Outage-Optimal Relaying

In the Low SNR Regime

Salman Avestimehr

and

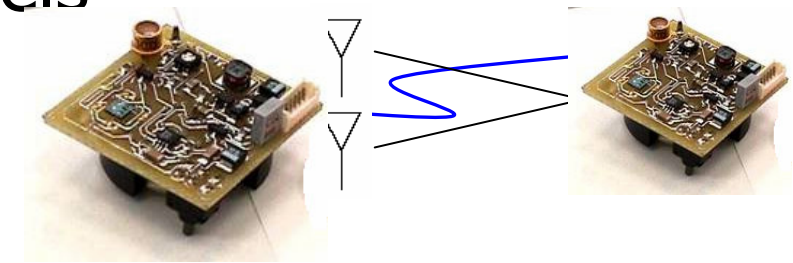
David Tse

University of California, Berkeley

Diversity

- Unreliability of fading channels

  - Diversity gain

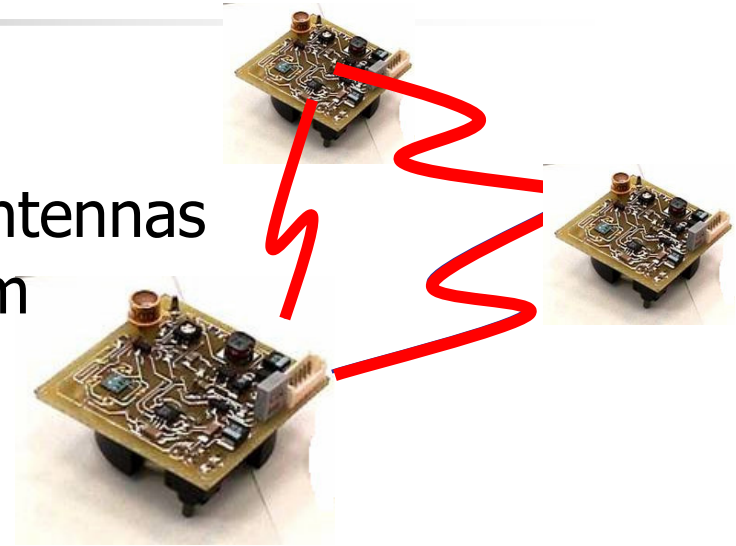


- Transmit Diversity
  - Creates more diversity

    Diversity gain=2

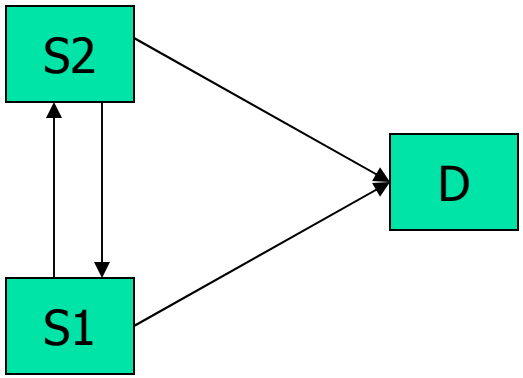
- Transmit diversity: may not be practical
  - most handsets (size)
  - wireless sensor network (size, power)

Cooperative communication

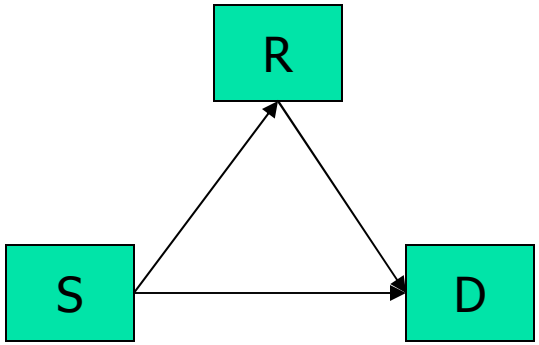


- Single-antenna mobiles share their antennas
  - Virtual Multiple-Antenna system

- Two scenarios:



