Structured Superposition for Backhaul Constrained Cellular Uplink

Bobak Nazer*, Amichai Sanderovich†, Michael Gastpar*, and Shlomo Shamai (Shitz)†

***Abstract*—In this paper, we demonstrate the advantage of the inherent algebraic structure of lattice codes, for the uplink channel of a cellular deployment. The out-of-cell interference is assumed to be symmetric, as in Wyner's model. We employ a new relaying technique, compute-and-forward, which allows cell-sites to decode equations of the transmitted bits by exploiting the channel interference. However, the standard compute-and-forward technique is penalized whenever the channel coefficients are non-integer. We develop a superposition strategy to mitigate this penalty. By using part of the power towards a private message, we can effectively modify the channel seen by compute-and-forward. We demonstrate that, in certain regimes, this mixed strategy significantly outperforms decode-and-forward, compress-and-forward, and ordinary compute-and-forward.**

I. INTRODUCTION

A major challenge in cellular communication is efficient interference management for the uplink channel. Each basestation in a cellular deployment receives signals from users within its cell as well as interference in the form of signals from users in adjacent cell sites. Classical communication schemes attempt to mitigate the effects of this interference either by orthogonalization or interference averaging. Significant gains are possible over these strategies by allowing some degree of cooperation between the basestations, which is known as *joint multiple cell-site processing*.

Wyner's original paper on cellular systems considers full cooperation between the basestations and finds the capacity in the nonfading, symmetric case [1]. Somekh *et al.* extended this result to the flat fading case and demonstrated that out-of-cell interference can be completely eliminated by joint processing [2]. Both of these works assume that the "backhaul capacity" from each cell-site to the remote central processor (RCP) is infinite. For most networks, this assumption may not hold; however, it does establish a promising target for the limited backhaul case. Recent work by Sanderovich *et al.* gave an achievable strategy based on compress-and-forward (also known as estimate-and-forward [3]) and decode-and-forward for the limited backhaul case [4], [5]. They also showed that compress-and-forward carries an additional benefit: basestations can be oblivious to the codebooks of neighboring cell-sites. An overview of this work can be found in [6]. Marsch and Fettweis have also recently extended this strategy to include superposition coding [7].

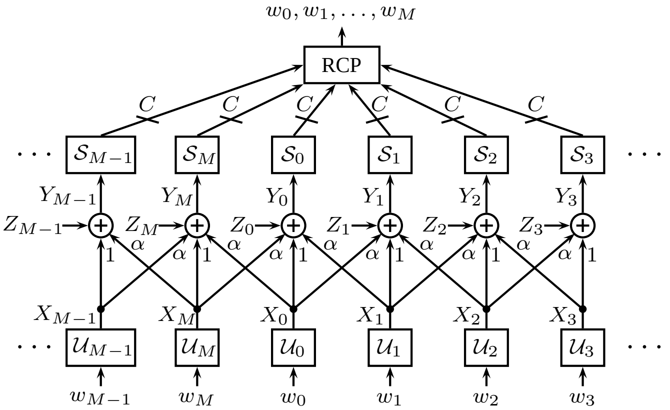

Fig. 1. Idealized cellular uplink network from users to cell-sites to the RCP.

Recent work has shown that structured random codes can, in certain network scenarios, outperform standard random coding approaches based on single-letter information expressions [8]–[12]. Here, we develop a lattice-based strategy that allows the cell-sites to decode equations of codewords (or lattice points) and forward these to the RCP, which, given a full rank system of such equations, can recover the original messages. This strategy strikes a balance between noise removal at the basestations and joint decoding at the RCP. Our strategy is based on Nazer and Gastpar's recent *compute-and-forward* scheme [13]. In compute-and-forward, each receiver decodes an equation with integer coefficients that approximate the channel coefficients. The remainder from this integer approximation acts like additional noise at the receiver. Here, we develop a superposition strategy that significantly reduces this approximation penalty for Wyner's cellular model. In our scheme, every other user employs some of its power towards a private message. By changing the power allocation, we can effectively steer the interference parameter to increase the compute-and-forward rate.

