Theta_{gi} | b_{gij} \leq \theta_{gij} \leq c_{gij}, 1 \leq j \leq m_{gi}\}$ and $\Omega_{gi} \triangleq \{\Theta_{gi} | b_{gij} - \delta_{gi} \leq \theta_{gij} \leq c_{gij} + \delta_{gi}, 1 \leq j \leq m_{gi}\}$ where the values of b_{gij} , c_{gij} and $\delta_{gi} > 0$ can be arbitrarily specified. Let $\Phi_{gi} \triangleq u_i \zeta_{gi}(Pe)_i$, $i = 1, 2, 3$ where $(Pe)_i$ denotes the i element of Pe . Therefore, the smooth projection algorithm of Θ_{gi} with respect to Ω_{gi} is

$$\text{Proj}[\Phi_{gi}, \Theta_{gi}] \triangleq \begin{cases} \bar{\phi}_{gij}, & \text{if } (\theta_{gij} > c_{gij} \& \phi_{gij} > 0) \\ \tilde{\phi}_{gij}, & \text{if } (\theta_{gij} < b_{gij} \& \phi_{gij} < 0) \\ \phi_{gij}, & \text{otherwise, } 1 \leq j \leq m_{gi} \end{cases} \quad (29)$$

where $\bar{\phi}_{gij} \triangleq (1 + (c_{gij} - \theta_{gij}/\delta_{gi}))\phi_{gij}$, $\tilde{\phi}_{gij} \triangleq (1 + (\theta_{gij} - b_{gij}/\delta_{gi}))\phi_{gij}$ and ϕ_{gij} denotes the j element of Φ_{gi} .

Let Θ_{gi}^* , $i = 1, 2, 3$ be the optimal approximation parameters. Define the minimum approximation errors $\Delta g_i(t, x) = g_i(t, x) -$

$\hat{g}_i(x, \Theta_{gi}^*)$ and the estimated errors $\tilde{\Theta}_{gi} = \Theta_{gi} - \Theta_{gi}^*$. After some simple manipulations, the input matrix $G(t, x)$ is equal to

$$G(t, x) = \hat{G}(x, \Theta_g) - Y_g(x) \tilde{\Theta}_g + \Delta G(t, x) \quad (30)$$

where

$$\begin{aligned} \hat{G}(x, \Theta_g) &= \text{diag}[\hat{g}_1(x, \Theta_{g1}), \hat{g}_2(x, \Theta_{g2}), \hat{g}_3(x, \Theta_{g3})] \\ Y_g(x) &= \text{diag}[\zeta_{g1}^T(x), \zeta_{g2}^T(x), \zeta_{g3}^T(x)] \\ \tilde{\Theta}_g &= \text{diag}[\tilde{\Theta}_{g1}, \tilde{\Theta}_{g2}, \tilde{\Theta}_{g3}] \\ \Delta G(t, x) &= \text{diag}[\Delta g_1(t, x), \Delta g_2(t, x), \Delta g_3(t, x)]. \end{aligned}$$

In addition, we make the following assumption.

A4 There is a constant $0 < \kappa_g < 1$ such that $|\Delta g_i(t, x) \hat{g}_i^{-1}(x, \Theta_{gi})| \leq \kappa_g$, $\forall 1 \leq i \leq 3$. $\square$

Theorem 3: Consider the uncertain nonlinear chaotic Chua's circuit (1)–(3). Suppose **A2**–**A4** are satisfied. If there exists a symmetric positive matrix $P = P^T > 0$ satisfying the Riccati-like equation

$$-K^T P - PK + Q + P \left(\frac{1}{\rho^2} I_{3 \times 3} - (1 - \kappa_g) R^{-1} \right) P = 0 \quad (31)$$

then the following VSS adaptive fuzzy-based control law

$$u = \hat{G}^{-1}(x, \Theta_g)(-Y(x)\Theta + \dot{x}_d - Ke + u_h + u_s) \quad (32)$$

