

## Chapter 2

# Electronic Circuits [cir]

In this chapter we introduce the schematic diagrams as a powerful “language” to represent electronic circuits. We also discuss

### 2.1 Circuit Analysis

Electronic circuit design relies on the ability to analyze and predict circuit behavior. In general this requires the calculation of electromagnetic fields, a complex task even for very simple circuits. Fortunately in many cases we can rely on the following approximations to greatly simplify circuit analysis:

1. Electrical interactions happen instantaneously.
2. Electrical components interact solely through wires.
3. The net charge on electrical components is always zero.

Let’s check the validity of each of these approximations. Electrical fields propagate at or near the speed of light, 300,000 km/ sec. While this exceeds most everyday life experiences, electronic circuits can operate at very high speed and it is therefore prudent to check the assumption of instantaneous interaction.

For example, a computer operating at 1 GHz clock rate executes  $10^9$  operations per second, or one operation in every nano-second. During this time electrical signals propagate 30 cm. Since integrated circuit chips are much smaller and most signals travel only through part of a chip, the delay is indeed usually negligible. We conclude that for systems operating at frequencies less than about 1 GHz and with their longest dimension no larger than about ten centimeters, the assumption of instantaneous interaction is valid.

The second assumption, interaction solely through wires, is obviously violated by many electronic circuits. Depending on the situation, such interaction is variously referred to wireless communication or interference. Since however the signals generated by such remote action are usually small, the effects can often be neglected. Having said that we certainly will observe interference in the laboratory and learn about some simple precautions.

The final approximation is a consequence of the electrostatic force between two charged bodies. This force grows very rapidly and quickly counteracts any temporary charge buildup. See Problem 2.6 on page 28 for an example.

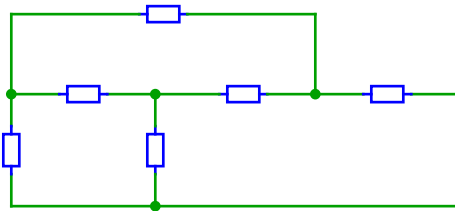
In practice and unless we work e.g. on wireless devices we can usually assume that the assumptions listed above are valid but should be ready to check if we observe otherwise inexplicable phenomena.

## 2.2 Schematic Diagrams

Electronic circuit components and interconnects come in a wide variety. Even simple devices such as batteries or switches exist in many forms and colors. Although these aspects are certainly important, they do not affect the electrical function. For effective circuit design we need a representation that only captures the operation of the electronic circuits.

Schematic diagrams meet this requirement. Figure 2.1 shows an example. The rectangles indicate the components; interconnect wires are represented by lines. The coloring, blue for components and green for interconnects, is for clarification only.

Just like the electronic circuits they represent, schematic diagrams are hierarchical: except for basic components such as batteries or resistors, the components themselves are electronic circuits and can be represented by their own schematic diagram.



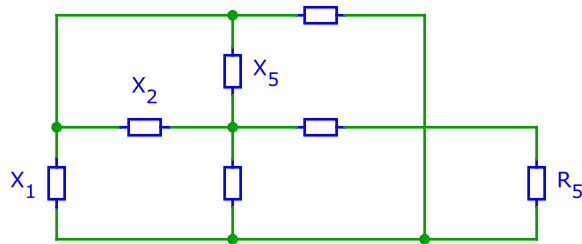
**Figure 2.1** Schematic circuit diagram showing electronic components (blue) and their interconnects (green).

In the example, each component has only two connections. Practical components have anywhere from two (e.g. resistors or single pole switches) to several thousand connections (some microprocessors). An optional label, such as  $X_1$  or  $R_5$  names the component. Often it also indicates its function. The prefix  $R$ , for example, is usually reserved for resistors.

The orientation and position of the circuit components in the schematic is arbitrary, although neat arrangements that for example minimize crossings of interconnects are preferable. It is also good practice to indicate which interconnects are connected with a dot, as shown in the sample diagram. This is particularly important for crossings. The schematic in Figure 2.2 has two crossings, one with and the other without dot, indicating that the wires are connected on one case but not in the other. Of course it would be better to redraw the schematic to avoid the crossing without connection to avoid possible misunderstandings.

Many free and commercial tools are available for drawing schematic diagrams. Often they are integrated with other programs e.g. for printed circuit board layout or electronic circuit simulators used for verification.

Although versatile, schematic diagrams are not the only solution for representing circuits. Especially when interacting with computer tools netlists listing all components and

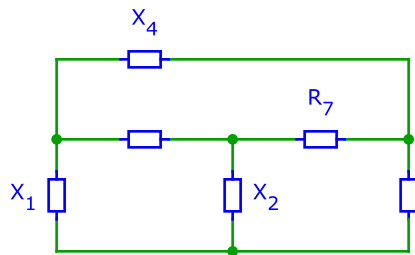


**Figure 2.2** Schematic diagram with wire crossings.

their interconnections in a textual format are often preferred. Fortunately most schematic capture programs automatically convert the graphical to a textual representation.

## 2.3 Schematic Diagrams

Figure 2.3 shows a schematic diagram consisting of branches and nodes. In this diagram, branches are shown in blue and nodes are shown in green. This is for clarity only, often circuit diagrams are in a single color. In electronics, the terms schematic diagram and circuit diagram are used interchangeably.



**Figure 2.3** Schematic circuit diagram showing circuit components (blue) and interconnects (green).

The branches in the diagram represent circuit components such as batteries, switches, transistor, etc. Optionally, components are given names such as  $X_1$ ,  $X_2$ , or  $R_7$ .

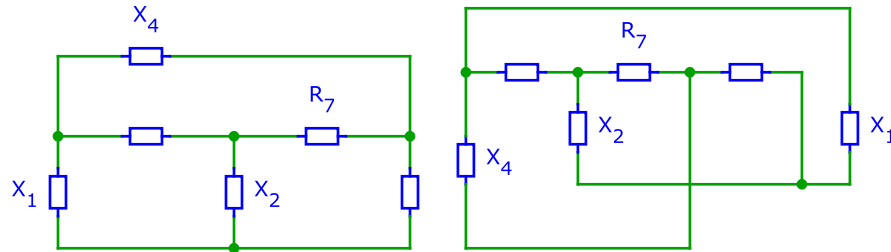
Nodes represent electrical interconnects such as wires or copper traces on printed circuit boards or integrated circuits. Each contiguous green trace represents a node, regardless of the number of branches connected to it. The circuit in Figure 2.3 has four nodes. Dots indicate where interconnects are tied together.

The schematics shown in Figure 2.4 are functionally equivalent since they contain the same components and interconnected in the same way, as can be seen by redrawing the schematic on the right. For circuit analysis, simple arrangements are preferable. For example, it is much easier to count the number of nodes in the schematic on the left than in the functionally equivalent schematic on the right.

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### EXAMPLE 2-1: Number of branches and nodes.

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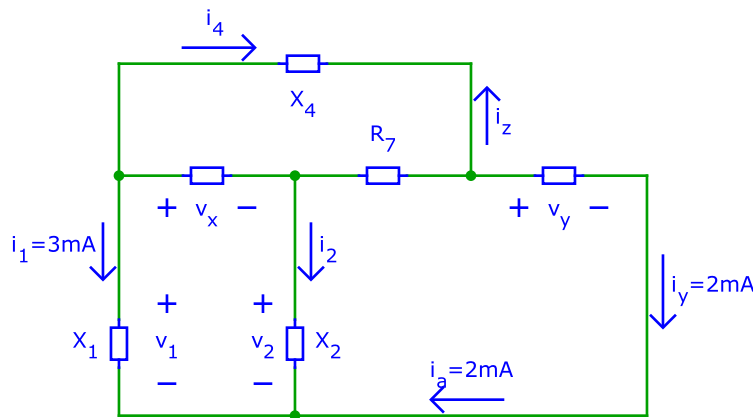
**Figure 2.4** Two identical circuit diagrams: The circuit on the right can be transformed into the one on the left by rearranging the components.

**EXAMPLE 2-2: Equivalent circuits.**

**2.3.1 Voltage and Current**

Circuit components interact with each other via currents flowing in the interconnects. Current is electronic charge passing through the interconnects. In most electronic systems the charges are electrons. Voltage, also known as electromotive force, drives the current flow and is established for example by a battery.

```
define I, V, units
charge
q_e
velocity of electrons
```



**Figure 2.5** Currents and voltages in a schematic diagram.

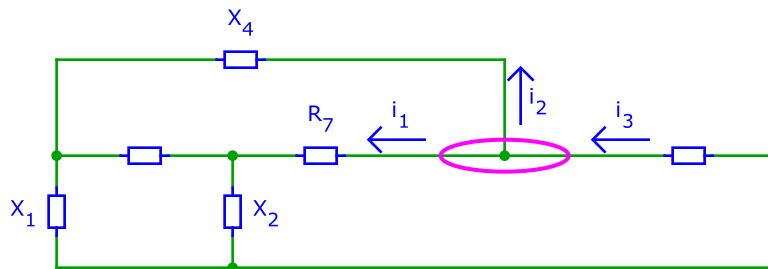
Current and voltage are analogous to velocity and pressure in a hydraulic system, with water molecules taking the place of electrons, pipes corresponding to interconnects, and valves and pumps assuming the roles of switches and voltage sources.

Figure 2.5 shows how currents ( $i_1, i_2, i_3, \dots$ ) and voltages ( $v_1, v_2, v_y, \dots$ ) are indicated in a schematic diagram. The direction of the symbols is arbitrary: redrawing the arrow indicating current  $i_1$  to point simply corresponds to changing the sign of the current from +3 mA to -3 mA.

Currents  $i_a$  and  $i_y$  are identical. Such redundancy is sometimes convenient, but can also result in confusion.

### 2.3.2 Kirchhoff's Laws

Voltages and currents in electronic circuits obey the same laws of conservation as flow and pressure in hydraulic systems. In electronic circuits the resulting relationships are referred to as Kirchhoff's Current Law (KCL) and Kirchhoff's Voltage Law (KVL).



**Figure 2.6** Circuit illustrating for Kirchhoff's Current Law (KCL). The sum of all currents exiting any closed shape, such as the ellipse shown in the diagram, is zero.

#### Kirchhoff's Current Law (KCL)

Kirchhoff's Current Law expresses formally the fact that the total current flowing out of a node must equal zero. Consider the circuit in Figure 2.6. The sum of all currents exiting the region indicated by the ellipse must be zero. Otherwise, excess charges would continue to accumulate in the wires, a situation that is physically impossible. The sum of all currents exiting the ellipse is

$$i_1 + i_2 + (-i_3) = 0 \quad (2.1)$$

Current  $i_3$  appears in the equation with a negative sign since it is entering rather than leaving the ellipse.

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#### EXAMPLE 2-3: Applying KCL to find unknown currents.

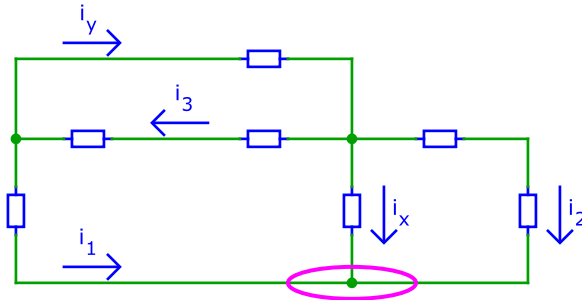
In the circuit diagram in Figure 2.7, currents  $i_1 = 3 \text{ mA}$ ,  $i_2 = -7 \text{ mA}$  and  $i_3 = 5 \text{ mA}$  are known. Determine the unknown currents  $i_x$  and  $i_y$ .

We first apply Kirchhoff's Current Law to the region indicated in the schematic by the ellipse.