II. PROBLEM STATEMENT: WYNER'S CELLULAR MODEL

We will focus on the model introduced by Sanderovich, Somekh and Shamai in [4], [5] (see Figure 1) which is a variation on the classical Wyner model [1]. Consider an idealized cellular uplink network where M cell-sites are equally spaced on a circle. We assume that only one user is active in a cell per time slot and that cell-sites only see interference from a neighboring cells' users. The users are given n channel uses to reliably convey their messages. The complex signal $X_m(t)$ is transmitted by the m^{th} user at time t and is power-limited

---

* B. Nazer and M. Gastpar are with the Wireless Foundations Center, EECS Department, University of California, Berkeley, CA 94720-1770, USA. Email: bobak@eecs.berkeley.edu, gastpar@eecs.berkeley.edu

† A. Sanderovich and S. Shamai are with the Department of Electrical Engineering, Technion, Haifa, 32000, Israel. Email: amichi@techunix.technion.ac.il, sshlomo@ee.technion.ac.il

in the usual way, $\frac{1}{n}\sum_{t=1}^{n}\|X_m(t)\|^2 \leq P$. The signal seen by the m^{th} cell-site at time t is:

$$Y_m(t) = X_m(t) + \alpha(X_{[m-1]_M}(t) + X_{[m+1]_M}(t)) + Z_m(t)$$

where $[m]_M = m \mod M$, $\alpha \in [0,1]$ is the inter-cell interference level, and $Z_m(t)$ is i.i.d. $\mathcal{CN}(0,N)$ noise. For notational convenience, we define the signal-to-noise ratio, $\mathsf{SNR} \triangleq \frac{P}{N}$. Unlike the Wyner model, where unlimited backhaul is available, in the Sanderovich *et al.* model we use here, the cell-sites have a lossless backhaul link to the RCP with rate C bits per channel use [4], [5]. In this paper, we only look at the symmetric case assuming equal rate users. We say that a *per user rate R* is achievable if each user's message can be decoded by the RCP at rate of at least R with vanishing probability of error (in the blocklength). We do not consider time-varying fading in this paper. Note that this model can also be written in matrix form:

$$\mathbf{Y} = \mathbf{HX} + \mathbf{Z}$$

$$\mathbf{H} = \{h_{im}\}, \quad h_{im} = \begin{cases} 1 & \text{if } i = m, \\ \alpha & \text{if } i = [m+1]_M \text{ or } [m-1]_M, \\ 0 & \text{otherwise.} \end{cases}$$

III. UPPER BOUND: CUT-SET

We will use a simple cut-set bound to upper bound the capacity. Since we are only interested in symmetric rate tuples and the channel is symmetric, the sum rate is upper bounded by the dominant cut set bound. Recall that for a MIMO channel with independent (and equal power) channel inputs the capacity is given by:

$$C_{\text{MIMO}} = \log\det\left(\mathbf{I} + \mathsf{SNR}\ \mathbf{HH}^*\right) \tag{1}$$

where $\mathbf{H}^*$ denotes the Hermetian transpose of $\mathbf{H}$. Clearly, the rate of information flow between the cell-sites and the RCP is upper bounded by MC. Also, if the cell-sites could cooperate freely, then the channel from the users becomes a MIMO channel. Thus, our cut-set bound is the minimum between C_{MIMO} and MC, normalized by the number of users:

$$C_{\text{UPPER}} = \min\left(C, \frac{1}{M}\log\det\left(\mathbf{I} + \mathsf{SNR}\ \mathbf{HH}^*\right)\right) \tag{2}$$

This upper bound reveals two potential information bottlenecks. One is the MIMO behavior of the channel and the other is the finite backhaul. An ideal scheme for this problem will cope with both bottlenecks simultaneously.

For the special circularly symmetric matrix at hand, Wyner showed in [1] that as M tends to infinity we get:

$$C_{\text{MIMO}} = \int_0^1 \log\left(1 + \mathsf{SNR}(1 + 2\alpha\cos 2\pi\theta)^2\right) d\theta. \tag{3}$$

IV. CLASSICAL RANDOM CODING STRATEGIES

We first review the performance of two known strategies, compress-and-forward (CF) and decode-and-forward (DF).