Symmetrical



Relay

Outline

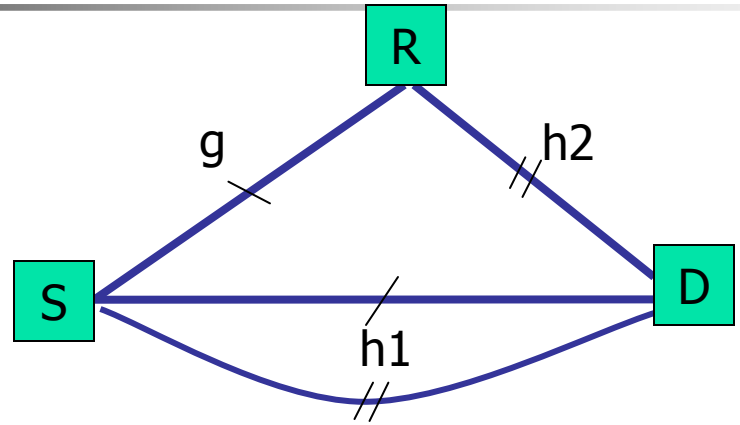
- Explaining the Relay Scenario
- Motivations
- Upper bound on the outage capacity
- Different protocols:

  Decode-Forward

  Amplify-Forward

  Bursty Amplify-Forward
- Further Directions

The Relay Scenario



- Rayleigh fading channels + AWGN noise
- Slow fading
- Half-Duplex constraint
- Channel State Information available at the receiver only
- Average power constraint (P) on both S and R

The Relay Scenario (cont.)

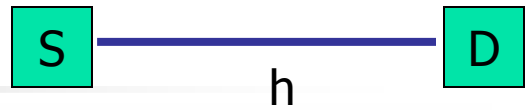
- The problem is hard: Look at different regimes
- Two Regimes of interest: high SNR, low SNR
- High SNR (Laneman et. al. & Azarian et. al.):