The sum of all currents leaving the ellipse is

$$(-i_1) + (-i_x) + (-i_2) = 0.$$

Solving for  $i_x$  we get  $i_x = -i_1 - i_2 = 4 \text{ mA}$ .

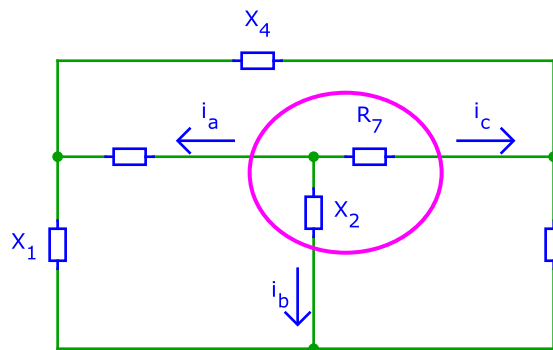


**Figure 2.7** Example illustrating how to use KCL to determine unknown currents.

Similarly we can find an equation for  $i_y$ , for example by summing the currents leaving a shape that includes the three components at the top right of the diagram to get

$$(-i_y) + i_3 + i_x + i_2 = 0.$$

Solving for  $i_y$  and substituting the result from above for  $i_x$  yields  $i_y = i_3 + i_x + i_2 = 2 \text{ mA}$ .



**Figure 2.8** Applying Kirchhoff's Current Law to a region that includes circuit components.

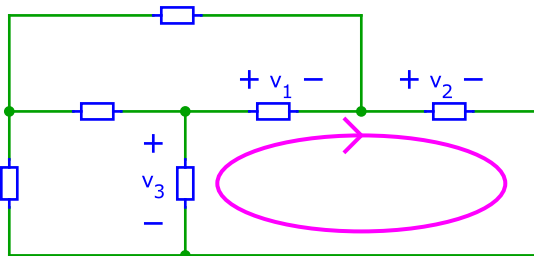
Figure 2.8 shows a situation where the region with net zero current outflow includes circuit components. Kirchhoff's law applies here also because of the assumption that the net charge on circuit components is always zero. Applying KCL yields the following equation

$$i_a + i_b + i_c = 0. \tag{2.2}$$

For  $i_a = -3 \text{ A}$  and  $i_b = 2 \text{ A}$  we get  $i_c = -i_a - i_b = 1 \text{ A}$ .

### Kirchhoff's Voltage Law (KVL)

Analogous to Kirchhoff's Current Law, Kirchhoff's Voltage Law (KVL) states that the sum of the voltages along any closed path must be zero. This can be visualized with an analogy to staircases: No matter which stairs we take between floors in a building, the total height gain and loss must be equal after returning to the starting point. In the hydrology equivalent, the sum of all pressure drops along a closed path is zero.



**Figure 2.9** Circuit illustrating Kirchhoff's Voltage Law (KVL). The sum of the voltages along any closed path is zero.

Figure 2.9 shows an example. The sum of all voltages along the path indicated in red is

$$v_1 + v_2 + (-v_3) = 0. \quad (2.3)$$

Voltage  $v_3$  appears in the equation with a minus sign since when circling the ellipse in clockwise direction the negative sign marking the orientation of  $v_3$  is encountered first. For voltages  $v_1$  and  $v_2$  the plus sign is encountered first, consequently they appear with a plus sign in the sum.

## 2.4 Power

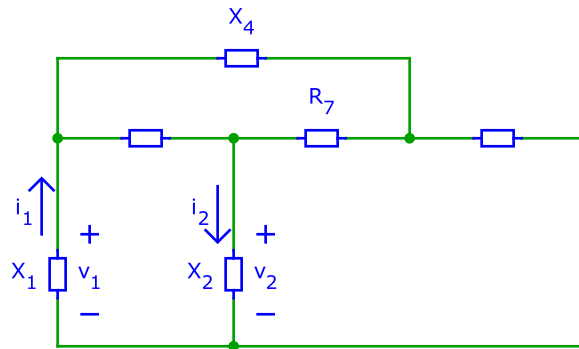
Power is the product of voltage times current and measured in Watts. A circuit component can either deliver power to the rest of the circuit or absorb power delivered by other components in the same circuits. Which of the two situations applies must be determined from the orientation of the voltage and current and the signs of their values.

To analyze the power of component  $X_1$  in Figure 2.10, consider a charge  $q$  passing through  $X_1$  in the direction of current arrow  $i_1$ . Since the voltage increases when passing through the component, the charge gains energy  $q \times v_1$  from  $X_1$ . Hence component  $X_1$  is *delivering* power  $p_d = v_1 i_1$ . For  $v_1 = 3\text{ V}$  and  $i_1 = 2\text{ mA}$ , the power delivered is  $p_d = 6\text{ mW}$ .

Similarly, in component  $X_2$  charge moving in the direction of current  $i_2$  loses energy, hence  $X_2$  is *absorbing* power  $p_a = v_2 i_2$ . If  $v_2 = 1\text{ V}$  and  $i_2 = -3\text{ mA}$ , the power dissipated is  $p_a = -3\text{ mW}$ , which is the same as saying that the power delivered by  $X_2$  is  $p_d = -v_2 i_2 = 3\text{ mW}$ .

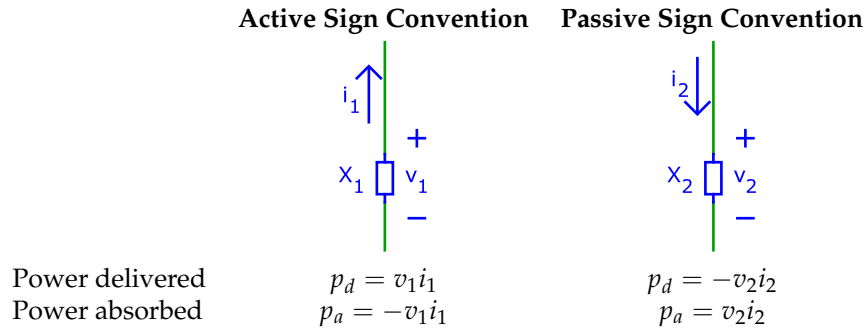
Figure 2.11 summarizes the different cases for power delivered and absorbed. The situation on the left is usually referred to as "active sign convention", the one on the right "passive sign convention".

When labeling circuit diagrams we typically use the active sign convention for sources such as batteries, and the passive sign convention for dissipating elements such as resistors.



**Figure 2.10** Illustration of Kirchhoff's Voltage Law (KVL). The sum of all voltages along any closed path is zero.

This is only a convention, the direction of power flow must always be determined from the direction of voltage and current and the signs of their values. For example, a rechargeable battery can both deliver and absorb power.

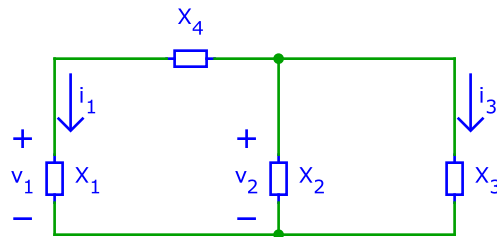


**Figure 2.11** Summary of the signs for power delivered and absorbed.

From conservation of energy it follows that the sum of the power delivered (or, analogously, absorbed) by all components in a circuit must equal to zero.

**EXAMPLE 2-4: Power delivered and absorbed**

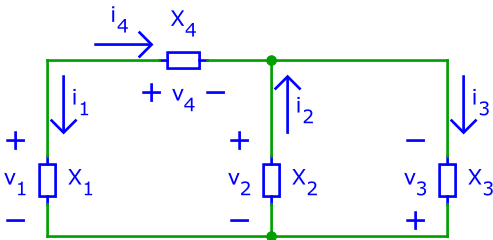
Determine the power delivered by each component in the circuit in Figure 2.12 for  $v_1 = 3\text{ V}$ ,  $v_2 = -2\text{ V}$ ,  $i_1 = 3\text{ mA}$ , and  $i_3 = -5\text{ mA}$ .



**Figure 2.12** Circuit for power analysis example.



First we label the voltages and currents in all components. The directions are arbitrary. Here we use the choices indicated in Figure 2.13; you are encouraged to redo the example for different directions and verify that you get the same final result.



**Figure 2.13** Labeled circuit for computing power delivered and absorbed.

Next we use KVL and KCL to determine the unknown voltages and currents to get  $v_3 = -v_2 = 2\text{ V}$ ,  $v_4 = v_1 - v_2 = 5\text{ V}$ ,  $i_2 = i_1 + i_3 = -2\text{ mA}$ , and  $i_4 = -i_1 = -3\text{ mA}$ .

Now we use Figure 2.11 to determine the sign convention for each component and appropriate equation to determine the power dissipated. Finally we verify that the sum of the power delivered by all components is zero. The table below summarizes the results.

Component	Sign Convention	Power Delivered
$X_1$	passive	$p_{X_1,d} = -v_1 i_1 = -9\text{ mW}$
$X_2$	active	$p_{X_2,d} = v_2 i_2 = 4\text{ mW}$
$X_3$	active	$p_{X_3,d} = v_3 i_3 = -10\text{ mW}$
$X_4$	passive	$p_{X_4,d} = -v_4 i_4 = 15\text{ mW}$
All		$\sum_i p_{X_i,d} = 0\text{ mW}$

## 2.5 Energy

## 2.6 Skills

### Problems

- max chip size for  $f_s$
- number of electrons in universe
- force holding atomic nuclei together

1. What is the SI symbol for the unit of temperature?

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2. What is the SI symbol for  $10^{-18}$ ?

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3. At what frequency does the power system in the United States operate?

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4. A wire carries a  $8.8 \mu\text{A}$  current. Calculate the number of electrons passing per second.

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5. In this problem we investigate charge neutrality.

The force between two electrical charges  $q_1$  and  $q_2$  at distance  $r$  can be calculated using Coulomb's law,

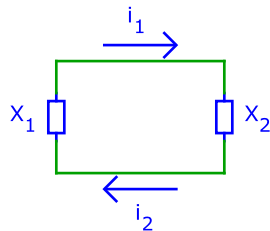
$$F_c = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2}$$

- a) Calculate the absolute value of the electrostatic force between an electron and a proton at distance  $r = 9 \text{ mm}$ .

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- b) From the assumption that the net charge on every circuit component remains zero at all times, it follows that currents  $i_1$  and  $i_2$  are equal in the circuit shown below. To get a better feel for the assumption, let's assume instead that  $i_1 = 0 \text{ A}$  and  $i_2 = 9 \text{ mA}$ . Then negatively charged electrons accumulate on  $X_2$ , leaving behind positively charged atomic nuclei on  $X_1$ . Calculate the attractive force (it is positive) between  $X_1$  and  $X_2$  after  $t = 1 \text{ s}$  for  $X_1$  and  $X_2$  at a distance of  $6 \text{ mm}$  (treat  $X_1$  and  $X_2$  as point charges).

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- c) How many loaded trucks weighing  $m = 40000 \text{ kg}$  each can you lift with this force?

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6. The number of free electrons (i.e. available to conduct current) in copper is about  $3 \times 10^{24} \text{cm}^{-3}$ . Calculate the carrier velocity in a copper wire with radius 4.9 mm that conducts 8.4 A. The SI unit for velocity is m/s. E.g. 1.4nm/s.

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The information is carried by the electric field which propagates at or near the speed of light, not by the electrons which travel much more slowly.

7. A computer operating at frequency  $f_s = 4.8 \text{GHz}$  executes instructions in  $T = 1/f_s$  seconds. Assuming that electrical signals propagate at the speed of light, how far do signals propagate in one cycle  $T$ ?