A. Compress-and-Forward

Sanderovich *et al.* showed in [4], [5] that compress-and-forward achieves the following rate per user:

$$R_{\text{CF}} = \frac{1}{M}\max_{0\leq r}\min_{S\subseteq[M]} \left\{|S|(C-r) + \log\det\left(\mathbf{I} + \mathsf{SNR}(1-2^{-r})\mathbf{H}_{S^c}\mathbf{H}_{S^c}^*\right)\right\} \tag{4}$$

where $\mathbf{H}_{S^c}$ is the submatrix of $\mathbf{H}$ that includes only rows in the subset S^c (which is the complement of the subset S). This rate in the asymptotic case, where $M \to \infty$ is [4]:

$$R_{\text{CF}} = F(r^*) \text{ where } r^* \text{ is the solution of } F(r^*) = C - r^*,$$

$$F(r) = \int_0^1 \log\left(1 + \mathsf{SNR}(1-2^{-r})(1+2\alpha\cos 2\pi\theta)^2\right) d\theta.$$

The approach of [4], [5] is oblivious in terms of assuming no decoding at cell-sites and is optimal if C or SNR tends to infinity. As C tends to infinity, the rate converges to the infinite backhaul capacity case given by Equation (3). As SNR tends to infinity, this rate converges to C. For moderate SNR and C, compress-and-forward roughly follows the outer cut-set bound (with α) but never touches it. This gap from the outer bound occurs since the scheme forwards the entire channel output, including the noise.

B. Decode-and-Forward

Sanderovich *et al.* also derive the performance of decode-and-forward in [4], [5]. For the no fading case, the channel to each cell-site is equivalent to a three input multiple-access channel, so that the decode-and-forward rate is:

$$R_1 = \log\left(1 + \frac{\mathsf{SNR}}{1 + 2\alpha^2\mathsf{SNR}}\right) \tag{5}$$

$$R_2 = \min\left\{\frac{1}{2}\log\left(1 + 2\alpha^2\mathsf{SNR}\right), \frac{1}{3}\log\left(1 + (1+2\alpha^2)\mathsf{SNR}\right)\right\} \tag{6}$$

$$R_{\text{DF}} = \min\left\{\max\left(R_1, R_2\right), C\right\} \tag{7}$$

R_1 models decoding when the other signals are treated as noise, while R_2 assumes full reliable decoding of all three data streams received at the cell-site. Note that there is no joint processing gain. When the backhaul capacity C is small compared to the rates between the users and the cell-sites, or when the interference level is low, decode-and-forward is optimal.

V. COMPUTE-AND-FORWARD

Each cell-site sees a linear combination of its user's transmission as well as the transmissions of two neighboring users plus noise. Ideally, we could remove the noise and send the error-free linear combination of codewords to the RCP. If the RCP can collect enough linear equations to form a full rank system, it can recover the original messages. Recent work has shown that this type of decoding, called *compute-and-forward*, is indeed possible by employing lattice codes [13], [14].

A. Decoding Equations

An n-dimensional *lattice*, Λ, is a set of points in $\mathbb{R}^n$ such that if $\mathbf{x}, \mathbf{y} \in \Lambda$, then $\mathbf{x}+\mathbf{y} \in \Lambda$, and if $\mathbf{x} \in \Lambda$, then $-\mathbf{x} \in \Lambda$. A lattice can always be expressed in terms of a generator matrix $\mathbf{G} \in \mathbb{R}^{n \times n}$:

$$\Lambda = \{\mathbf{x} = \mathbf{z}\mathbf{G} : \mathbf{z} \in \mathbb{Z}^n\} \tag{8}$$

where $\mathbb{Z}$ represents the integers. Let $[\mathbf{x}] \bmod \Lambda = \mathbf{x} - Q_\Lambda(\mathbf{x})$ where $Q_\Lambda(\mathbf{x})$ quantizes $\mathbf{x}$ to the nearest lattice point in Euclidean distance.

Urbanke and Rimoldi showed that a lattice code can approach the AWGN capacity [15]. Further work by Erez and Zamir showed that lattice decoding is sufficient for nested lattice codes [16]. Zamir, Shamai, and Erez demonstrated that nested lattice constructions are appropriate for many multiterminal problems [17].

Our compute-and-forward strategy depends crucially on the algebraic structure of the channel and the employed codebook. We choose (nested) lattice codes for their inherent linearity which matches the MIMO structure of our problem. Namely, we select a good nested lattice using the results of Erez and Zamir [16] and have each transmitter use the same lattice code. The goal of each cell-site is to decode a linear equation of the transmitted lattice points that is close to that given by the channel. However, it is easy to see that only linear combinations with *integer coefficients* will result in another lattice point. The remainder from this approximation shows up as an additional noise term. At a first glance, this seems to suggest that non-integer channel coefficients always result in a rate penalty. Fortunately, this is not the case and the situation can be significantly improved by a simple trick. Essentially, the cell-site can scale its channel observation by a constant, $g \in \mathbb{R}$, prior to decoding. This results in a set of “new” channel coefficients, $\tilde{h}_{im} = g h_{im}$, that may be closer to an integer point at the expense of increasing the noise variance from N to $g^2 N$. The analysis of this scheme can be found in [13], [18] and the main theorem is reproduced below.

