Soft X-Rays and Extreme Ultraviolet Radiation: Principles and Applications

Chapter 8 Homework Problems

8.1 (a) Use Heisenberg's uncertainty principle for transverse position and momentum to derive the limiting phase space condition for radiation of wavelength λ

$$d \cdot \theta \geq \lambda/2\pi$$

where $d = 2\Delta x$ is the uncertainty in emitted photon position, $\theta = \Delta\theta$ is the detection cone half-angle, and where Δx and $\Delta\theta$ are single-sided (half width) root-mean-square ($1/\sqrt{e}$) measures of Gaussian spatial and angular distributions. (b) Explain under what circumstances this is a condition for spatial coherence. (c) Convert this to an expression involving full-width at half maximum (FWHM) measures of source size and divergence.

8.2 Consider the lowest order laser cavity mode (TEM00), described by a Gaussian intensity distribution

$$\frac{I}{I_0} = e^{-r^2/r_0^2}$$

where $r(z) = r_0\sqrt{1 + (\lambda z/4\pi r_0^2)^2}$, z is measured along the direction of propagation (symmetry axis), and λ is the wavelength. (a) Show that in the far-field, where $z \gg 4\pi r_0^2/\lambda$, that

$$d \cdot \theta = \lambda/2\pi$$

where $\theta = r(z)/z$ is the far field half-angle and $d = 2r_0$ is the "waist" diameter, i.e., the smallest diameter as a function of z. (b) Why is this condition associated with spatial coherence? (c) How is this achieved in a laser? (d) What happens in a multimode laser, and how does this affect spatial coherence of the resultant radiation field?

8.3 (a) Describe the effects of a monochromator and a pinhole spatial filter as they are used to improve the coherence properties of partially coherent radiation. (b) Relate the improved spatial and temporal coherence to reductions in photon flux (or power). (c) For the monochromator, separate the effects of narrowed spectral width from the "insertion loss" of the monochromator due to component reflectivities and grating (or crystal) efficiency. (d) For the pinhole spatial filter draw a diagram to illustrate how this works as a "phase-space filter", reducing some combination of acceptance angle and size. (e) Describe three types of experiments where these techniques would be utilized.

8.4 Derive an expression relating coherent power, $P_{coh, \lambda/\Delta\lambda}$ to spectral brightness $B_{\Delta\omega/\omega}$ for an arbitrary source of radiation. Start with an expression such as Eq. (5.58), as given in Chapter 5, for spectral brightness, an expression to convert photon flux F to power, such as $P=(h\nu/\text{photon})F = (hc/\lambda\cdot\text{photon})F$, and a suitable definition of coherent power in terms of $d \cdot \theta$ and $\lambda/\Delta\lambda$ (or $\lambda^2/2\Delta\lambda$). Your expression should be equally valid for undulator and bending magnet radiation, laser radiation, and emissions from extended sources such as plasma discharges and laser produced plasmas.

8.5 Consider the diffraction of radiation by a pinhole illuminated with varying degrees of partial coherence, as first studied in detail by R.A. Shore, B.J. Thompson, and R.E. Whitney [*JOSA* <u>56</u>, 733-738 (1966)]. In that paper the authors report both theoretical modeling involving the van Cittert-Zernike theorm, and experiments involving mercury arc lamp emission in the visible at 546.1 nm. In their experiment emission from the arc lamp uniformly illuminates a small pinhole of diameter d which serves as the first element in a pinhole-angular-aperture spatial filter. A lens of focal length f is then used to collimate radiation from this first pinhole. A second pinhole of diameter $D = 2a$ then intercepts the quasi-collimated radiation, setting the angular acceptance of the spatial filter at half-angle $\theta = a/f$. The second pinhole also serves as the "diffractive aperture" for these studies, with the intensity distribution of its diffracted radiation recorded in the far-field as a function of the partial coherence parameter $C = \pi d\theta/\lambda$. The figure below shows the far-field intensity distribution as a function of the normalized diffraction angle $ka\psi$, for various values of the parameter C. As reported in the cited article, the measured diffraction pattern closely matches the theoretical predictions. (a) Discuss the theoretical limits of the diffraction pattern for small and large C. Which corresponds to essentially coherent illumination, and which to largely incoherent illumination? (b) What is the value of C for which the normalized degree of spatial coherence μ (see Figure 8.21 of the text) goes to zero just along the circumference of the second pinhole? (c) How does the corresponding curve in the figure below compare to your intuitive expectation for pinhole diffraction with this degree of coherence across the area of the pinhole? (d) To observe the sharp null features associated with an Airy pattern, what correlation among fields would one expect across the aperture? (e) How does this relate to values of C or C^2? (f) The authors refer to C as the "number of correlation intervals contained in the aperture radius"; relate this quantitatively to the cases where $C = 2$, 1 and 1/4.
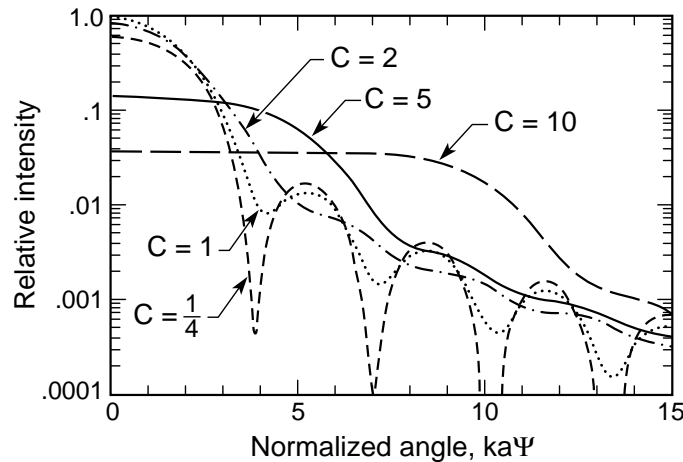