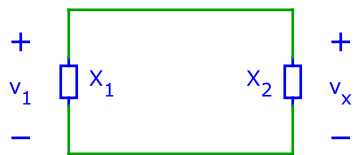
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Finite propagation delay must be considered in the design of computers whose size exceeds a few percent of this value.

8. In the circuit below find  $v_x$  for

$$v_1 = 1.3 \text{V} \quad \underline{\hspace{2cm}}$$

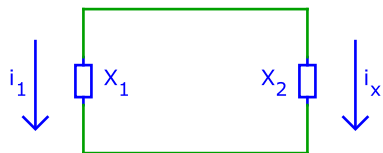
$$v_1 = -4.7 \text{V} \quad \underline{\hspace{2cm}}$$



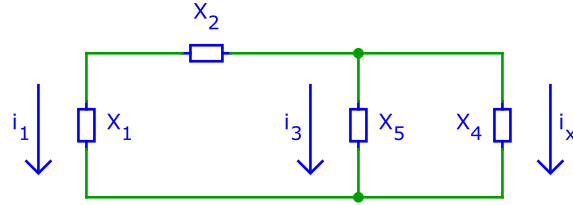
9. In the circuit below find  $i_x$  for

$$i_1 = 6.6 \text{mA} \quad i_x = \underline{\hspace{2cm}}$$

$$i_1 = -1.4 \mu\text{A} \quad i_x = \underline{\hspace{2cm}}$$



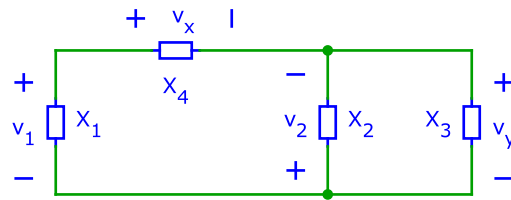
10. a) What is the number of nodes in the circuit below? \_\_\_\_\_  
 b) What is the number of branches? \_\_\_\_\_  
 c) For  $i_1=9.5 \mu\text{A}$  and  $i_3=-1.9 \mu\text{A}$  find  $i_x =$  \_\_\_\_\_



11. In the circuit below  $v_1 = -9.8 \text{ kV}$  and  $v_2 = -7.8 \text{ kV}$ . Find

$$v_x = \underline{\hspace{2cm}}$$

$$v_y = \underline{\hspace{2cm}}$$



12. For  $i_1 = 0.9 \text{ mA}$ ,  $i_2 = 5.6 \text{ mA}$ ,  $v_1 = 7.9 \text{ V}$  and  $v_2 = -4.5 \text{ V}$  calculate

$$i_x = \underline{\hspace{2cm}}$$

$$i_y = \underline{\hspace{2cm}}$$

$$i_z = \underline{\hspace{2cm}}$$

$$P_{X_1} = \underline{\hspace{2cm}}$$

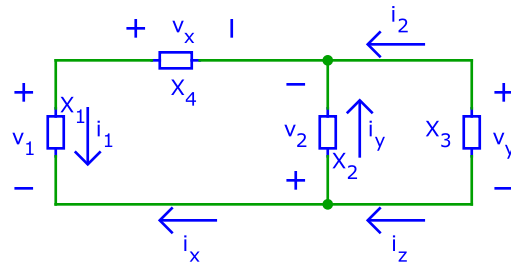
$$P_{X_2} = \underline{\hspace{2cm}}$$

$$P_{X_3} = \underline{\hspace{2cm}}$$

$$P_{X_4} = \underline{\hspace{2cm}}$$

$$\sum_{i=1}^4 P_{X_i} = \underline{\hspace{2cm}}$$

The notation  $P_{X_i}$  stands for the power dissipated in component  $X_i$ .



13. For how long will a 5.2 V battery with 6.8 kJ capacity power a flash light consuming

6.8 mW? Report your result in seconds, i.e. s, ks, Ms, etc.

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14. Rechargeable batteries are often rated in “Ampere-hours”, the number of hours that the battery can deliver a current of 1 A at its nominal voltage. Note that in reality the voltage would drop gradually, and small batteries cannot even deliver 1 A. Despite these shortcomings, the measure is convenient and popular.

A 12 V car battery is rated for 54 Ah. Assuming that the battery is initially fully charged and needs to be 20 percent full to start the engine, how long can the lights consuming 28 W total be left on (with motor off) before the car will no longer start? The SI unit for time is s, e.g. 34.7 ks.

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15. What are the power  $P_{V_1}$  and  $P_{I_2}$  dissipated in sources  $V_1$  and  $I_2$  for

$V_1$	$I_2$	$P_{V_1}$	$P_{I_2}$
9.0 mV	7.5 A	_____	_____
8.0 V	-7.6 $\mu$ A	_____	_____
-2.7 V	0.4 mA	_____	_____



16. What are the minimum and maximum positive voltages  $V_{\min}$  and  $V_{\max}$  that can be synthesized using either one or both voltage sources with values  $V_1 = 6.6$  V and  $V_2 = 2.3$  V?

$$V_{\min} = \underline{\hspace{2cm}}$$

$$V_{\max} = \underline{\hspace{2cm}}$$

## Chapter 5

# Operational Amplifiers [amp]

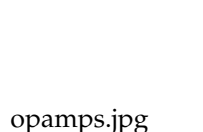
### 5.1 Introduction

The need for amplification is a common occurrence in sensor interfaces. For example, the output from a microphone cannot drive a speaker directly without prior amplification.

These amplifiers are electronic circuits themselves, usually made from transistors, resistors, and a few other circuit components. Amplifier design is a complex subject, but fortunately ready-made circuits are available called operational amplifiers. Operational amplifiers are very general devices that can be configured for many different applications using a technique called “feedback”.

### 5.2 Ideal Operational Amplifiers

Operational amplifiers, “opamps” for short, are very versatile building blocks that can be configured for many different functions with just a few external components. Figure 5.1 shows pictures of packaged operational amplifiers.

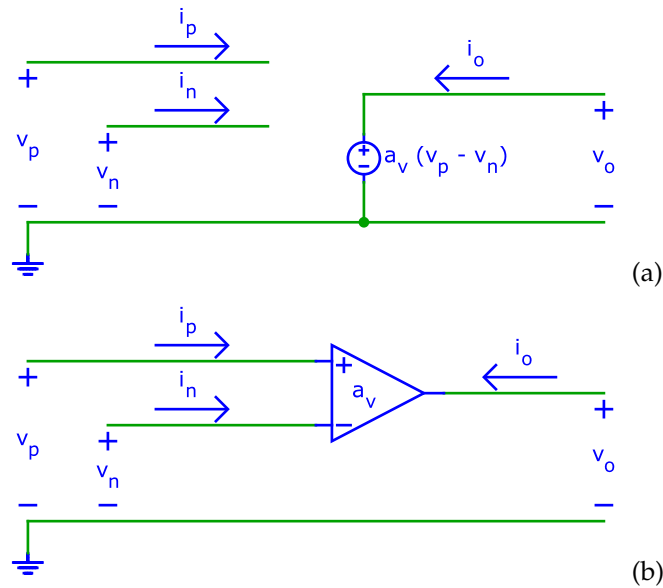
 **Figure 5.1** Operational amplifiers come in packages containing one or several individual amplifiers, each consisting of ten or more individual components. Details such as connection diagrams are described in data sheets and available from the manufacturer’s websites. Typical opamps about one Dollar in quantities of 1000.

Although differing in their details, the basic characteristics of all operational amplifiers are the same and described by the circuit in Figure 5.2a. The output  $v_o$  is set by the input voltages  $v_p$  and  $v_n$ :

$$v_o = a_v (v_p - v_n) \tag{5.1}$$

where  $a_v$  is the gain of the operational amplifier.

Ideal operational amplifiers have input currents  $i_p$  and  $i_n$  equal to zero and infinite gain



**Figure 5.2** (a) Equivalent circuit describing the operation of an operational amplifier with inputs  $v_p$  and  $v_n$  and output  $v_o = a_v(v_p - v_n)$ . (b) Symbol for an ideal opamp.

$a_v$ , i.e.

$$\begin{aligned} i_p = i_n = 0 \text{ A} \quad \text{and} \\ a_v \rightarrow \infty \end{aligned} \tag{5.2}$$

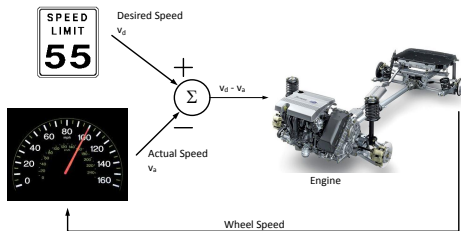
Real operational amplifiers come close to the ideal. Typical values for the input current are  $50 \text{ fA} \dots 10 \text{ }\mu\text{A}$ ; gains range from  $10^4$  to  $10^5$ , depending on the specific device used. In many applications, the errors arising from non-ideal opamp behavior are negligible.

In circuit diagrams, opamps are usually represented with the symbol shown in Figure 5.2b. Note that the symbol omits the ground terminal from the equivalent circuit. This sometimes causes confusion, since the output current  $i_o$  seems to come from nowhere. If of course comes from the ground terminal, as illustrated by the equivalent circuit.

Amplifiers with infinite or near infinite gain are not terribly useful. Fortunately a technique called “feedback” is available that lowers their gain to any desired value. We will study feedback in the next section and then apply it to operational amplifiers.

### 5.3 Feedback

Feedback is very common. For example, nature uses feedback to control population growth: a species taking overhand results in the decimation of its food source, which in turn results in famine and reduction of the population. Thermostats turn on the heater when the temperature falls below a threshold, and turn it off when the desired temperature is reached or exceeded. Governments try feedback to reduce greenhouse emissions with taxes on carbon dioxide. Any system that observes the consequences of its actions to adjust the actions themselves is using feedback.



**Figure 5.3** Example illustrating the use of feedback for driving a vehicle at a desired speed  $v_d$ . The driver compares the  $v_d$  to the actual speed  $v_a$  measured by the speedometer. If the difference  $v_d - v_a$  is positive, he accelerates the engine and decelerates otherwise. This in turn changes the wheel speed, which in turn results in an updated speedometer reading. Adjusting actions based on their result is called feedback.

Let's consider a concrete example. Suppose you are driving and would like to go at the speed limit. To do this, you compare the speed limit to the speedometer reading, as shown in Figure 5.3. Depending on which speed is larger, you accelerate or decelerate in an attempt of keeping the two speeds as close as possible. Since you are observing the outcome of your action (accelerating) to determine the action, you are using feedback to control your vehicles velocity.

Since you are subtracting the outcome (the odometer reading) from the desired speed, this type of feedback is called *negative feedback*. Switching the sign results in *positive feedback* and has completely different characteristics. Here we focus on negative feedback, returning to positive feedback only at the end of the chapter in Section 5.7 on page 44.

In our example we can make a few observations that apply to (negative) feedback systems in general:

- Feedback minimizes the difference between a desired quantity (the speed limit) and the feedback signal (the speedometer reading), ideally driving it to zero.
- Feedback uses amplification (the car engine) to accomplish this goal. The higher the gain (more powerful the engine), the smaller the difference. For example, a sports car with a very powerful engine is capable to quickly adjust to increases of the speed limit.
- The exact value of the amplification (power of the engine) is not important, as long as it is high. Any car with a powerful engine is capable of keeping a speed of 60 km/h, independent of its specific horsepower rating.