Theorem 1: Each cell-site sees a noisy linear combination of the transmitted codewords, $Y_m(t) = \sum_{i=1}^{M} h_{im} X_i(t) + Z_m(t)$, and chooses a linear equation to decode, $U_m = \sum_{i=1}^{M} q_{im} X_i,\ q_{im} \in \mathbb{Z} + j\mathbb{Z}$. The rate from each cell-site to the RCP is C. Assume that the matrix of cell-site equations, $\mathbf{Q} = \{q_{im}\}_{im}$, is full rank. Then, the following symmetric rate per user is achievable using a compute-and-forward strategy:

$$\begin{aligned} R &= \min\{R_{\text{COMP}}, C\} \\ R_{\text{COMP}} &= \min_{m} \max_{\mathbf{q}_m \in \mathbb{Z}^M} -\log\left(\|\mathbf{q}_m\|^2 - \frac{\mathsf{SNR}|\mathbf{q}_m^* \mathbf{h}_m|^2}{1 + \mathsf{SNR}\ \|\mathbf{h}_m\|^2}\right) \end{aligned} \tag{9}$$

where $\mathbf{h}_m = [h_{1m}\ \cdots\ h_{Mm}]^T$ and $\mathbf{q}_m = [q_{1m}\ \cdots\ q_{Mm}]^T$. See [18] for a proof. Note that we only need to consider $\mathbf{q}_m$ that satisfy $\|\mathbf{q}_m\|^2 \leq 1 + \mathsf{SNR}(1 + \|\mathbf{h}_m\|^2)$ since other choices trivially give zero rate.

B. Basic Strategy

Due to the symmetry in the inter-cell interference, we can have each cell-site use the same recovery coefficients for its linear function and guarantee that the resulting matrix is full rank. We choose our integer decoding coefficients q_{im} to match the structure of the channel coefficients:

$$q_{im} = \begin{cases} b_1 & \text{if } i = m, \\ b_2 & \text{if } i = [m+1]_M \text{ or } [m-1]_M, \\ 0 & \text{otherwise.} \end{cases}$$

where $b_1, b_2 \in \mathbb{Z},\ b_1 \neq 0$. The diagonal structure of the channel matrix guarantees the full rank condition. We now give a simplified version of the achievable rate expression.

Theorem 2: The following rate per user is achievable with backhaul rate C using the compute-and-forward strategy:

$$\begin{aligned} R &= \min\{R_{\text{COMP}}, C\} \\ R_{\text{COMP}} &= \max_{b_1, b_2 \in \mathcal{B}} -\log\left(b_1^2 + 2b_2^2 - \frac{\mathsf{SNR}(b_1 + 2\alpha b_2)^2}{1 + \mathsf{SNR}(1 + 2\alpha^2)}\right) \\ \mathcal{B} &= \{(b_1, b_2) : b_1, b_2 \in \mathbb{Z},\ b_1 \neq 0, \\ &\qquad b_1^2 + 2b_2^2 \leq 1 + \mathsf{SNR}(1 + 2\alpha^2)\}. \end{aligned} \tag{10}$$

The restriction of b_1 and b_2 to $\mathcal{B}$ makes it possible to evaluate the rate expression exactly since $\mathcal{B}$ is a finite set. Note that all (b_1, b_2) pairs outside $\mathcal{B}$ trivially give zero rate (as well as many pairs inside the set). The performance of this strategy at $\mathsf{SNR} = 25$dB with infinite backhaul capacity is plotted as the dotted line in Figure 2. Note that as α tends to an integer (either 0 or 1) the performance dramatically improves. There are also local maxima in performance at $\alpha = \frac{1}{4}, \frac{1}{3}, \frac{1}{2}, \frac{2}{3}, \frac{3}{4}$. These interference values are ratios of small integers and can be exactly captured by the compute-and-forward decoding coefficients at this SNR. As we increase the SNR, more of these local maxima will appear as the range of integers that give positive rates increases. For instance, in Figure 3, only $\alpha = \frac{1}{2}$ exhibits a local maximum since here the SNR is only 15dB.