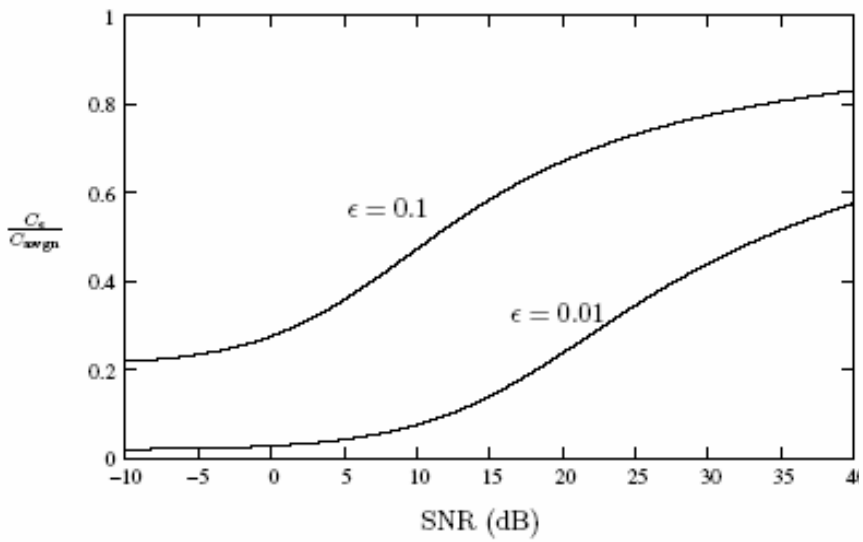
  Diversity-Multiplexing tradeoff

- Low SNR

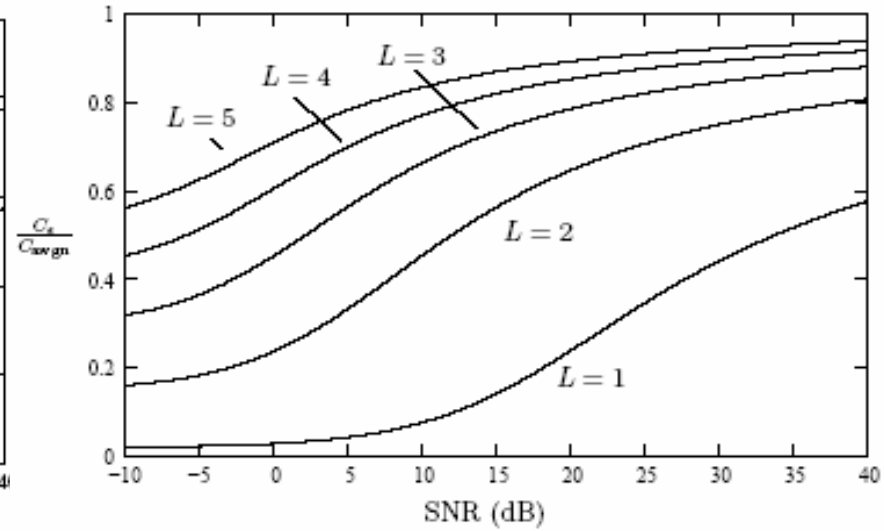
  - CDMA
  - Sensor Networks

Why Low SNR





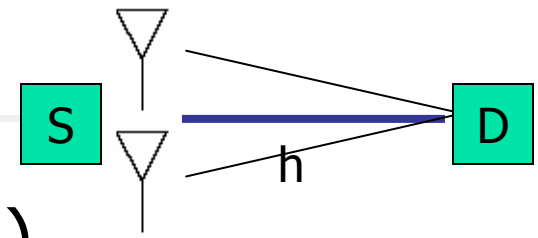
The impact of fading



The impact of diversity (ε=0.01)

- The capacity loss from fading is more significant at low SNR
- The impact of diversity on capacity is more significant at low SNR

Outage Capacity



- $C = \ln(1+|h|^2\ SNR)$ nats/s (fixed h)
- Outage event at rate R: $\ln(1+|h|^2\ SNR) < R$
- For outage probability, ε:

$$\Pr_{outage} = \Pr\{\ln(1+|h|^2\ SNR) < R\}$$
$$\approx \Pr\{|h|^2\ SNR < R\} \approx \frac{R}{SNR} = \varepsilon$$

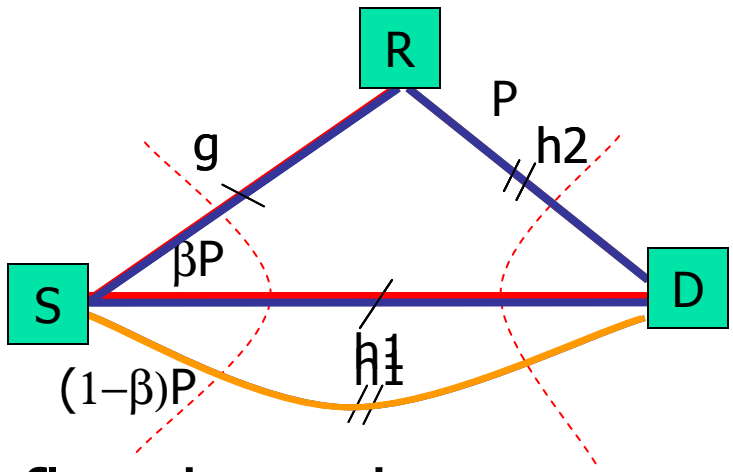
- Outage Capacity: $C_{\varepsilon,1-1} \approx \varepsilon SNR$

- For MISO channel: $C_{\varepsilon,2-1} \approx \sqrt{2\varepsilon}SNR$

$$\Pr_{outage} = \Pr\{\ln(1+(|h_1|^2+|h_2|^2)SNR) < R\} \approx \frac{1}{2}(\frac{R}{SNR})^2$$

For example at ε=0.01: $C_{\varepsilon,2-1} \approx 14 \times C_{\varepsilon,1-1}$

Upper Bound on the Performance?



- Min-cut max-flow bound:

$$C_{relay}(g,h_1,h_2) \le \max_{\beta}\{|h_1|^2 P + \min\{|g|^2 \beta P, |h_2|^2 P\}\}/N$$

$$= (|h_1|^2 P + \min\{|g|^2 P, |h_2|^2 P\})/N$$

- The corresponding bound on the outage capacity,

$$C_{\varepsilon,relay} \le \sqrt{\varepsilon} SNR$$

Different Protocols

1. Decode-Forward:

- First Time-slot: Source transmits the vector of encoded data.
- Second time-slot: If relay was able to decode data, it will re-encode it and transmit it.

$$R_{\varepsilon,DF} \approx \sqrt{\frac{2}{3}\varepsilon SNR}$$:Exploits Diversity

MFMC upper bound	$C_{\varepsilon,relay} \le \sqrt{\varepsilon} SNR$
Decode-Forward	$R_{\varepsilon,DF} \approx \sqrt{\frac{2}{3}\varepsilon SNR}$

- Not optimal, why?

cut set: $C_{relay}(g,h_1,h_2) \le (|h_1|^2 P + \min\{|g|^2 P, |h_2|^2 P\})/N$

Amplify-Forward

- First Time-slot: Source transmits the vector of encoded data, x.
- Second time-slot: Relay retransmits the received vector by amplifying.

$$R_{\varepsilon,AF} \approx \varepsilon SNR$$ No Diversity !