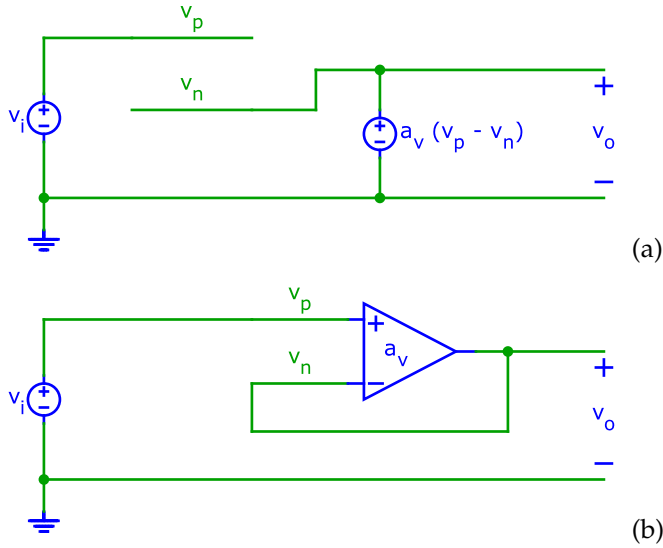
We will now apply feedback to electronic amplifiers, observing the same basic principles summarized above.

## 5.4 Feedback Amplifiers

### 5.4.1 Non-inverting Operational Amplifier

Figure 5.4 shows an ideal operational amplifier with negative feedback: the output  $v_o$  is brought back to the negative summing terminal  $v_n = v_o$  and subtracted from the input  $v_p = v_i$ . If we assume that initially  $v_i = v_o = 0\text{ V}$ , the difference  $v_d = v_i - v_o$  at the





**Figure 5.4** Operational amplifier with negative feedback. (a) Equivalent circuit, (b) using opamp symbol.

amplifier input is zero, resulting in  $v_o = 0\text{ V}$  regardless of the amplifier gain  $a_v$ . If now the input  $v_i$  is increased to  $1\text{ V}$   $v_d$  increases also. The amplifier amplifies  $v_d$ , in turn increasing  $v_o$ . This continues until  $v_o$  reaches the value of  $v_i$ . Now  $v_d = 0\text{ V}$ , and the amplifier output no longer changes. The same happens for any other value of  $v_i$ : because of the feedback the output  $v_o$  follows the input  $v_i$ . The voltage gain of this circuit is

$$A_v = \frac{v_o}{v_i} = 1. \quad (5.3)$$

This value is very different from the gain of the amplifier,  $a_v$ , which is very large (ideally infinite). The amplifier gain  $a_v$  without feedback is often referred to as *open-loop gain*, while the resulting gain  $A_v$  with feedback applied is the *closed-loop gain*.

A puzzle remains: how can the output  $v_o$  be  $1\text{ V}$  when its input,  $v_d = 0\text{ V}$ ? We can gain insight by analyzing the circuit assuming finite open-loop gain  $a_v$ . Substituting  $v_i^+ = v_i$  and  $v_i^- = v_o$  in Equation 5.1 we get

$$v_o = a_v(v_i - v_o) = a_v v_d \quad (5.4)$$

or

$$v_d = v_p - v_o = \frac{v_o}{a_v} \quad (5.5)$$

The voltage difference  $v_d$  at the amplifier input goes to zero as  $a_v \rightarrow \infty$ . For a typical opamp with gain  $a_v = 100,000$ ,  $v_d = 10\ \mu\text{V}$  for  $v_o = 1\text{ V}$  and is relevant only in very high precision circuits.

Finite open-loop gain  $a_v$  also causes an error in the closed-loop gain  $A_v$ . Solving Equa-

tion 5.4 for  $v_o$  and dividing by  $v_i$  we get

$$\begin{aligned}
 A_v &= \frac{v_o}{v_i} = \frac{a_v}{1 + a_v} = \frac{1}{1 + \frac{1}{a_v}} \\
 &\approx \underbrace{1}_{\text{Correct Result}} - \underbrace{\frac{1}{a_v}}_{\text{Error}} \quad (a_v \gg 1)
 \end{aligned} \tag{5.6}$$

For open-loop gain  $a_v$  in excess of 100, the error of the closed-loop gain  $A_v$  is less than one percent. Since most practical operational amplifiers have much higher gain, the gain error is negligible in most applications and the approximation  $a_v \rightarrow \infty$  is justified. Note that the exact value of  $a_v$  does not need to be known to obtain a precise value of  $A_v$ .

This is a very important property of feedback: accurate closed-loop gain only requires the amplifier gain to be high, without requiring it to have a precise or even constant value. The data sheets of operational amplifiers typically specify a range for the open-loop gain, e.g. 50,000 ... 250,000 that takes into account factors such as manufacturing tolerances and temperature change. Practical electronic systems make liberal use of feedback to reduce their sensitivity to these variations.

Insensitivity of the closed-loop to the precise value of the amplification is important also in the example of driving at a constant speed discussed in Section 5.3 on page 38: imagine how impractical it would be if you needed to know the engine power to be able to drive at the speed limit. Indeed, this would be the case without feedback: if you could not observe the odometer or another effect informing you about your actual speed, it would be next to impossible to maintain a desired velocity.

Unless otherwise marked, we assume that operational amplifiers are ideal and hence

$$v_d = v_p - v_n = 0, \tag{5.7}$$

except when noted otherwise. In general, it is often a good idea to first investigate circuit operation assuming ideal opamps, followed by an analysis of the effect of nonidealities. Such verification can frequently be performed by circuit simulation. Vendors of operational amplifiers usually offer SPICE models for their designs that include nonidealities such as non-zero input current and finite gain.

The amplifier discussed above and shown in Figure 5.4 has unity closed-loop gain,  $A_v = 1$ . Often of course a larger gain is required, and Figure 5.5 shows a circuit that accomplishes this.

Resistor ratio sets gain, very precise if resistors alike and at same temperature

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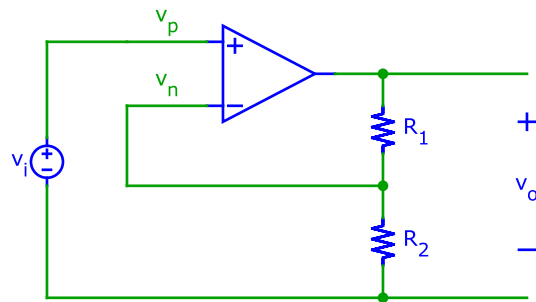
#### EXAMPLE 5-1: Temperature Sensor

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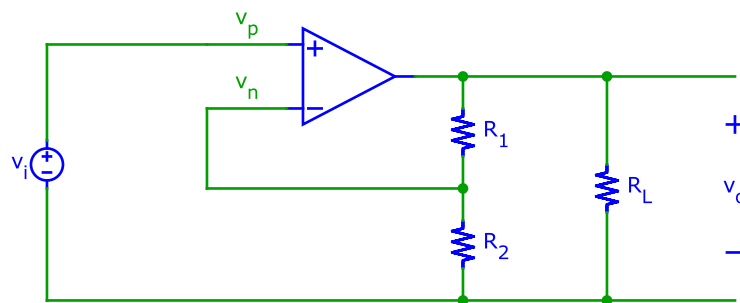


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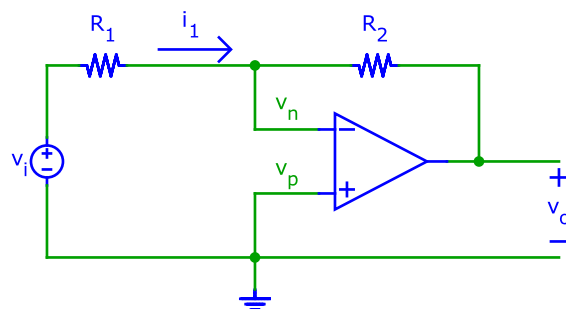
#### EXAMPLE 5-2: Addition, Subtraction, and Scaling



**Figure 5.5** non-inverting opamp example, gain = 10



**Figure 5.6** design example:  $A_v = 5 \rightarrow R_1/R_2 = 4$ , consider power diss in fb network &  $i_{out}$



**Figure 5.7** inverting

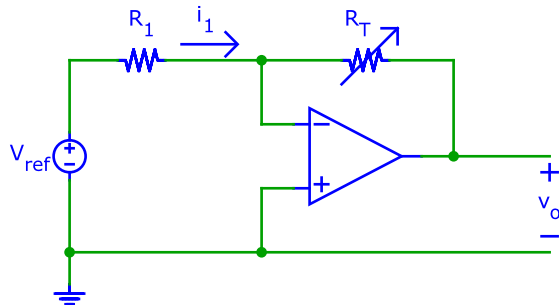


Figure 5.8 Temperature sensor circuit

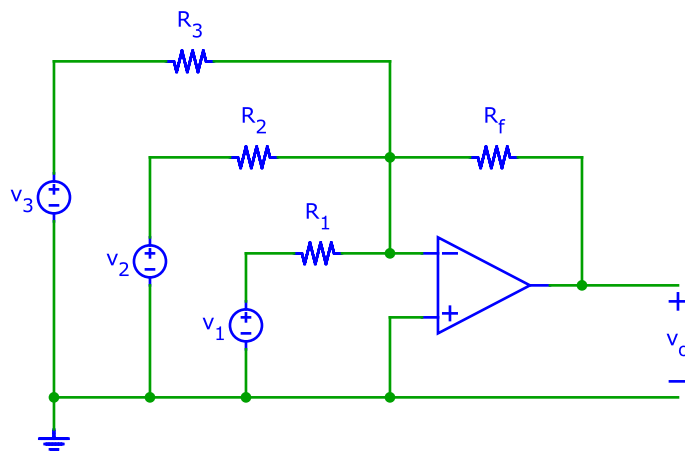


Figure 5.9 multiple inputs

Design an amplifier with four inputs  $v_1, v_2, v_3$  and  $v_4$  and output

$$v_o = A_1 v_1 + A_2 v_2 - A_3 v_3 - A_4 v_4 \tag{5.8}$$

where  $A_1 = 5, A_2 = 3, A_3 = 2$  and  $A_4 = 7$ .

$$v_o = v_1 \frac{R_a || R_2}{R_1 + R_a || R_2} \frac{R_b + R_3 || R_4}{R_3 || R_4} + v_2 \frac{R_a || R_1}{R_2 + R_a || R_1} \frac{R_b + R_3 || R_4}{R_3 || R_4} - v_3 \frac{R_3}{R_b} - v_4 \frac{R_4}{R_b} \tag{5.9}$$

In this circuit, the summation of  $v_1$  and  $v_2$  is accomplished by the resistive circuit formed by  $R_1, R_2,$  and  $R_a$ . Since the network is passive, it attenuates  $v_1$  and  $v_2$ . The amplifier then a the resulting attenuated signal  $v_{i1} \dots$

NVA example

## 5.5 Amplifier Input and Output Resistance

model input as resistor model output as Thevenin equivalent finite  $r_o$  instrumentation amplifier current source input example pos resistor example

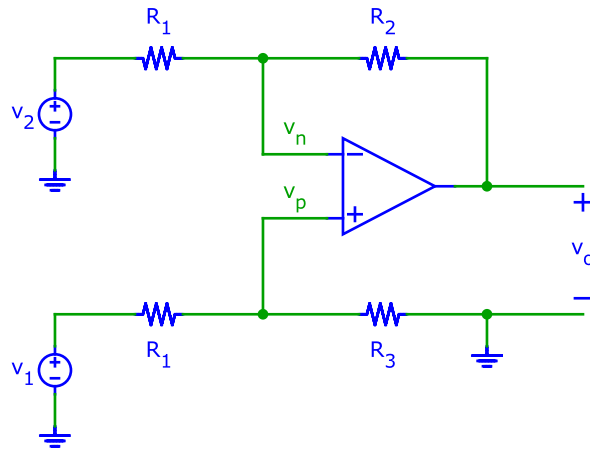


Figure 5.10 Difference Amplifier.