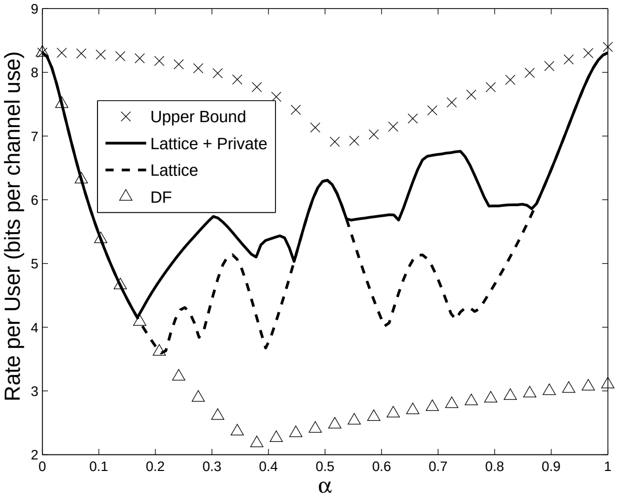

Fig. 2. $\mathsf{SNR} = 25$dB. Achievable rates per user as a function of the inter-cell interference levels α, for infinite backhaul capacity. In this case, compress-and-forward matches the upper bound.

VI. SUPERPOSITION STRATEGY

While the compute-and-forward performance in Figure 2 is promising, the significant variations due to integer effects do not seem fundamental to the problem. That is, the rate should not sharply degrade with only a small change in α. We now develop a superposition strategy that decreases the effective interference seen by the even users and increases the effective interference seen by the odd users. With this strategy, we can avoid the local minima in α. Usually, superposition coding is used to provide different messages (or rates) to users of varying quality. Here, we are using superposition to adjust the interference structure of the problem itself. The resulting performance boost to compute-and-forward is a new phenomenon: traditional strategies would not benefit at all from such a scheme.

We now provide a brief outline of the superposition lattice scheme. Each odd-numbered user will use a lattice codeword with power γP where $\gamma \in [0,1]$. Each even-numbered user will use a lattice codeword with power $(1+\delta)P$ where $\delta \in [0, 1-\gamma]$ and superimpose a lattice codeword with power $(1-\gamma-\delta)P$ on top. Let R_C denote the desired compute-and-forward rate and R_S the superimposed message rate. From [16], there exist lattices $\Lambda, \Lambda_C, \Lambda_S \subset \mathbb{R}^n$ such that $\Lambda \subset \Lambda_C$, $\Lambda \subset \Lambda_S$, $\frac{1}{n}\log|\Lambda_C \bmod \Lambda| = \frac{1}{2}R_C$, and $\frac{1}{n}\log|\Lambda_S \bmod \Lambda| = \frac{1}{2}R_S$. Let $\mathbf{d}_{mC}, \mathbf{d}_{mS}$ denote dither random vectors that are independently and uniformly distributed over the fundamental Voronoi region of Λ. These dither vectors are made available to all users as common randomness.

Since $\alpha \in \mathbb{R}$, the real and imaginary channel inputs remain separated in the channel output. Thus, we can use the same encoding and decoding process separately on the real and imaginary components. We describe the real part below. Every user i chooses a lattice point $\lambda_{mC} \in \Lambda_C \bmod \Lambda$. Every even-numbered user also chooses a lattice point $\lambda_{mS} \in \Lambda_S \bmod \Lambda$. Every user generates $\mathbf{v}_{mC}$ and the even-numbered users also generate $\mathbf{v}_{mS}$:

$$\mathbf{v}_{mC} = [\lambda_{mC} - \mathbf{d}_{mC}] \bmod \Lambda \tag{11}$$

$$\mathbf{v}_{mS} = [\lambda_{mS} - \mathbf{d}_{mS}] \bmod \Lambda \tag{12}$$

The channel input, $\mathbf{x}_m$, from each user is:

$$\mathbf{x}_m = \begin{cases} \sqrt{1+\delta}\,\mathbf{v}_{mC} + \sqrt{1-\gamma-\delta}\,\mathbf{v}_{mS} & \text{if } m \text{ is even,} \\ \sqrt{\gamma}\,\mathbf{v}_{mC} & \text{if } m \text{ is odd.} \end{cases}$$