Fig. 3. (Continued) The responses for the nonlinear chaotic Chua's circuit using the developed controller. (e) Control input $u_2(t)$. (f) Control input $u_3(t)$. (g) phase-plane trajectory $v_{c1} - v_{c2}$. (h) Phase-plane trajectory $v_{c2} - i_L$.

with u_h in (13), $u_s = -(\epsilon_f + \kappa_g M_e) \text{sgn}(Pe)/(1 - \kappa_g)$, $\dot{\Theta}$ in (15), and $\dot{\Theta}_{gi} = \gamma_{gi} \text{Proj}[\Phi_{gi}, \Theta_{gi}]$ in (29) with $b_{gij} - \delta_{gi} > 0 \forall 1 \leq j \leq m_{gi}$, $1 \leq i \leq 3$ where $\gamma_{gi} > 0$ are the adaptive gains, guarantees all the variables of the closed-loop system are bounded and an H^∞ tracking performance is achieved. $\square$

Proof: From (30) and (32), the error dynamic equation (4) can be rewritten as

$$\begin{aligned} \dot{e} = & -Ke - Y(x)\tilde{\Theta} + \Delta F(x) + \Delta G(t, x)u \\ & - Y_g(x)\tilde{\Theta}_g u + u_h + u_s + d. \end{aligned} \quad (33)$$

Choose a Lyapunov function as

$$W = \frac{1}{2} e^T P e + \frac{1}{2\gamma} \tilde{\Theta}^T \tilde{\Theta} + \sum_{i=1}^3 \frac{1}{2\gamma_{gi}} \tilde{\Theta}_{gi}^T \tilde{\Theta}_{gi}. \quad (34)$$

As in the proof of Theorem 1, the derivative $\dot{W}$ can be bounded as

$$\begin{aligned} \dot{W} \leq & -\frac{1}{2} e^T Q e + \frac{1}{2} \rho^2 d^T d \\ & - \sum_{i=1}^3 u_i \tilde{\Theta}_{gi}^T \zeta_{gi}(Pe)_i + \sum_{i=1}^3 \frac{1}{\gamma_{gi}} \dot{\Theta}_{gi}^T \tilde{\Theta}_{gi}. \end{aligned} \quad (35)$$

It can be shown that $(1/\gamma_{gi})\dot{\Theta}_{gi}^T \tilde{\Theta}_{gi} - u_i \tilde{\Theta}_{gi}^T \zeta_{gi}(Pe)_i \leq 0$ and $\Theta_{gi}(t) \in \Omega_{\theta_{gi}} \forall t \geq 0$. Similarly to the proof of Theorem 1, an H^∞ performance is achieved and all the variables are bounded.

Finally, we shall show that the matrix $\hat{G}(x, \Theta_g)$ is invertible. Let $\underline{b}_{gi} \triangleq \min_{1 \leq j \leq m_{gi}} (b_{gij} - \delta_{gi})$ and $\bar{c}_{gi} \triangleq \max_{1 \leq j \leq m_{gi}} (c_{gij} + \delta_{gi})$. As in Remark 3, it can be concluded that $\underline{b}_{gi} \leq \hat{g}_i(x, \Theta_{gi}) \leq \bar{c}_{gi}$, $i = 1, 2, 3$. Then, if we suitably choose b_{gij} and δ_{gi} such that $\underline{b}_{gi} > 0$, $\hat{G}(x, \Theta_g)$ is invertible and so the controller is well-defined. $\square$

IV. SIMULATION EXAMPLE

Consider the tracking control problem of the uncertain chaotic Chua's circuit shown in Fig. 1. For the convenience of simulation, the nominal parameters are given as $C_{10} = 1$, $C_{20} = 0.5$ and $L_0 = 1$, and the perturbations are given as $\Delta C_1 = 0.1 + 0.1 \cos(t/2)$, $\Delta C_2 = 0.1$ and $\Delta L = 0.15$. The perturbed resistors are characterized by $R = 5 + \sin(t/2)$ and $g(v_{c1}) = -v_{c1} + 0.02v_{c1}^3$. The exogenous disturbances are $d_1 = 0.25 \sin(2t) \exp(-0.1t)$, $d_2 = 0.2 \cos(2t) \exp(-0.1t)$ and $d_3 = 0.2 \sin(2t) \exp(-0.1t)$.