Figure for 8.5 Far-field intensity distribution of a circular aperture (pinhole) illuminated uniformly, but with varying degrees of spatial coherence. (Courtesy of B.J. Thompson, University of Rochester.)

8.6 (a) Calculate P_{cen} and $P_{coh,N}$ for a 5 cm period undulator at the Advanced Light Source in Berkeley. Assume $N = 89$, $E = 1.9$ GeV, $I = 400$ mA, and beam parameters (σ_x, σ_y, σ'_x, σ'_y) from Chapter 5, Table 5.1, p.128. Display your results in linear-linear scaled graphs (x - photon energy, y - power) for magnetic deflection parameters $0 \leq K \leq 3.0$. (b) Furthermore, calculate $P_{coh, \lambda/\Delta\lambda}$ following a monochromator set for $\Delta\lambda/\lambda = 2.5 \times 10^{-4}$, with an insertion efficiency of $\eta = 10\%$. (c) By what factor is the single pulse peak coherent power greater than the time averaged coherent power if all buckets are full and the RF is set to provide 100 ps FWHM pulses every 2 ns?

8.7 (a) Calculate the spatially coherent power within a $1/N$ relative spectral power for the 52 mm period undulator at MAX II in Lund, Sweden, as described in the homework problem 5.16 of Chapter 5, for $0 \leq K \leq 2.7$. (b) Also calculate the spatially coherent power within a relative spectral bandwidth of $\Delta\lambda/\lambda = 2.5 \times 10^{-4}$, assuming a monochromator insertion loss of 0.1, i.e., an efficiency of 10%. (c) What type of experiments would benefit from spatially filtered radiation such as this? Consult the website [http://www.maxlab.lu.se] for current beam parameters.

8.8 Calculate the spatially coherent power within a relative spectral bandwidth $\Delta\lambda/\lambda = 2.5 \times 10^{-4}$ for the X-1 undulator at Brookhaven National Laboratory's NSLS. The undulator has an 8 cm period, $N = 35$, and operates in the range $1 \leq K \leq 2.5$. Assume a beamline efficiency of 5%. The x-ray ring operates at 2.584 GeV with a maximum current of 300 mA. Assume beam parameters of $\sigma_x = 400$ μm, $\sigma_y = 5.9$ μm, $\sigma'_x = 250$ μr, $\sigma'_y = 17$ μr. Current beam parameters can be confirmed at the website [http://www.nsls.bnl.gov] under x-ray ring parameters. Also consult the NSLS website section "Accelerator Physics" for formulae regarding beam parameters in terms of emittance and betatron function.

8.9 (a) Calculate spatially coherent power within a relative spectral bandwidth of 5×10^{-4} for the $N = 84$, 49 mm period undulator at BESSY II, operating at a beam energy of 1.7 GeV and a current of 200 mA. (b) Graph your results versus photon energy, for K-values from 0 to 2.5. Assume beam parameters of $\sigma_x = 314$ μm, $\sigma_y = 24$ μm, $\sigma'_x = 18$ μr, $\sigma'_y = 2$ μr, or go to the website [http://www.bessy.de/oracle] for updated values. (c) What experiments at BESSY II would benefit from the use of spatially coherent radiation? (d) What type of experiments would require this spectral resolution?

8.10 Calculate spatially coherent power within a relative spectral bandwidth of 2×10^{-4} for the 3.30 cm period, $N = 72$, undulator at the Advanced Photon Source at Argonne National Laboratory near Chicago, operating at a beam energy of 7 GeV and a current of 100 mA. Assume $0 \leq K \leq 2.6$, a beamline efficiency of 10%, and beam parameters as given in Table 5.1 of the text on p.128.