Each basestation observes $\mathbf{y}_m$:

$$\mathbf{y}_m = \mathbf{x}_m + \alpha(\mathbf{x}_{[m-1]_M} + \mathbf{x}_{[m+1]_M}) + \mathbf{z}_m \tag{13}$$

The even-numbered cell-sites first decode the superimposed lattice point. Upon successful decoding, they cancel out $\mathbf{v}_{mS}$ and use the standard compute-and-forward strategy on the resulting channel output. The effective channel gains for compute-and-forward after this successive cancellation step are:

$$h_{im}^{\text{EVEN}} = \begin{cases} \sqrt{1+\delta} & \text{if } i = m, \\ \alpha\sqrt{\gamma} & \text{if } i = [m+1]_M \text{ or } [m-1]_M, \\ 0 & \text{otherwise.} \end{cases}$$

The odd-numbered cell-sites either decode and remove both superimposed messages from their two neighbors or treat them as additional noise. They then use compute-and-forward on the output which has the following effective channel coefficients:

$$h_{im}^{\text{ODD}} = \begin{cases} \sqrt{\gamma} & \text{if } i = m, \\ \alpha\sqrt{1+\delta} & \text{if } i = [m+1]_M \text{ or } [m-1]_M, \\ 0 & \text{otherwise.} \end{cases}$$

The odd and even cell-stes need to maximize over integer coefficients to best fit the channel as before. We run this strategy for half of our channel uses and then reverse the roles of the odd and even users for the second half. This time-sharing scheme results in equal rates per user.

Theorem 3: The following rate is achievable using the superposition strategy:

$$R = \min\left\{ \max_{\gamma\in[0,1]} \max_{\delta\in[0,1-\gamma]} \max\left(R^D, R^I\right),\ C \right\}$$

$$R^D = \min\left(\frac{R_S^E}{2}, \frac{R_S^{DO}}{2}\right) + \min\left(\max_{b_1,b_2} R_C^E,\ \max_{b_1,b_2} R_C^{DO}\right)$$

$$R_S^E = \log\left(1 + \frac{\mathsf{SNR}(1-\gamma-\delta)}{1+\mathsf{SNR}(1+\delta+2\alpha^2\gamma)}\right)$$

$$R_S^{DO} = \frac{1}{2}\log\left(1 + \frac{2\alpha^2\mathsf{SNR}(1-\gamma-\delta)}{1+\mathsf{SNR}(\gamma+2\alpha^2(1+\delta))}\right)$$

$$R_C^E = -\log\left(b_1^2 + 2b_2^2 - \frac{\mathsf{SNR}(\sqrt{1+\delta}\,b_1 + 2\alpha\sqrt{\gamma}\,b_2)^2}{1+\mathsf{SNR}(1+\delta+2\alpha^2\gamma)}\right)$$

$$R_C^{DO} = -\log\left(b_1^2 + 2b_2^2 - \frac{\mathsf{SNR}(\sqrt{\gamma}\,b_1 + 2\alpha\sqrt{1+\delta}\,b_2)^2}{1+\mathsf{SNR}(\gamma+2\alpha^2(1-\gamma))}\right)$$

$$R^I = \frac{R_S^E}{2} + \min\left\{\max_{b_1,b_2} R_C^E,\ \max_{b_1,b_2} R_C^{IO}\right\}$$

$$R_C^{IO} = -\log\left(b_1^2 + 2b_2^2 - \frac{\mathsf{SNR}(\sqrt{\gamma}\,b_1 + 2\alpha\sqrt{1+\delta}\,b_2)^2}{1+\mathsf{SNR}(\gamma+2\alpha^2(1-\gamma))}\right)$$

where R_S^E is the rate at which the even cell-sites can recover the superimposed message, R_S^{DO} is the rate at which the odd cell-sites can recover the superimposed message, R_C^E is the compute-and-forward rate for the even cell-sites, R_C^{DO} is the compute-and-forward rate for the odd cell-sites when they first decode the superimposed message, and R_C^{IO} is the compute-and-forward rate for the odd cell-sites when they ignore the superimposed message as noise.

