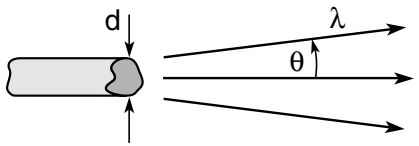
Spatial and Temporal Coherence; Coherent Undulator Radiation

David Attwood

University of California, Berkeley

(http://www.coe.berkeley.edu/AST/srms)

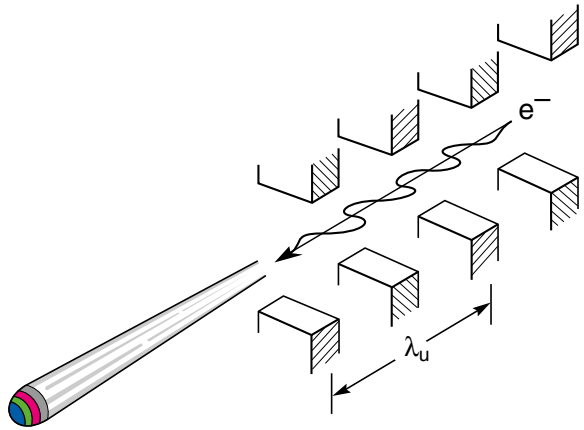
COHERENCE AT SHORT WAVELENGTHS

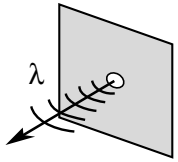


$$l_{coh} = \lambda^2/2\Delta\lambda \text{ \{temporal (longitudinal) coherence\}} \quad (8.3)$$

$$d \cdot \theta = \lambda/2\pi \text{ \{spatial (transverse) coherence\}} \quad (8.5)$$

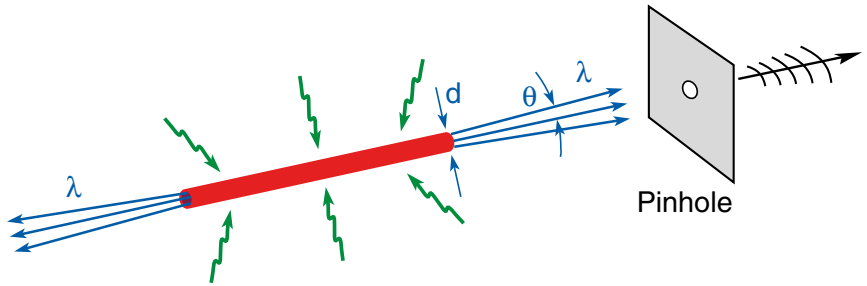
$$\text{or } d \cdot 2\theta|_{FWHM} = 0.44\,\lambda \quad (8.5^*)$$





$$\bar{P}_{coh,N} = \frac{(\lambda/2\pi)^2}{(d_x\theta_x)(d_y\theta_y)}\bar{P}_{cen} \quad (8.6)$$

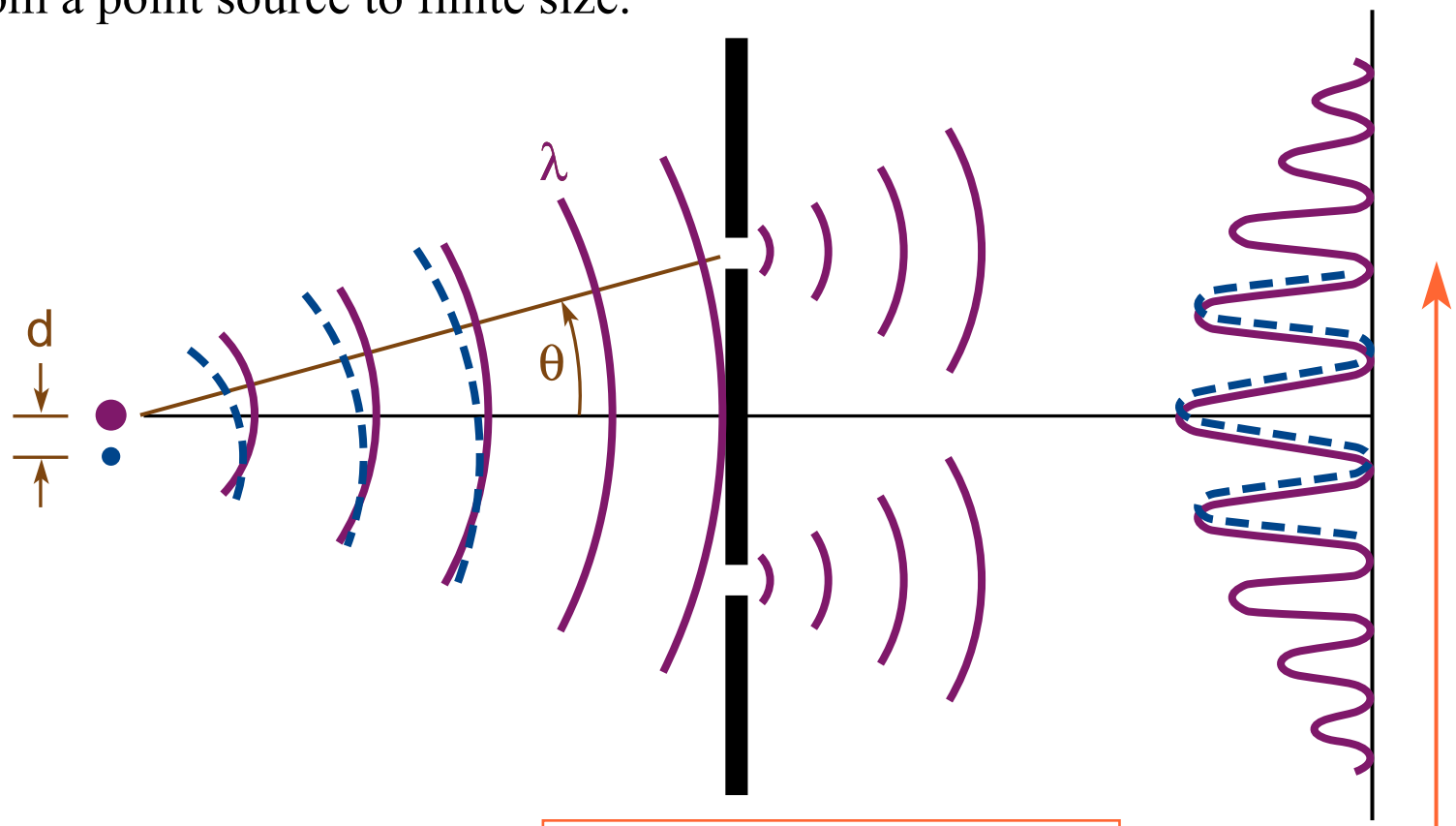
$$\bar{P}_{coh,\lambda/\Delta\lambda} = \frac{e\lambda_u I\eta(\Delta\lambda/\lambda)N^2}{8\pi\epsilon_0 d_x d_y} \cdot \left[1 - \frac{\hbar\omega}{\hbar\omega_0}\right] f(K) \quad (8.9)$$