8.11 (a) Calculate the spatially coherent power within a relative spectral bandwidth of 3×10^{-4} for the 42.0 mm period undulator at the ESRF in Grenoble, France, operating at a beam energy of 6.04 GeV and a current of 200 mA. Assume $N = 38$, $0 \leq K \leq 2.1$, and a beamline efficiency of 10%. Use as beam parameters $\sigma_x = 395$ μm, $\sigma_y = 9.9$ μm, $\sigma'_x = 10.5$ μr, $\sigma'_y = 3.9$ μr, or consult the website [http://www.esrf.fr] for updated values of the parameters. (b) What type of experiments would benefit from use of spatially coherent power over this wavelength region?

8.12 (a) Calculate the spatially coherent power in a relative spectral bandwidth of 1×10^{-4} for the 32 mm period, $N = 140$ undulator at the SPring-8 synchrotron facility in Harima, Japan, operating at 8 GeV and 100 mA. Do the calculations and graph your results for $0 \leq K \leq 2.45$. Assume a beamline efficiency of 10%. For beam parameters assume $\sigma_x = 393$ μm, $\sigma_y = 5.0$ μm, $\sigma'_x = 16$ μr, $\sigma'_y = 1.2$ μr, or consult the updated parameters at the website [http://www.spring8.or.jp]. (b) What applications might take advantage of coherent radiation at these short wavelengths?

8.13 Calculate $P_{coh, \lambda/\Delta\lambda}$ for bending magnet radiation at the ALS, using a monochromator set for $\Delta\lambda/\lambda = 10^{-3}$, and a beamline efficiency of 10%. For bending magnet beam parameters assume $\sigma_x = 53$ μm, $\sigma_y = 36$ μm, $\sigma'_x = 150$ μr, $\sigma'_y = 12$ μr. (Hint: Start with beam size, then choose angles, then calculate photon flux and power versus photon energy on a log-log scale.)

8.14 The Ne-like argon laser discussed in Chapter 7, Section 7.5, operates at a wavelength of 46.86 nm (26.46 eV photon energy) in experiments at Colorado State University. It generates an average power of about 1 mW in a relative spectral bandwidth $\Delta\lambda/\lambda = 10^{-4}$. Its measured source size and divergence, which depend on gas pressure in the discharge tube, are approximately 200 µm FWHM and 5 mr (2θ) FWHM at a background argon pressure of 650 mtorr. (a) What is the spatially coherent power $P_{coh,\ \lambda/\Delta\lambda}$, and (b) what is the (longitudinal) coherence length? In place of a monochromator, assume that a multilayer mirror of 30% reflectivity is available to suppress emission at other wavelengths from entering your experimental chamber.

8.15 Consider a nickel-like Nd laser as discussed in Chapter 7, Section 7.4, operating at Osaka University's Institute of Laser Engineering at a wavelength of 7.905 nm (photon energy of 165.8 eV). (a) If the output energy is 40 µJ per 130 ps FWHM pulse, what is the peak power in watts? (b) If the radiation emerges from a 40 µm diameter circular region into an angular cone of 3 mr (2θ) FWHM, what is the spatially coherent peak power per pulse? (c) If pulses are generated every 400 ps, what is the average coherent power? If the laser linewidth is Doppler broadened by an ion temperature of 220 eV, what is the (longitudinal) coherence length?

8.16 A 1 joule, 1 ns Nd laser pulse at 1.064 µm wavelength is focused to a 200 µm diameter spot size (all quantities FWHM) on a suitable target. (a) If the conversion efficiency is 2×10^{-3} into radiation within a desired spectral width $\Delta\lambda$, at $\lambda = 13$ nm, and if this radiation is emitted into a solid angle of 2π steradians, what fraction of the emitted radiation is spatially coherent? (b) If the repetition rate of the laser is 1 kHz, what is the average coherent power? (c) Into what angular emission cone is the spatially coherent power radiated (from the 200 µm diameter spot)?

8.17 Calculate the average and peak coherent power of a Ni-like Cd laser at 13.2 nm with an average power of 1 µW. Treat the laser output as originating from a source of 10 µm (FWHM) in size and the divergence angle of the beam is 10 mrad (FWHM). Assume the laser operates at 5 Hz and EUV pulse duration is 8 ps. Furthermore, calculate the average and peak spectral brightness of the laser. Express the answer in the typical unit of photons/s·mm^2·mrad^2·(0.1% bandwidth) and include your understanding of the laser's spectral bandwidth.