Due to space limitations, we only show that the even-numbered cell-sites can decode the superimposed message at rate R_S^E and remove it from the channel output. The analysis for the odd-numbered cell-sites is very similar. Each even-numbered cell-site m computes:

$$\begin{aligned} g &= \frac{P\sqrt{1-\gamma-\delta}}{N + P((1+\delta) + (1-\gamma-\delta) + 2\alpha^2\gamma)} \\ \mathbf{t}_m &= [g\mathbf{y}_m + \mathbf{d}_{mS}] \bmod \Lambda \\ &= [\mathbf{v}_{mS} + \mathbf{d}_{mS} + g\alpha(\mathbf{x}_{[m-1]_M} + \mathbf{x}_{[m+1]_M}) + g\mathbf{z}_m \cdots \\ &\quad \cdots - (1 - g\sqrt{1-\gamma-\delta})\mathbf{v}_{mS} + g\sqrt{1+\delta}\,\mathbf{v}_{mC}] \bmod \Lambda \\ &= [\lambda_{mS} + \tilde{\mathbf{z}}_m] \bmod \Lambda \end{aligned}$$

where $\tilde{\mathbf{z}}_m$ is an equivalent noise term with variance given by:

$$N_{eq} = \frac{P^2(1+\delta+2\alpha^2\gamma)+PN}{P(1+\delta+2\alpha^2\gamma+1-\gamma-\delta)+N} \qquad (14)$$

From [16], we know that the lattice point λ_{mS} can be recovered (for sufficiently large n) at any rate below $R_S^E = \log\left(\frac{P}{N_{eq}}\right)$. Plugging N_{eq} into this formula gives the desired rate. Finally, the cell-site uses its estimate $\hat{\lambda}_{mS}$ to form $\hat{\mathbf{v}}_{mS} = [\hat{\lambda}_{mS} - \mathbf{d}_{mS}] \bmod \Lambda$ and subtracts $\sqrt{1-\gamma-\delta}\hat{\mathbf{v}}_{mS}$ from the channel output. Conditioned on successful decoding, the channel output now looks as if its just a linear combination of two lattice codewords according to the channel coefficient given by h_{im}^{EVEN}. It can also be shown that as $M \to \infty$ we can solve for the original messages from the odd and even equations so long as $b_1 \neq 0$ for both.

VII. Performance Comparisons

We now compare the rates achieved by the superposition strategy, compute-and-forward, decode-and-forward, and compress-and-forward in the asymptotic $M \to \infty$ regime. In Figure 2, we plot the performance in the infinite backhaul case with $\mathsf{SNR} = 25$dB to get a sense for how compute-and-forward behaves with α. As discussed earlier, the basic compute-and-forward strategy varies significantly with α with local minima at values of α that are hard to approximate with small integers. The superposition strategy does an excellent job of filling in these "valleys" and has significantly higher rates for the middle range of α. Note that the performance of compute-and-forward matches that of decode-and-forward for low values of α. In this regime, the best equation to decode is the unit vector which amounts to treating interference as noise.

In Figure 3, we plot the performance at $\mathsf{SNR} = 15$dB with $C = 3.5$ bits per channel use. Here, our scheme outperforms every other strategy except for a small range of α around 0.3. It also exactly meets the upper bound for a significant range of α (for $C = 3$ it meets the upper bound for an even larger range of α.)

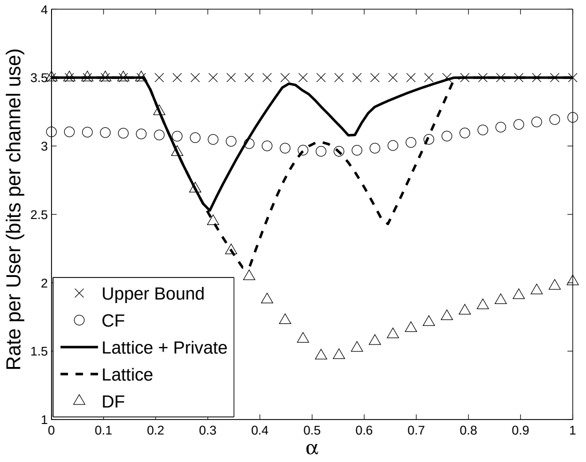

Fig. 3. $\mathsf{SNR} = 15$dB. Achievable rates per user as a function of the inter-cell interference levels α, for backhaul $C = 3.5$ bits per channel use.

