Soft X-Rays and Extreme Ultraviolet Radiation: Principles and Applications

Chapter 5 Homework Problems

5.1 (a) Give a physical explanation as to why synchrotron radiation from relativistic electrons is observed at very short wavelengths and in a narrow forward radiation cone. (b) How do both "scale", that is, how do observed wavelength and cone half angle vary with exponent of the major parameters (excluding numerical coefficients and more slowly varying parameters, if any).

5.2 Give two important technical differences between "third generation" (modern) storage rings and older rings.

5.3 Describe important features of (a) bending magnet radiation, (b) undulator radiation, and (c) wiggler radiation. Each offers very important advantages, even for users at modern storage rings. Be sure to capture these.

5.4 The Advanced Light Source (ALS) at Berkeley, and the Advanced Photon Source (APS) near Chicago, provide the US with two complimentary state of the art research facilities for materials characterization, biological, chemical, and environmental research, as well as various technological studies. The ALS operates at 1.90 GeV, covering the soft x-ray and extreme ultraviolet regions of the spectrum. The APS operates at 7.00 GeV, covering the multi-keV to hard x-ray spectral regions.

(1) The ALS has 24 ports to collect the spectral continuum of bending magnet radiation. For 1.90 GeV electron beam energy, an average current of 400 mA, and a bending magnet field of 1.27 tesla (a) What is the critical photon energy and what is its significance. What is the significance of $4E_c$? (b) Graph the photon flux vs. photon energy from 10 eV to 10 keV, assuming a relative spectral bandwidth of 0.1%, a horizontal collection angle of 1 mrad, and a vertical collection angle large enough to collect most of the radiation. Show your results as log-log plot. (c) What vertical collection angle is required to collect 80% of the radiation at photon energies of 0.1, 1, and 3 E_c? (d) What is the power radiated into a 2% spectral bandwidth at 2.5 nm wavelength, as collected by a 2 mrad optic? (e) If the electron beam consists of a large number of "electron bunches," each having a near-Gaussian axial distribution characterized by σ_z = 1.1 cm, separated from each other by a distance of 60 cm as they move through the storage ring, what is the time structure of the radiation, and what is the ratio of peak to average photon flux?

(2) Repeat calculations (a) through (e) for the APS, which operates at 7.00 GeV and 100 mA, and has a bending magnet field of 0.60 tesla. For Part (2)(b) plot the curve in the region from 70 eV to 70 keV. For Part (2)(d) do the calculation for a wavelength of 1 Å (0.1 nm) and a collection angle of 1 mrad. For Part (2)(e) assume σ_z = 5.3 cm and a bunch-to-bunch separation of 85 cm.

5.5 (a) Explain how dipole radiation, with relativistic transformations, is used to explain undulator radiation. (b) How do the two factors of γ enter the undulator equation? (c) Explain the physical significance of each term in the undulator equation. (d) Why is K sometimes called the "deflection parameter"? (e) What is the significance of the central radiation cone? (f) Explain the interdependence between selection of angular acceptance cone and spectral bandpass for undulator radiation. (g) How does the finite number of magnet periods affect the interdependence for acceptance cones narrower than the central radiation cone?

5.6 Radiated power in the central cone is given by Eq. (5.39). For low-K undulator operation the power scales as K^2. (a) Why is this so? (b) At what K-value does power in the central cone peak? (c) Why does power decrease for large K? (d) Why is there no dependence on N? (e) What does the slowly varying K-dependent correction factor, $f(K)$ in Eq. (5.41a), represent? (f) What is the usefulness of replacing the K-dependence in Eq. (5.39) by a dependence on $\hbar\omega/\hbar\omega_0$? (g) What does $\hbar\omega_0$ represent? (h) Why is the spectrum of undulator radiation within the central radiation cone, as illustrated in Figure 5.23, generally skewed to lower photon energies? (i) Why is spectrum not symmetric about the central photon energy?

5.7 (a) Use the undulator equation (5.28) to transform the K-dependent expression for power in the central radiation cone, as given in Eq. (5.41a), to a photon energy dependence as in Eq. (5.41c). (b) What is the significance of $\hbar\omega_0$, and what does it represent?