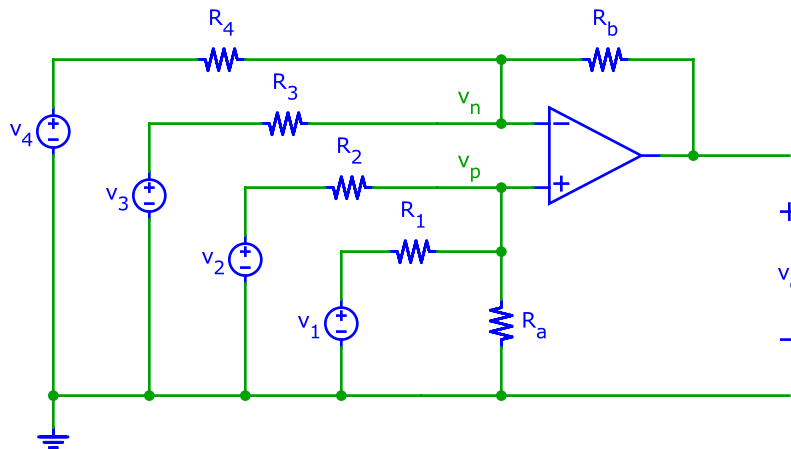


Figure 5.11 Circuit for voltage addition, subtraction, and scaling.

## 5.6 Practical Amplifiers

supply offset

## 5.7 Positive Feedback

compare w/ negative rule of thumb: if loop goes back to - terminal, it's probably neg feedback complex issue, entire courses on this alone

Usually we recognize negative feedback by a connection that feeds all or a fraction of the output of an amplifier back to its negative input. In general the analysis of feedback is more complicated, as the next example shows.

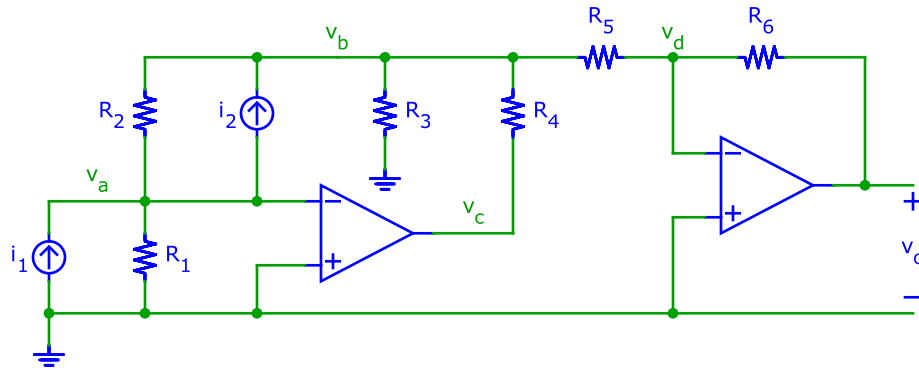


Figure 5.12 NVA example

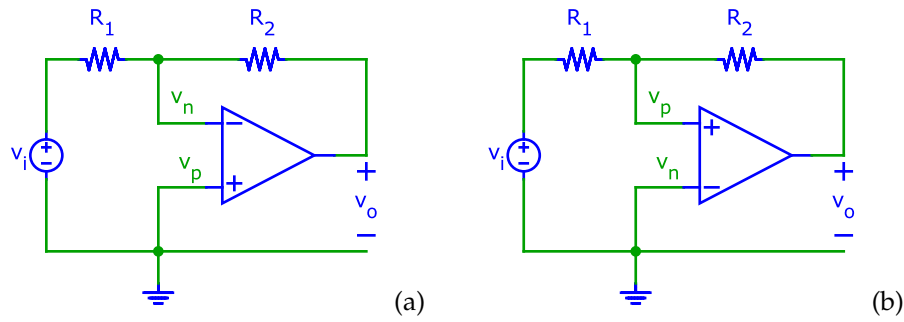


Figure 5.13 pos vs neg feedback

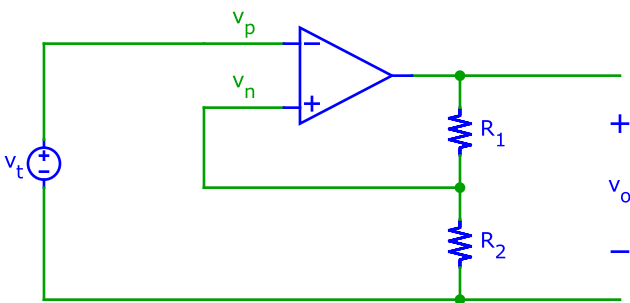
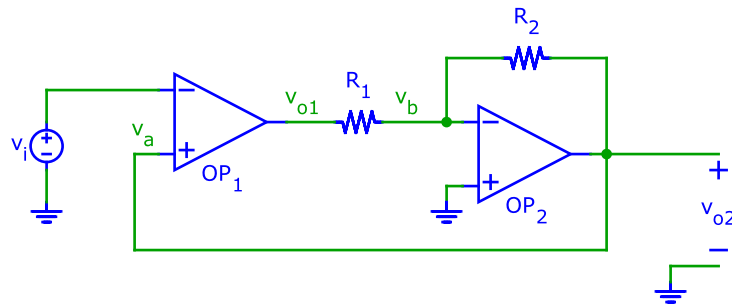


Figure 5.14 Thermostat example

**EXAMPLE 5-3: Negative Feedback Example?**

**Figure 5.15** Two opamps with neg feedback

Find the value of the closed-loop gains of the circuit in Figure 5.15,  $A_{v1} = v_{o1}/v_i$  and  $A_{v2} = v_{o2}/v_i$ .

In the first step, we need to find all feedback loops and determine for each one if it is positive or negative. Resistor  $R_2$  samples the output  $v_{o2}$  of amplifier  $OP_2$  and returns a portion back to its negative input terminal  $v_b$ . The feedback around this loop is negative.

The output  $v_{o1}$  from  $OP_1$  is amplified by  $OP_2$  and then fed back to input  $v_a$  of  $OP_1$ . Since the feedback is to the positive input of  $OP_1$ , it seems as if this feedback is positive. This, however, is not the case since  $OP_2$  inverts the output  $v_{o1}$  from  $OP_1$  before feeding it back to the positive input  $v_a$  of  $OP_1$ . Feeding back to the negative input of  $OP_1$  would result in a second inversion and overall positive feedback.

Now that we have found the feedback loops and determined that they are both negative, we can proceed in the usual fashion, exploiting that the difference between the inputs of operational amplifiers is zero for negative feedback, i.e.  $v_{o2} = v_a = v_i$  and  $v_b = 0\text{V}$ . Since the gain of  $OP_2$  is  $A_2 = v_{o2}/v_{o1} = -R_2/R_1$ , we get  $v_{o1} = v_{o2}/A_2 = -v_{o2}R_2/R_1$ . Using  $v_{o2} = v_{o1}$  and dividing by  $v_{o1}$  we get  $A_{v1} = v_{o1}/v_i = -R_2/R_1$ , the inverse of the gain of  $OP_2$  alone.

## Problems

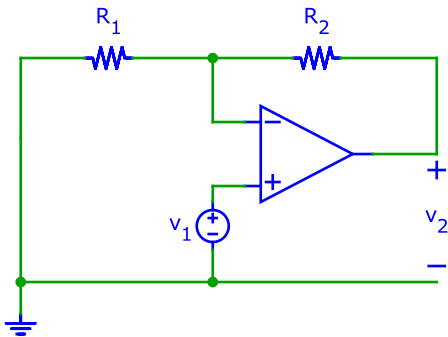
Assume that all operational amplifiers are ideal, unless specified otherwise.

- add tons of problem analyzing gain of many configurations
- calculate iout
- finite av with  $f \neq 1$ , error for large Av gets big  $\rightarrow$  limit Av to 100 typ
- min av for  $<0.1\%$  gain error, vary  $R_2$  (Av)
- error from finite Rin, vary av, Av, R1
- $R_{in} = f(av)$  for trans-R amp
- Current source

- Neg Rin

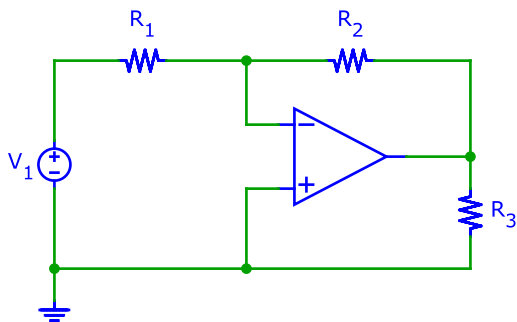
1. Calculate the ratio  $v_2/v_1$ . Assume that the operational amplifier is ideal. Use  $R_1 = 4.5 \text{ k}\Omega$ ,  $R_2 = 6.9 \text{ k}\Omega$ .

$$v_2/v_1 = \underline{\hspace{2cm}}$$



2. Calculate the power dissipated in resistor  $R_3$ . Assume that the operational amplifier is ideal. Parameter:  $V_1 = 6.4 \text{ V}$ ,  $R_1 = 7.5 \text{ k}\Omega$ ,  $R_2 = 8.6 \text{ k}\Omega$ ,  $R_3 = 4.9 \text{ k}\Omega$ .

$$P_{R_3} = \underline{\hspace{2cm}}$$



3. In Section 5.4.1 on page 39 we derived an expression of for the error in the closed-loop gain  $A_v$  as a function of finite open-loop gain  $a_v$  for a unity gain amplifier. Now we generalize this result to arbitrary closed-loop gain.

- a) Derive an algebraic expression of the closed-loop gain  $A_v = v_o/v_i$  of the circuit below as a function of the open-loop gain  $a_v$  and feedback resistors  $R_1$  and  $R_2$ . Write your result in the form

$$A_v = \frac{R_2}{R_1 + R_2} \times f(a_v, R_1, R_2)$$



You will recognize the first factor in the equation as the closed-loop gain  $A_{v\infty}$  of the circuit for  $a_v \rightarrow \infty$ . The term  $f(a_v, R_1, R_2)$  models the error incurred for finite  $a_v$  and you need to find it in terms of  $a_v$ ,  $R_1$ , and  $R_2$ .

b) Bring your result into the form

$$A_v \approx \frac{R_2}{R_1 + R_2} (1 - \epsilon) \quad (a_v \gg 1)$$

using the approximation

$$\frac{1}{1+x} \approx 1 - x \quad (x \ll 1).$$

Find the relative gain error  $\epsilon$  as a function of  $a_v$ ,  $R_1$ , and  $R_2$ .

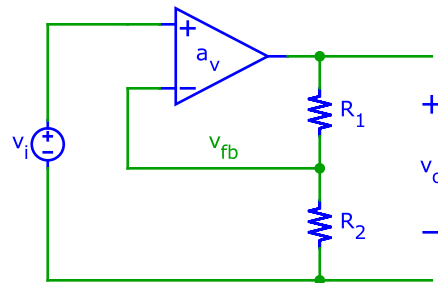
c) In the final step, replace the dependence on  $R_1$  and  $R_2$  in the above result by rewriting  $\epsilon$  as a function only of the closed-loop gain  $A_{v\infty}$  of the circuit for  $a_v \rightarrow \infty$  and the open-loop gain  $a_v$  of the amplifier. You have now found a very useful result: evidently, keeping the closed-loop gain error  $\epsilon$  small requires not only a large open-loop gain  $a_v$ , but large "excessive gain",  $a_v / A_{v\infty}$ . In other words, the open-loop gain must be much larger than the desired closed-loop gain, i.e.  $a_v \gg A_{v\infty}$ .