VIII. Conclusions

For moderate values of SNR and backhaul capacity C, our superposition strategy seems like a promising *digital* scheme to approach the system capacity. By using a superimposed private message, we were able to adjust the effective interference to be closer to a decodable equation. Our future work will focus on the fading case as well as a more advanced superposition structure to further improve the compute-and-forward performance.

Acknowledgement

The work of A. Sanderovich and S. Shamai has been supported by the REMON Consortium and by the Network of Excellence in Wireless COMmunications NEWCOM++ of the European Commission. B. Nazer and M. Gastpar were supported by NSF grants CCR 0347298, CNS 0627024, and CCF 0830428.

References

[1] A. D. Wyner, "Shannon-theoretic approach to a Gaussian cellular multiple-access channel," *IEEE Trans. Info. Theory*, vol. 40, pp. 1713–1727, Nov. 1994.

[2] O. Somekh, B. M. Zaidel, and S. Shamai, "Sum rate characterization of joint multiple cell-site processing," *IEEE Trans. Info. Theory*, vol. 53, pp. 4473–4497, Dec. 2007.

[3] R. Dabora and S. D. Servetto, "On the role of estimate-and-forward with time sharing in cooperative communication," *IEEE Trans. Info. Theory*, vol. 54, pp. 4409–4431, Oct. 2008.

[4] A. Sanderovich, O. Somekh, and S. Shamai, "Uplink macro diversity with limited backhaul capacity," *ISIT 2007*, Nice, France.

[5] A. Sanderovich, O. Somekh, H. V. Poor, and S. Shamai, "Uplink macro diversity of limited backhaul cellular network," *IEEE Trans. Info. Theory*, to appear. See http://arxiv.org/abs/0805.4620

[6] S. Shamai, O. Simeone, O. Somekh, A. Sanderovich, B. Zaidel, and H. V. Poor, "Information theoretic implications of constrained cooperation in simple cellular models," *PIMRC 2008*, Cannes, France.

[7] P. Marsch and G. Fettweis, "On backhaul-constrained multi-cell cooperative detection based on superposition coding," *PIMRC 2008*, Cannes, France.

[8] J. Körner and K. Marton, "How to encode the modulo-two sum of binary sources," *IEEE Trans. Info. Theory*, vol. 25, pp. 219–221, Mar. 1979.

[9] K. Narayanan, M. P. Wilson, and A. Sprintson, "Joint physical layer coding and network coding for bi-directional relaying," in *Allerton 2007*, Monticello, IL.

[10] D. Krithivasan and S. Pradhan, "Lattices for distributed source coding: Jointly Gaussian sources and reconstruction of a linear function," *IEEE Trans. Info. Theory*, Submitted Jul. 2007. See http://arxiv.org/abs/0707.3461

[11] B. Nazer and M. Gastpar, "The case for structured random codes in network capacity theorems," *Euro. Trans. Telecomm.*, vol. 19, pp. 455–474, Jun. 2008.

[12] A. Sanderovich, M. Peleg, and S. Shamai, "Scaling laws in decentralized processing of interfered Gaussian channels," in *IZS 2008*, Zurich, Switzerland. See http://arxiv.org/abs/0801.4129

[13] B. Nazer and M. Gastpar, "Compute-and-forward: Harnessing interference with structured codes," in *ISIT 2008*, Toronto, Canada.

[14] B. Nazer and M. Gastpar, "Computation over multiple-access channels," *IEEE Trans. Info. Theory*, vol. 53, pp. 3498–3516, Oct. 2007.

[15] R. Urbanke and B. Rimoldi, "Lattice codes can achieve capacity on the AWGN channel," *IEEE Trans. Info. Theory*, vol. 44, pp. 273–278, Jan. 1998.

[16] U. Erez and R. Zamir, "Achieving $\frac{1}{2}\log(1+\text{SNR})$ on the AWGN channel with lattice encoding and decoding," *IEEE Trans. Info. Theory*, vol. 50, pp. 2293–2314, Oct. 2004.

[17] R. Zamir, S. Shamai, and U. Erez, "Nested linear/lattice codes for structured multiterminal binning," *IEEE Trans. Info. Theory*, vol. 48, pp. 1250–1276, Jun. 2002.

[18] B. Nazer and M. Gastpar, "Compute-and-forward: A novel strategy for cooperative networks," in *Asilomar 2008*, Pacific Grove, CA.