Assignment 0: Describe a Parallel Application

Jianwei Xiao

1 Biography

I am a first year PhD student in applied mathematics at University of California, Berkeley. My current research interests are numerical linear algebra and numerical solutions to PDE. I would like to learn parallel programming skills and how can parallel programming solve certain computational mathematics problems. I also would like to learn more examples about application of parallel programming.

2 Parallel Application: Implementation of Weighted Essentially Non-oscillatory Schemes

2.1 Introduction to WENO Schemes

Weighted essentially non-oscillatory (WENO) schemes are effective numerical methods to compute flows having shocks and steep gradients. WENO schemes are based on the successful essentially non-oscillatory (ENO) schemes introduced in [9] and [10]. The initial WENO scheme is constructed in [6], and further research in WENO can be found in papers such as [2], [4] and [5].

Both ENO and WENO schemes use adaptive stencils in the reconstruction procedure based on the local smoothness of the numerical solution to automatically achieve high order accuracy and non-oscillatory property near discontinuities. Instead of choosing the optimal stencil candidate to pick an interpolating polynomial in the reconstruction procedure, WENO schemes use a convex combination of all candidates to achieve the essentially non-oscillatory property. The coefficient of each candidate is a nonlinear weight which depends on the local smoothness of the numerical solution based on the stencil.
WENO schemes have many advantages over ENO schemes. WENO schemes improves ENO schemes in robustness, better steady state convergence, better smoothness of the fluxes and more efficiency. WENO schemes are widely used in applications like gas dynamics, aeroacoustics and incompressible flow problems.

2.2 Application of Parallel Programming in WENO Schemes

When the PDE solution contains both discontinuities and complex solution structures, high order WENO schemes can be more economical in CPU time than many traditional numerical methods such as PPM [1] or other TVD [3] methods. In [7], the authors verified such efficiency through two representative numerical experiments: the double Mach reflection problem and the Rayleigh-Taylor instability problem.

The effectiveness of WENO schemes for computing solutions containing both discontinuities and complex solution features have many reasons. **One of the most important reasons is WENO schemes have excellent parallel efficiency.** As explicit methods, WENO schemes are easily implemented on parallel computers and relatively easy to code. All computation of those two numerical experiments in [7] are performed on the IBM SP parallel computer using up to 48 processors at the Technology Center for Advanced Scientific Computing and Visualization of Brown University.

The following table is parallel efficiency of ninth order WENO schemes in computing Rayleigh-Taylor instability problem in [7].

<table>
<thead>
<tr>
<th>number of processors</th>
<th>h=1/240</th>
<th></th>
<th>h=1/480</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>CPU time(s)</td>
<td>efficiency</td>
<td>CPU time(s)</td>
<td>efficiency</td>
</tr>
<tr>
<td>1</td>
<td>5823</td>
<td></td>
<td>48527</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>788</td>
<td>92.37%</td>
<td>6111</td>
<td>99.26%</td>
</tr>
<tr>
<td>12</td>
<td>565</td>
<td>85.88%</td>
<td>4205</td>
<td>96.17%</td>
</tr>
<tr>
<td>24</td>
<td>368</td>
<td>65.93%</td>
<td>2323</td>
<td>87.04%</td>
</tr>
<tr>
<td>48</td>
<td>260</td>
<td>46.66%</td>
<td>1398</td>
<td>72.32%</td>
</tr>
</tbody>
</table>

In this table, authors of [7] list the parallel efficiency for ninth order WENO schemes using 8, 12, 24, 48 processors for two different mesh sizes $h = \frac{1}{240}, \frac{1}{480}$. We can see the parallel efficiency is over 90% when the operation for each processor is kept constant, i.e. when the number of proces-
sors increases together with a mesh refinement. This data means the application “scale” to large problems on many processors.

Moreover, parallel computing can also make some other numerical methods such as Discontinuous Galerkin method and multigrid method more effective. Finding application of parallel computing in numerical methods is an interesting and important topic for applied mathematician.

References