This is one reason why it is rarely possible to achieve a gain in excess of about 100 with a single operational amplifier. For higher gain, several amplifiers must be cascaded. For example, the overall gain of two cascaded amplifiers with gain 50 each is  $50^2 = 2500$ .

d) What is the minimum open-loop gain  $a_{v,\min}$  required to keep the error of the closed-loop gain  $A_v = 66$  less than 0.1%?

$$a_{v,\min} = \underline{\hspace{2cm}}$$

Adjusting the values of  $R_1$  and  $R_2$  slightly to compensate for finite  $a_v$  is impractical because of the variation of  $a_v$  with temperature and other uncontrollable effects.



4. Calculate  $v_o$  for Assessment Problem 5.4 in N & R with  $R_x = 11.7 \text{ k}\Omega$ . For simplicity assume that the amplifier is ideal and that the output voltage does not saturate due to the finite supply voltage.

$$v_o = \underline{\hspace{2cm}}$$

5. The open-loop gain  $a_v$  of an operational amplifier is defined as the ratio of output voltage  $v_o$  over the input differential voltage  $v_1 - v_2$ ,

$$a_v = \frac{v_o}{v_1 - v_2} \quad (5.10)$$

Ideal operational amplifiers have infinite gain,  $a_v \rightarrow \infty$ . Then the closed loop gain

$$G_v = \frac{v_o}{v_i} \quad (5.11)$$

is a function only of the feedback network ( $R_1$  and  $R_2$  in the circuit below), and does not depend on  $a_v$ . Typical gains of real operational amplifiers are in the range  $10^4 \dots 10^7$ . Because of the finite open loop gain, the closed loop gain  $G_v(a_v)$  is slightly off from its ideal value,  $G_{v\infty} = G_v(a_v \rightarrow \infty)$ .

Let's analyze this effect. Derive first an expression for  $G_v(a_v)$  as a function of  $R_1$ ,  $R_2$ , and  $a_v$ . Verify that for  $a_v \rightarrow \infty$  you obtain the expected expression for the gain for a non-inverting amplifier. Then solve for the gain error  $\epsilon$  as a function of finite open loop gain,  $a_v$ , which is defined as

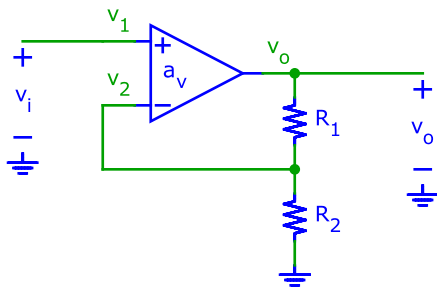
$$\epsilon = \left| \frac{G_v(a_v)}{G_{v\infty}} - 1 \right| \quad (5.12)$$

Substitute  $G_{v\infty}$  for  $R_1$  and  $R_2$  so that the result is a function only of  $a_v$  and  $G_{v\infty}$ .

From the equation we conclude that for small error we want  $a_v \gg G_{v\infty}$ , a condition that because of the high value of  $a_v$  for most operational amplifiers is usually met and justifies common practice to evaluate circuits assuming  $a_v \rightarrow \infty$ . The validity of the assumptions can be checked for example by simulating the circuit with an operational amplifier with the actual gain. Use  $G_{v\infty} = 94$  to compute the values in the table below.

$a_v = 10^6$	$\epsilon =$	_____
$a_v = 10^4$	$\epsilon =$	_____
$a_v = 10^2$	$\epsilon =$	_____

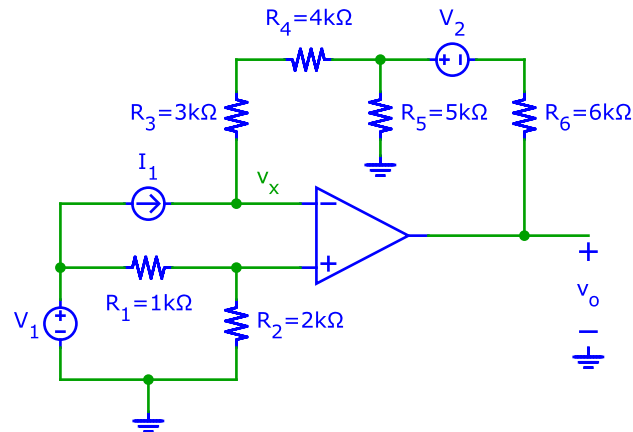
Unless otherwise specified you may always assume in EE40 that the gain of operational amplifiers is "infinite".



6. Find the voltage at node  $v_x$  for  $V_1 = 1.0\text{ V}$ ,  $V_2 = 3.5\text{ V}$ ,  $I_1 = 1.4\text{ mA}$ .

Note: the solution of this problem is trivial!

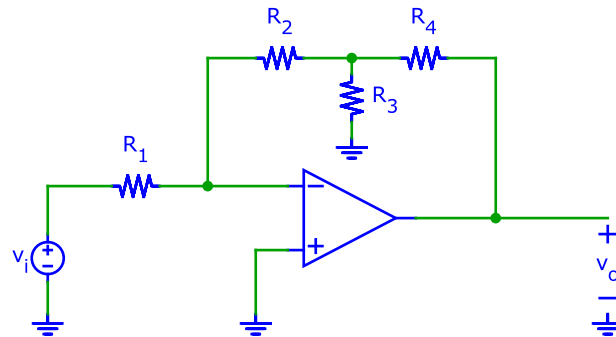
$v_x =$  \_\_\_\_\_



7. Large resistor ratios are required if a big closed-loop gain  $|v_o/v_i|$  is desired in an operational amplifier circuit. E.g. setting the gain of an inverting amplifier to -1000 requires resistors with values  $10\text{ k}\Omega$  and  $10\text{ M}\Omega$  or multiples thereof. Such large resistor values and ratios often exhibit unacceptably large variation, causing error in the closed-loop gain. In integrated circuits, large resistor values occupy a large area, increasing fabrication cost.

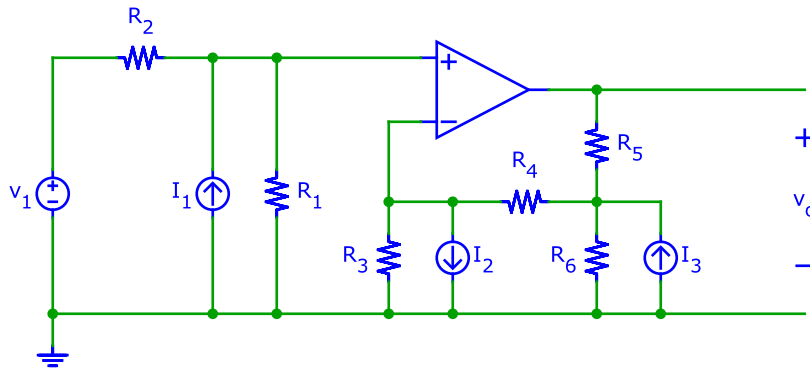
The circuit shown below requires resistor ratios that are much smaller than the closed-loop gain. Find the ratio  $R_2/R_1$  such that  $v_o/v_i = -550$ . Use  $R_1 = R_3 = 2\text{ k}\Omega$  and  $R_2 = R_4$ .

$R_2/R_1 =$  \_\_\_\_\_



8. Find the value of  $v_o$  for  $V_1 = 4.6\text{ V}$ ,  $I_1 = 9.8\text{ mA}$ ,  $I_2 = 3.4\text{ mA}$ ,  $I_3 = 5.1\text{ mA}$  and  $R_1 = R_2 = R_3 = R_4 = R_5 = R_6 = 1\text{ k}\Omega$ .  
Suggestion: use superposition.

$v_o =$  \_\_\_\_\_

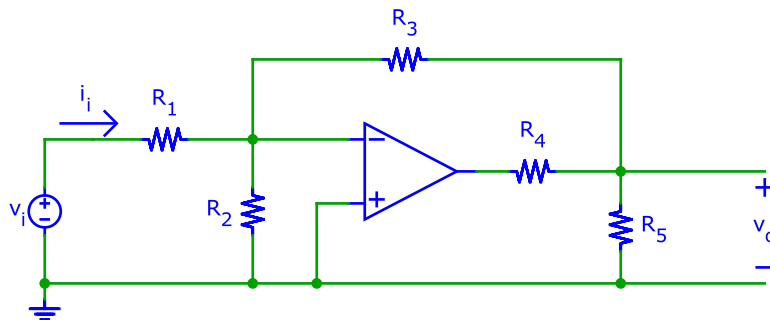


9. Calculate the closed-loop gain  $A_v$  and input resistance  $R_i$  of the circuit below. Use  $R_1 = 4.9\text{ k}\Omega$ ,  $R_2 = 7.0\text{ k}\Omega$ ,  $R_3 = 4.5\text{ k}\Omega$ ,  $R_4 = 6.3\text{ k}\Omega$ ,  $R_5 = 5.9\text{ k}\Omega$ .

Note: this problem can be solved by inspection.

$$A_v = v_o/v_i = \underline{\hspace{2cm}}$$

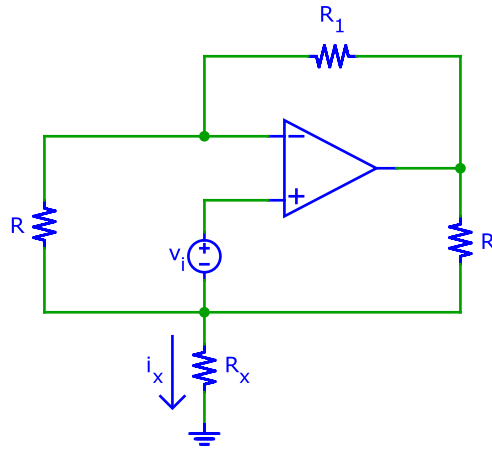
$$R_i = v_i/i_i = \underline{\hspace{2cm}}$$



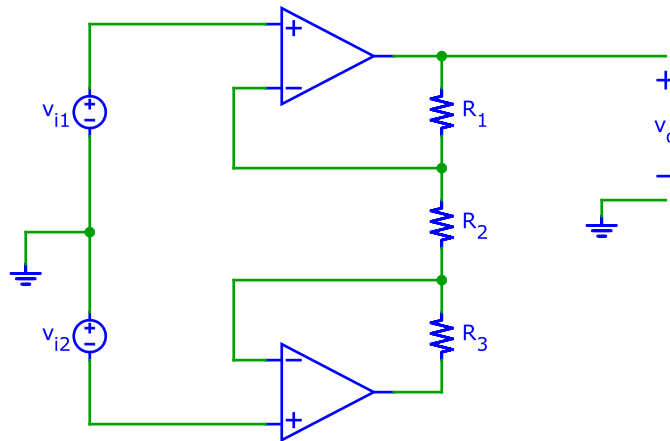
10. The current  $i_x$  in the circuit below is independent of the value of  $R_x$ , i.e. the circuit realizes a current source  $i_x$ . Find  $i_x/v_i$  for  $R = 7.2\text{ k}\Omega$  and  $R_1 = 35\text{ k}\Omega$ .

Note: the unit of the result is  $[S] = [1/\Omega]$  (Siemens).