5.8 Harmonics are an important feature of undulator radiation, providing access to high brightness, partially coherent radiation at higher photon energies than otherwise available in the fundamental (n = 1) at a given facility. (a) What is the physical basis for harmonic wavelengths being shorter by a factor of $1/n$, as indicated by Eq. (5.30)? (b) Why is the concomitant relative spectral bandwidth within the central radiation cone equal to $1 / nN$? Consider the number of cycles in a Fourier decomposition of the motion. (c) What storage ring or beam parameters might limit the achievement of high harmonics or broaden the anticipated spectral bandwidth within the narrow cone?

5.9 Calculate and graph power in the central cone (n = 1 only) as a function of photon energy for two undulators at the Advanced Light Source in Berkeley (a) for a 5.00 cm period undulator with 89 periods and a magnetic tuning range $0 \le K \le 2.5$, and (b) for an 8.00 cm period undulator with 55 periods and a magnetic tuning range $0 \le K \le 3.0$. Assume a beam energy of 1.90 GeV and an average current of 400 mA. Present your results in a linear-linear scaled graph. (c) What is the half-angle of the central radiation cone, in each case for K = 1? (d) What is the relative spectral bandwidth in each case?

5.10 Repeat problem 5.9 for an undulator at the Advanced Photon Source near Chicago in Illinois. Do your calculation for the 3.30 cm period undulator, with N = 72, a tuning range of $0 \le K \le 2.60$, a beam energy of 7.00 GeV, and an average current of 100 mA.

5.11 (a) Calculate central cone power vs. photon energy for the 4.90 cm undulator at BESSY II in Berlin. Assume 84 periods, a beam energy of 1.70 GeV, an average current of 200 mA, and a magnetic tuning range of $0 \le K \le 2.5$. (b) What is the central cone half-angle for K = 1? (c) At what value of K does the power curve peak?

5.12 Calculate power in the central radiation cone vs. photon energy for the 42.0 mm undulator at the ESRF in Grenoble, France. Assume N = 38, a beam energy of 6.04 GeV, a beam current of 200 mA, and a magnetic tuning range of $0 \le K \le 2.1$. (b) What is the half-angle of the central radiation cone, photon energy, relative spectral bandwidth and power within the central (n = 1 only) at K = 1? (c) Calculate and graph the spectral brightness on a log-log scale assuming horizontal and vertical beam sizes (σ) of 395 μm and 9.9 μm respectively, and horizontal and vertical beam divergences (σ') of 10.5 μr and 3.9 μr respectively. Note that at ERSF undulators are also available with three times this length. Consult the website http://www.esrf.fr/machine/support/ids/Public/Sizes/sizes.html for updated beam parameters, and http://www.esrf.fr/machine/support/ids/Public/Ids/installed_IDs.html for undulator parameters.

5.13 (a) Calculate power in the central cone vs. photon energy for a 3.20 cm, 140 period undulator at SPring-8 in Harima, Japan, assuming a beam energy of 8.00 GeV, and operating beam current of 100 mA, and a magnetic tuning range of $0 \le K \le 2.45$. (b) At K = 1, what is the power in the central radiation cone

($n = 1$ only), its half-angle, center photon energy, and relative spectral bandwidth within this cone?

5.14 (a) Calculate the on-axis spectral brightness at 100 eV and 300 eV photon energies for an 8.00 cm period, $N = 55$, undulator at the ALS in Berkeley, assuming 1.90 GeV, 400 mA operation, and σ, σ' values given in Table 5.1, p. 128. Use K-dependent values for the central radiation cone. Express your answer in units of photons/s·mm^2·mrad^2·(0.1% bandwidth). (b) Repeat this for the 5.00 cm period, $N = 89$, undulator at the ALS in Berkeley, for photon energies of 300 eV and 550 eV.

5.15 (a) Calculate the on-axis spectral brightness for a 3.30 cm period, $N = 72$, undulator at the APS in Illinois, assuming beam parameters given in Table 5.1, p. 128. (b) Is the undulator condition satisfied in this case? (c) How might the calculation be corrected? (d) What is the photon tuning range for this undulator? (e) Where approximately does it peak (K, $\hbar\omega$)?