Why?

MFMC upper bound	$C_{\varepsilon,relay} \leq \sqrt{\varepsilon} SNR$
Decode-Forward	$R_{\varepsilon,DF} \approx \sqrt{\frac{2}{3}\varepsilon} SNR$
Amplify-Forward	$R_{\varepsilon,AF} \approx \varepsilon SNR$

A Better Protocol ?

- Bursty Amplify-Forward:

- To have less noisy observation at the Relay, source transmits rarely (α fraction of the time) but with high power:

$$SNR_{eff} = \frac{SNR}{\alpha}$$ be large

- In order to be power efficient, we should pick α such that the effective rate is small:

$$R_{eff} = \frac{R}{\alpha}$$ be small

- Can we pick such α ?

Yes, because at low outage probability $\frac{R}{SNR}$ is small.

- The achievable rate of this scheme: $R_{\varepsilon,BAF} \approx \sqrt{\varepsilon} SNR$

- Why optimal?

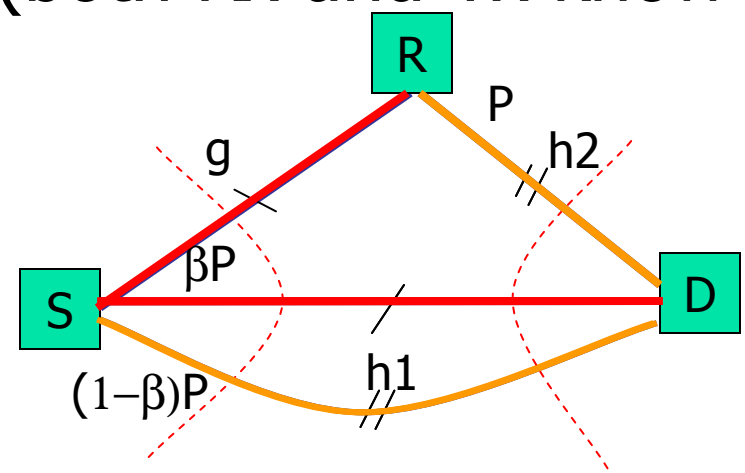
$$C_{relay}(g, h_1, h_2) \le (|h_1|^2 P + \min\{|g|^2 P, |h_2|^2 P\}) / N$$

Summary of results

MFMC upper bound	$C_{\varepsilon,relay} \leq \sqrt{\varepsilon} SNR$
Decode-Forward	$R_{\varepsilon,DF} \approx \sqrt{\frac{2}{3}\varepsilon} SNR$
Amplify-Forward	$R_{\varepsilon,AF} \approx \varepsilon SNR$
Bursty Amplify-Forward	$R_{\varepsilon,BAF} \approx \sqrt{\varepsilon} SNR$
Outage Capacity	$C_{\varepsilon,relay} \approx \sqrt{\varepsilon} SNR$

Further directions

1. Full CSI (both Rx and Tx know the channel):



- Allocate some power for beam forming

$$C_{relay}(g,h_1,h_2) \le \max_{\beta}\{|h_1|^2 \beta P + \min\{|g|^2 \beta P,$$

$$,|h_2|^2 P + |h_1|^2 (1-\beta)P + 2|h_1||h_2|\sqrt{1-\beta}P\}\}/N$$

- Can also be achieved by BAF + beam forming

The gain from Full CSI

- With full CSI:

$$C_{\varepsilon,relay} \approx 1.04 \times \sqrt{\varepsilon} SNR$$

the gain is small.

- The reason is that in the previous optimization, only small amount of power will be allocated for beam-forming (6%).
- In MISO the gain is 3dB, as all the power is used for beam-forming.