$i_x/v_i =$  \_\_\_\_\_

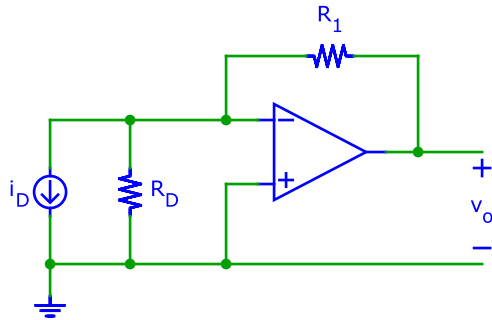


11. The circuit below is part of an instrumentation amplifier. Calculate  $v_o$  for  $R_1 = 5.2\text{ k}\Omega$ ,  $R_2 = 7.5\text{ k}\Omega$ ,  $R_3 = 9.0\text{ k}\Omega$  and  $V_{i1} = 9.2\text{ V}$ ,  $V_{i2} = 9.3\text{ V}$ .  
Suggestion: use circuit insight, not node voltage analysis.  
 $v_o =$  \_\_\_\_\_



12. Photodiodes are often used as light sensors (e.g. to measure ambient illumination or as receivers in fiberoptic communication systems). From a circuit perspective a photodiode behaves just like a current source  $i_D$  with output resistance  $R_D$ .  
In practice, a voltage output is usually preferred. Without the amplifier circuit shown below, the change of the voltage across the photodiode is small and further depends on the value of  $R_D$ , which itself is a function of the signal. The amplifier solves both problems. Find the value of  $R_1$  resulting in a transresistance  $v_o/i_D$  of  $5.8\text{ V}/\mu\text{A}$ .

$$R_1 = \underline{\hspace{2cm}}$$



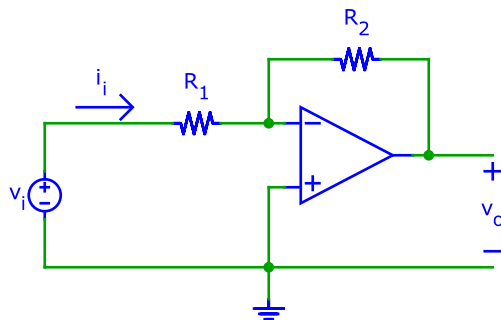
Such circuits have many applications, e.g. in electronic devices to measure ambient illumination and adjust the intensity of LCD back-lighting accordingly. Combined with an (LED) light source, this circuit can be used in an intruder alarm. Options for the class project.

13. The current into the input of an ideal operational amplifier is ideally zero. Many operational amplifiers come close to this ideal, with leakage currents in the pA or even fA range.<sup>1</sup> This corresponds to a very high input resistance (TΩ range) that is usually negligible.

High input resistance of the open-loop amplifier (i.e. without feedback resistors) does not always translate into the same characteristic for the closed-loop configuration. Specifically, non-inverting closed-loop configurations retain the high input resistance of the open-loop amplifier, but in inverting configurations the input resistance is determined by the feedback network and therefore much smaller.

The diagram below shows a test circuit. Calculate the input resistance  $R_i = v_i/i_i$  for  $R_1 = 2.5 \text{ k}\Omega$  and  $R_2 = 52 \text{ k}\Omega$ .

$$R_i = \underline{\hspace{2cm}}$$



<sup>1</sup>This depends on the type of amplifier. In particular, amplifiers with MOS or JFET inputs have very small input currents, while the input current of BJT amplifiers is much larger, often in the  $\mu\text{A}$  range. BJT amplifiers have other advantages, such as higher speed or output current capability and lower offset voltage. If low input resistance is critical, an operational amplifier with either MOS or JFET inputs is preferred.

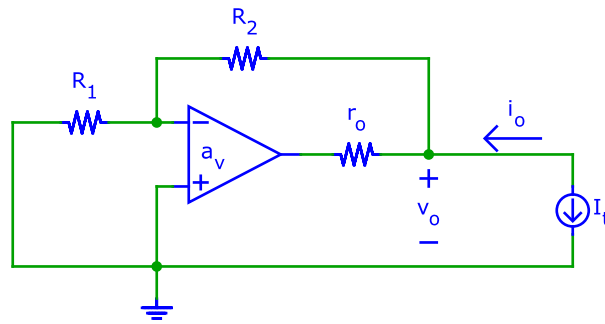
14. Ideally the output of an operational amplifier behaves like an ideal voltage source with zero output resistance, i.e. the output voltage is independent of the current supplied by the amplifier. In practice, the output resistance  $r_o$  of an operational amplifier is finite, as is its gain  $a_v$ . Here we examine the effect on the output resistance  $R_o = v_o/i_o$  of the closed-loop amplifier. To evaluate  $R_o$  we connect a test source  $I_t$  to the output of the amplifier, as shown in the diagram below. Calculate  $R_o$  for  $r_o = 184 \Omega$ ,  $R_1 = 14 \text{ k}\Omega$  and  $R_2 = 61 \text{ k}\Omega$ .

Note that  $r_o$  is the open-loop output resistance of the operational amplifier, i.e. the output resistance without feedback applied. Resistance  $R_o$  is the closed-loop output of the circuit with feedback.

$$\begin{array}{ll} a_v = 1\text{e}6 & R_o = \underline{\hspace{2cm}} \\ a_v = 1\text{e}4 & R_o = \underline{\hspace{2cm}} \\ a_v = 1\text{e}2 & R_o = \underline{\hspace{2cm}} \end{array}$$

If you solved this problem correctly, you will notice that high-gain feedback lowers the output resistance. This is a very desirable and often used property of feedback. In fact, it is difficult to design operational amplifiers with open loop output resistance  $r_o$  less than a few Ohms. Fortunately this value can be reduced to the  $\text{m}\Omega$  range or less using feedback.

For example, lab power supplies and voltage regulator chips use feedback to achieve very low output resistance and hence closely approximate the behavior of ideal voltage sources.

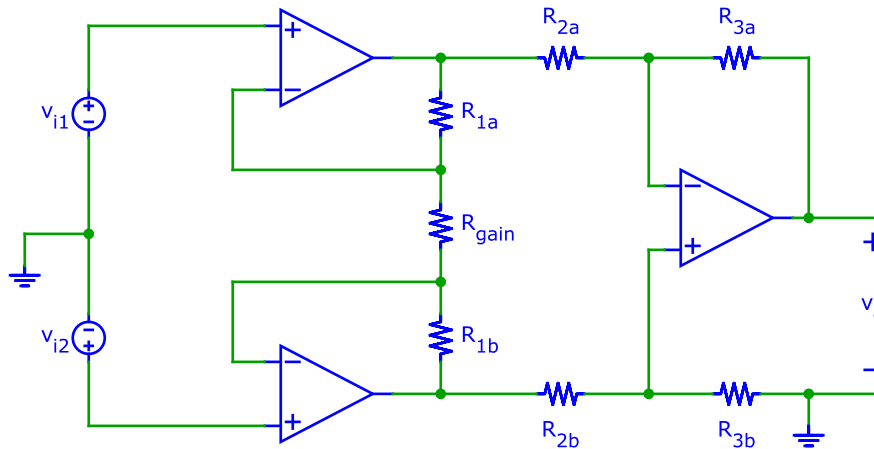


15. Sensor applications frequently call for amplification of a voltage difference. We have seen such a situation in the strain gage laboratory. Many sensors further have a high output resistance, requiring an amplifier with very large input resistance. The non-inverting amplifier has this characteristic, but the input resistance of the inverting amplifier is too small for many sensor applications and because of this cannot be used directly.

A solution is shown below that employs two non-inverting amplifiers to buffer the input signal, followed by an inverting stage that forms the difference. This configuration is usually referred to as “instrumentation amplifier”. Since it has many uses,

integrated circuits containing the complete structure are available from several manufacturers. The resistor  $R_{\text{gain}}$  is usually external and used to adjust the circuit gain. Calculate the gain  $A_v = v_o / (v_{i1} - v_{i2})$  for  $R_{\text{gain}} = 8 \text{ k}\Omega$ ,  $R_1 = R_{1a} = R_{1b} = 13 \text{ k}\Omega$ ,  $R_2 = R_{2a} = R_{2b} = 18 \text{ k}\Omega$  and  $R_3 = R_{3a} = R_{3b} = 80 \text{ k}\Omega$ .

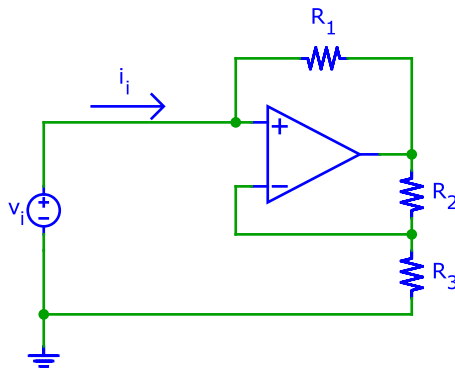
$A_v =$  \_\_\_\_\_



16. Calculate the input resistance  $R_i = v_i / i_i$  of the circuit below. Use  $R_1 = 7.7 \text{ k}\Omega$ ,  $R_2 = 19 \text{ k}\Omega$  and  $R_3 = 11 \text{ k}\Omega$ .

$R_i =$  \_\_\_\_\_

If you cannot get this right, check the sign of your answer.

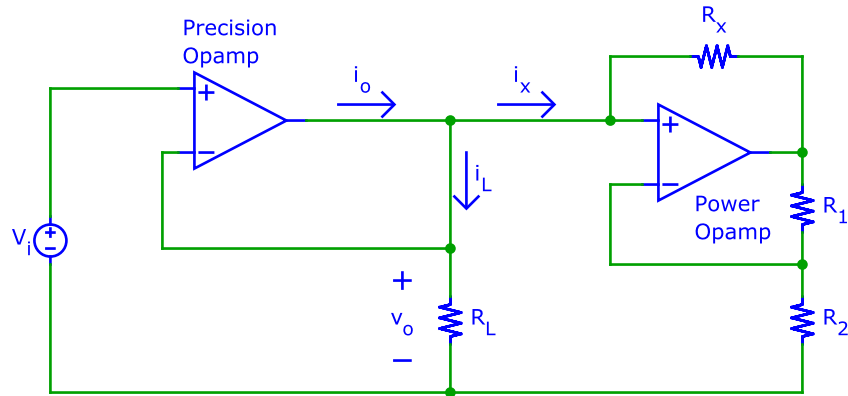


17. Precision opamps come particularly close to the specifications of “ideal” amplifiers, but usually cannot drive low resistance loads  $R_L$ . In the circuit below, the “precision opamp” sets  $v_o$ , while the “power opamp” delivers the load current  $i_L$ . Determine  $R_1$  such that  $i_o = 0$ . Hint: this condition is met when  $i_L + i_x = 0$ .



Parameter:  $R_2 = 4.4 \text{ k}\Omega$ ,  $R_L = 99.0 \Omega$ ,  $R_x = 75.3 \Omega$ .

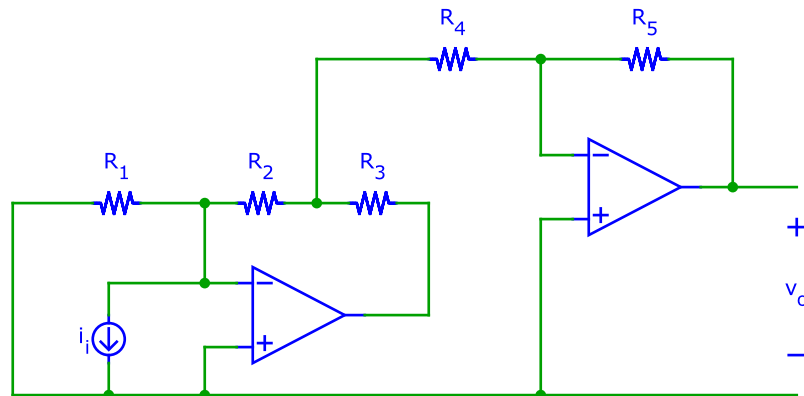
$R_1 =$  \_\_\_\_\_



18. Calculate the closed-loop gain  $\frac{v_o}{i_i}$  of the circuit below. Assume that the operational amplifier is ideal. Remember to include the correct unit with your result.