5.16 Calculate and graph spectral brightness vs. photon energy for the $N = 49$, 52 mm period undulator at MAX II in Lund, Sweden, assuming beam parameters of 1.50 GeV, 250 mA, horizontal and vertical beam sizes (σ) of 300 μm and 45 μm and horizontal and vertical beam divergences (σ') of 26 μr and 20 μr respectively. Allow K to vary from 0 to 2.7. Consult the website [http://www.maxlab.lu.se] for updated beam and undulator parameters.

5.17 (a) Explain the difference between a wiggler and an undulator. (b) What are the relevant K values? (c) Why is this the important parameter? (d) Which attributes are most important for each? (e) What features are less attractive? (f) How does each compare to bending magnet radiation? (g) Which could be retrofitted to an older storage ring? (h) Which would not reach its potential on an older storage ring, and why not? (i) Describe the evolution from undulator spectrum to wiggler spectrum as the magnet gap is closed. (j) Explain the concept of the "critical harmonic". (k) What is the significance of n_c? (l) Why do wigglers and bending magnet spectra have the same general spectral shape for high K? (m) How do the two differ?

5.18 (a) Calculate the total radiated power for a 16.0 cm period wiggler at the ALS ($N = 19$) with a 2.1 tesla maximum field on axis, a beam energy of 1.90 GeV and a current of 400 mA. (b) What is the K-value and the critical photon energy? (c) What is the significance of the critical photon energy? (d) Graph the on-axis photon flux in a log-log plot extending from 10 eV to 10 keV, in units of photons/s·mrad^2·(0.1% BW). (e) Also plot the photon flux per unit horizontal acceptance angle, over the same photon energy range, in units of photons/s·mrad·(0.1% BW). (f) What would be the photon flux at the critical photon energy within a 5 mrad acceptance angle and a 2% relative spectral bandwidth?

5.19 (a) Calculate the total power radiated, and the power per unit solid angle (kW/mrad^2) for the 8.5 cm, $N = 28$, APS wiggler when operating at its peak field on axis of 1.60 tesla. (b) What is the critical photon energy? (c) What is K? (d) Describe the angular radiation pattern. (e) What is the photon energy of the first harmonic? (f) What is n_c? Assume beam parameters of 7.00 GeV and 100 mA.

5.20 (a) Calculate total radiated power and radiated power per unit solid angle, in kW and kW/mrad^2 respectively, for the X17 wiggler on the x-ray ring (2.584 GeV, 300 mA) at Brookhaven National Laboratory's National Synchrotron Light Source. The X17 wiggler has a magnet period of 17.4 cm, $N = 2.75$, and $K = 81.3$. (b) Repeat these calculations for the X21 wiggler at NSLS, which has a magnet period of 12.0 cm, $N = 13.5$, and $K = 12.3$.

5.21 (a) Calculate the total power radiated by the 54 pole wiggler ($N = 27$, 7 cm period, 1.36 tesla peak field) at the Stanford Synchrotron Radiation Laboratory (SSRL), operating with beam parameters of 3.0

GeV and 100 mA. (b) What is the critical photon energy, (c) what is K, and (d) what is the angular extent of the radiation fan in the horizontal and vertical planes?

5.22 You are to design the radiation source for a biological microscope operating in the water window, between the absorption edges of carbon and oxygen. Assume you are working at a 2.0 GeV storage ring. (a) What are your main considerations? (b) Select major parameters of the radiation source. Consult Chapter 9 regarding microscope options if you wish.

5.23 You are to organize an experiment to study the magnetic properties of iron and cobalt. (a) What ring and what radiation source might you choose? (b) What would be your major technical considerations? (c) What major parameters of the ring or magnet structure are relevant to your experiments? (d) What additional equipment or components will you require? (e) Draw a sketch of your planned experiment.

5.24 You are asked to participate in an EUV lithography program. You have heard about laser plasma sources (Chapter 6) and discharge sources, but you first want to consider synchrotron radiation. Assume that you need to collect 8 W within a 2.5% relative spectral bandwidth at the center wavelength of a Mo/Be multilayer in the EUV (see Chapters 4 and 10). (a) Discuss the relative merits of bending magnet, undulator, and wiggler radiation. (b) Give details for your best choice. (c) How might the collected in-band EUV power be increased so as to provide a higher rate of wafer throughput? (See Chapter 10, Figure 10.5 of the text.) In Chapter 6 we will consider the laser plasma and discharge options.

Homework problems involving the spatial coherence of undulator radiation are addressed in Chapter 8.