Let the desired voltage and current be $x_{d1}(t) = \sin(t)$, $x_{d2}(t) = \cos(t)$ and $x_{d3}(t) = 1 + \sin(t)$. The VSS adaptive fuzzy-based H^∞ tracking control law developed in Theorem 2 is employed to treat this trajectory planning problem. We divide the design into three steps.

Step 1: Let $x_1 = v_{c1}$, $x_2 = v_{c2}$, $x_3 = i_L$ and $x_4 = v_{c1} - v_{c2}$. Define five fuzzy sets for each x_i , $i = 1, 2, 3, 4$ with labels F_i^1 (negative large), F_i^2 (near -0.5), F_i^3 (near 0), F_i^4 (near 0.5), and F_i^5 (positive large) which are characterized by the membership functions, respectively, $\mu_{F_i^1}(x_i) = 1/(1 + \exp(5(x_i + 1)))$, $\mu_{F_i^2}(x_i) = \exp(-2(x_i + 0.5)^2)$, $\mu_{F_i^3}(x_i) = \exp(-2x_i^2)$, $\mu_{F_i^4}(x_i) = \exp(-2(x_i - 0.5)^2)$, and $\mu_{F_i^5}(x_i) = 1/(1 + \exp(-5(x_i - 1)))$.

Now, construct the fuzzy approximator $\hat{f}_1(x, \Theta_1)$ in (7). 25 fuzzy rules are included

$$R_{ij}^{(l)}: \text{If } x_1 \text{ is } F_1^i \text{ and } x_4 \text{ is } F_4^j, \text{ Then } y \text{ is } F_{ij}^l, \\ \text{for } i, j = 1, \dots, 5 \quad l = 1, \dots, 25.$$

The fuzzy regressive vector in (7) is chosen to be $\zeta_1 = [\zeta_{11}, \zeta_{12}, \dots, \zeta_{125}]^T \in R^{25}$ with components as $\zeta_{11}(x) = \mu_{F_1^1}(x_1)\mu_{F_4^1}(x_4)/D_1$, $\zeta_{12}(x) = \mu_{F_1^1}(x_1)\mu_{F_4^2}(x_4)/D_1$, $\dots$, $\zeta_{124}(x) = \mu_{F_1^5}(x_1)\mu_{F_4^4}(x_4)/D_1$, and $\zeta_{125}(x) = \mu_{F_1^5}(x_1)\mu_{F_4^5}(x_4)/D_1$ where $D_1 = \sum_{i=1}^5 \sum_{j=1}^5 \mu_{F_1^i}(x_1)\mu_{F_4^j}(x_4)$. Hence, $\hat{f}_1(x)$ is approximated by $\hat{f}_1(x, \Theta_1) = \zeta_1^T(x)\Theta_1$ with $\Theta_1 \triangleq [\theta_{11}, \theta_{12}, \dots, \theta_{125}]^T \in R^{25}$. Next, 25 fuzzy rules are included in $\hat{f}_2(x, \Theta_2)$:

$$R_{ij}^{(l)}: \text{If } x_3 \text{ is } F_3^i \text{ and } x_4 \text{ is } F_4^j, \text{ Then } y \text{ is } F_{ij}^l, \\ \text{for } i, j = 1, \dots, 5 \quad l = 26, \dots, 50.$$