Parameter:  $R_1 = 2.0 \text{ k}\Omega$ ,  $R_2 = 2.6 \text{ k}\Omega$ ,  $R_3 = 6.7 \text{ k}\Omega$ ,  $R_4 = 4.5 \text{ k}\Omega$ ,  $R_5 = 2.1 \text{ k}\Omega$ .

$\frac{v_o}{i_i} =$  \_\_\_\_\_



19. In the circuit below,  $R_T$  is a temperature dependent resistor with value

$$R_T(T) = R_o (1 + \alpha T)$$

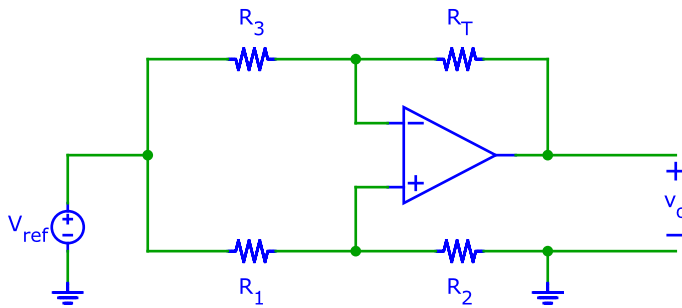
where  $T$  is the temperature in degrees Celsius [C],  $R_o = 4.3 \text{ k}\Omega$  and  $\alpha = 0.05 \text{ C}^{-1}$ .

Find the values of  $R_1$  and  $R_3$  such that the output voltage  $v_o$  of the circuit is

$$\begin{aligned} v_o(T) &= 1 \text{ V} && \text{for } T = 0 \text{ C and} \\ v_o(T) &= 2 \text{ V} && \text{for } T = 100 \text{ C.} \end{aligned}$$

Use  $R_2 = 7.9 \text{ k}\Omega$  and  $V_{ref} = 8.5 \text{ V}$ . Note: results may be negative.

$$\begin{aligned} R_1 &= \underline{\hspace{2cm}} \\ R_3 &= \underline{\hspace{2cm}} \end{aligned}$$



In a practical application you would connect  $v_o$  to the input of a microcontroller for display and function (e.g. thermostat).

20. A simple speaker model consists of a resistor  $R_1 = 9.3 \Omega$  in series with an inductor  $L_1 = 9.9 \text{ mH}$ . The speaker is driven by a “bad” amplifier that has a DC (constant) output  $V_{dc} = 8.6 \text{ V}$  superimposed on the audio signal. For the analysis below assume that the audio signal is a pure sinewave with frequency  $f_1 = 440 \text{ Hz}$  and amplitude  $V_1 = 3.1 \text{ V}$  superimposed on  $V_{dc}$ .

Calculate the average  $P$  and reactive  $Q$  power delivered by the amplifier.

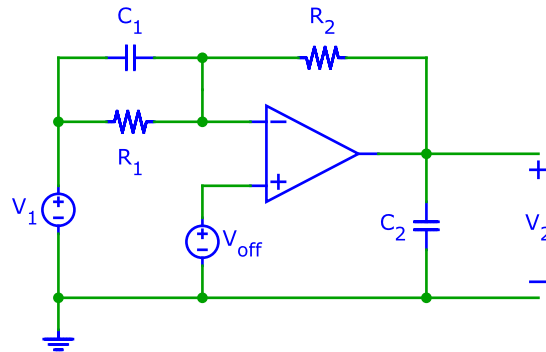
$$\begin{aligned} P &= \underline{\hspace{2cm}} \\ Q &= \underline{\hspace{2cm}} \end{aligned}$$

21. Practical operational amplifiers suffer from numerous nonidealities. In this problem we examine the effect of offset and finite bandwidth. We analyze both effects separately. Use the following component values:  $R_1 = 6.0 \text{ k}\Omega$ ,  $R_2 = 6.8 \text{ k}\Omega$ ,  $C_1 = 7.4 \text{ nF}$ ,  $C_2 = 8.1 \text{ nF}$ .

**Important:** Simplify your results for parts (b) and (c) so that they are in the form of a ratio of polynomials of  $s$ , i.e.

$$H(s) = G \frac{\sum_{i=0}^N a_i s^i}{\sum_{k=0}^M b_k s^k} \quad (5.13)$$

with  $a_0 = b_0 = 1$ . Simplify all your results.



- a) In this part we just consider offset, modeled by the source  $V_{\text{off}}$ . Convince yourself that, assuming that the opamp is otherwise ideal,  $V_2 = 0\text{ V}$  for  $V_1 = 0\text{ V}$  and  $V_{\text{off}} = 0\text{ V}$ . Practical opamps have  $V_{\text{off}} \neq 0\text{ V}$ . The actual value is random (varies from part to part); from measurements you find that for your particular opamp  $V_{\text{off}} = 6.2\text{ mV}$ . Calculate the value of  $V_1$  such that  $V_2 = 0\text{ V}$  at  $f = 0\text{ Hz}$ .

$V_1 =$  \_\_\_\_\_

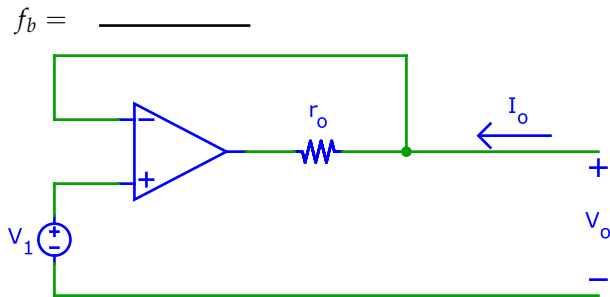
- b) Find an algebraic expression for  $H_b(s) = V_2(s)/V_1(s)$  assuming that the operational amplifier is ideal. Having dealt with  $V_{\text{off}}$  we use  $V_{\text{off}} = 0\text{ V}$ .

\_\_\_\_\_

- c) Find an algebraic expression for  $H_c(s) = V_2(s)/V_1(s)$  assuming that the operational amplifier has finite bandwidth  $\omega_b$ . Model the operational amplifier frequency response as  $a(s) = \omega_b/s$ . Use  $V_{\text{off}} = 0\text{ V}$ .

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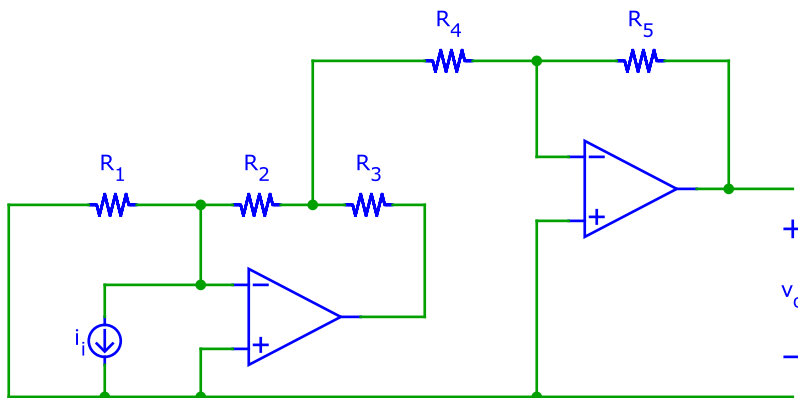
22. In the circuit below the operational amplifier is ideal except for finite unity-gain bandwidth  $f_b$  and output resistance  $r_o$ . Find the value of  $f_b$  for which the magnitude of the output impedance  $Z(s) = V_o(s)/I_o(s)$  equals  $69\text{ m}\Omega$  at  $f = 8\text{ kHz}$ . Use  $V_1 = 6.0\text{ V}$  and  $r_o = 2\text{ }\Omega$ .



23. Calculate the closed-loop gain  $\frac{v_o}{i_i}$  of the circuit below. Assume that the operational amplifier is ideal. Remember to include the correct unit with your result.

Parameter:  $R_1 = 7.8 \text{ k}\Omega$ ,  $R_2 = 6.4 \text{ k}\Omega$ ,  $R_3 = 7.0 \text{ k}\Omega$ ,  $R_4 = 2.2 \text{ k}\Omega$ ,  $R_5 = 6.0 \text{ k}\Omega$ .

$\frac{v_o}{i_i} =$  \_\_\_\_\_



24. In the circuit below,  $R_T$  is a temperature dependent resistor with value

$$R_T(T) = R_o (1 + \alpha T)$$

where  $T$  is the temperature in degrees Celsius [C],  $R_o = 8.4 \text{ k}\Omega$  and  $\alpha = 0.05 \text{ C}^{-1}$ .

Find the values of  $R_1$  and  $R_3$  such that the output voltage  $v_o$  of the circuit is

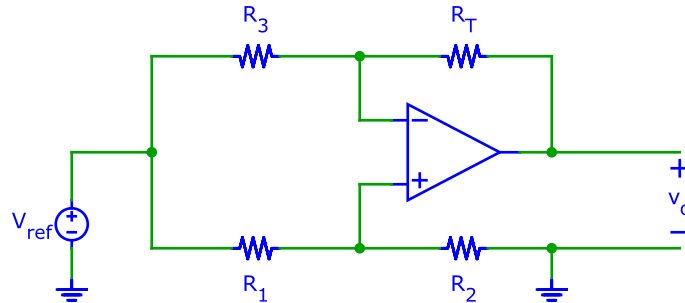
$$v_o(T) = 1 \text{ V} \quad \text{for } T = 0 \text{ C and}$$

$$v_o(T) = 2 \text{ V} \quad \text{for } T = 100 \text{ C.}$$

Use  $R_2 = 1.9 \text{ k}\Omega$  and  $V_{ref} = 7.7 \text{ V}$ . Note: results may be negative.

$R_1 =$  \_\_\_\_\_

$R_3 =$  \_\_\_\_\_

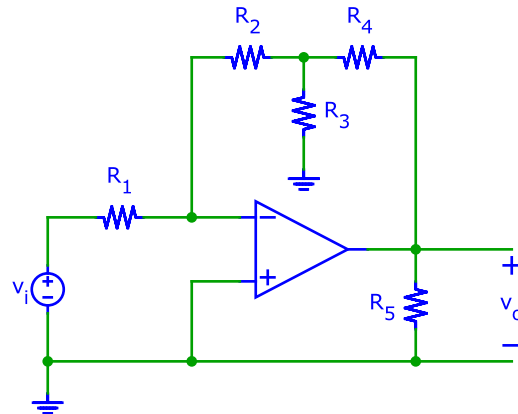


In a practical application you would connect  $v_o$  to the input of a microcontroller for display and function (e.g. thermostat).

25. Opamp circuits with high closed-loop gain require large resistor ratios. On integrated circuits these take up significant area and are therefore costly. The circuit below uses a so-called T-network to reduce the required resistor ratio.

Calculate the value of resistor  $R_3$  such that  $v_o/v_i = -85$ . Use  $R_1 = 2\text{ k}\Omega$ ,  $R_2 = R_4 = 5\text{ k}\Omega$  and  $R_5 = 1\text{ k}\Omega$ .

$R_3 =$  \_\_\_\_\_



26. The output voltage of a temperature sensor element is

$$v_t(T) = -2 \frac{\text{mV}}{^\circ\text{C}} \times T$$

where  $T$  is the temperature in degrees Celsius.

Design a thermometer circuit with output voltage

$$v_o(T) = 10 \frac{\text{mV}}{^\circ\text{C}} \times T$$

using the sensor, resistors, and ideal opamps. Your circuit should produce the correct output independent of the output resistance  $R_o$  of the temperature sensor, which is in the range  $50\text{ k}\Omega \dots 100\text{ k}\Omega$ . Draw the schematic diagram in the space provided below. Specify the values of all resistors (except  $R_o$ ).

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