Choose $\zeta_2 = [\zeta_{21}, \zeta_{22}, \dots, \zeta_{225}]^T \in R^{25}$ with $\zeta_{21}(x) = \mu_{F_3^1}(x_3)\mu_{F_4^1}(x_4)/D_2$, $\zeta_{22}(x) = \mu_{F_3^1}(x_3)\mu_{F_4^2}(x_4)/D_2$, $\dots$, $\zeta_{225}(x) = \mu_{F_3^5}(x_3)\mu_{F_4^5}(x_4)/D_2$, and $D_2 = \sum_{i=1}^5 \sum_{j=1}^5 \mu_{F_3^i}(x_3)\mu_{F_4^j}(x_4)$. Hence, $\hat{f}_2(x, \Theta_2) = \zeta_2^T(x)\Theta_2$ with $\Theta_2 \triangleq [\theta_{21}, \theta_{22}, \dots, \theta_{225}]^T \in R^{25}$. Finally, 5 fuzzy rules are included in $\hat{f}_3(x, \Theta_3)$:

$$R_i^{(l)}: \text{If } x_2 \text{ is } F_2^i, \text{ Then } y \text{ is } F_i^l \\ \text{for } i = 1, \dots, 5, l = 51, \dots, 55.$$

Choose $\zeta_3 = [\zeta_{31}, \dots, \zeta_{35}]^T \in R^5$ with $\zeta_{31}(x) = \mu_{F_2^1}(x_2)/D_3$, $\dots$, $\zeta_{35}(x) = \mu_{F_2^5}(x_2)/D_3$, and $D_3 = \sum_{i=1}^5 \mu_{F_2^i}(x_2)$. Hence, $\hat{f}_3(x, \Theta_3) = \zeta_3^T(x)\Theta_3$ with $\Theta_3 \triangleq [\theta_{31}, \dots, \theta_{35}]^T \in R^5$. Consequently, $F(x)$ is approximated by $\hat{F}(x, \Theta) = Y(x)\Theta$ with $Y(x) \triangleq \text{diag}[\zeta_1^T(x), \zeta_2^T(x), \zeta_3^T(x)]$ and $\Theta \triangleq [\Theta_1^T, \Theta_2^T, \Theta_3^T]^T$.

Step 2: Select $K = 2I_{3 \times 3}$, $\rho = 0.5$ and $Q = 2I_{3 \times 3}$. Set $\kappa = 0.2$. Consequently, take $R = (1 - \kappa)\rho^2 I_{3 \times 3} = 0.2I_{3 \times 3}$ and solving the equation in (11) yields $P = (1/2)K^{-1}Q = 0.5I_{3 \times 3}$.

Step 3: Obtain the tracking controller (12), (13), (15), (23) with $\epsilon_f = 0.2$, $\epsilon = 1$, $\nu = 0.1$, $\gamma = 200$, $b_{ij} = -5$, $c_{ij} = 5 \forall ij$, and $\delta = 0.5$.

Choose the initial conditions $v_{c1}(0) = -1$, $v_{c2}(0) = 0$, $i_L(0) = 0$, and $\Theta(0) = 0_{55 \times 1}$. The simulation results are shown in Fig. 3. The phase-plane trajectory $v_{c1} - v_{c2}$ converges to a circle of unit radius centered at (0, 0) and $v_{c2} - i_L$ converges to a circle of unit radius centered at (0, 1). These simulation results indicate that the tracking performance is nice and consequently the effects due to parametric uncertainties and external disturbances in chaotic Chua's control circuits can be efficiently diminished by the developed control algorithm.

V. CONCLUSIONS

An adaptive fuzzy-based tracking control design incorporating with a standard VSS control algorithm and a nonlinear H^∞ control algorithm has been proposed and solved for uncertain nonlinear chaotic Chua's circuits. The hybrid adaptive-robust tracking controller developed in this study guarantees that all the states and signals of the closed-loop system are bounded and an H^∞ tracking control is achieved. Compared with the previous investigations which also

address the problem of controlling Chua's circuits this paper can be extended to handle a broader class of nonlinear chaotic Chua's circuits in the presence of plant uncertainties and external disturbances, and so this design is quite useful from the viewpoint of practical applications. Finally, a simulation example is included to confirm the validity and performance of the developed control scheme.

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