$$P_{coh} = \frac{(\lambda/2\pi)^2}{(d_x\theta_x)(d_y\theta_y)} P_{laser} \quad (8.11)$$

Young's Double Slit Experiment: Spatial Coherence and the Persistence of Fringes

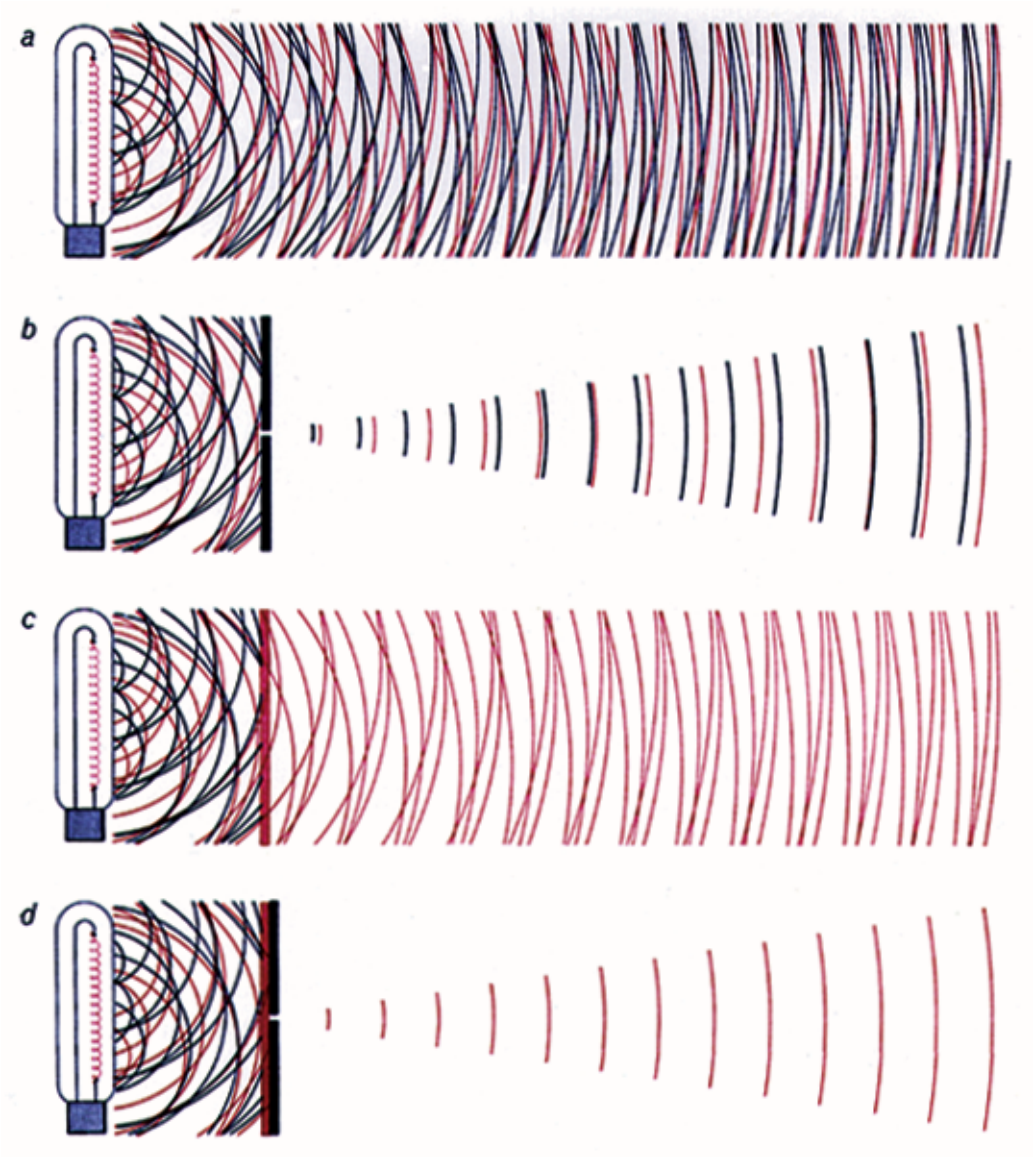
Persistence of fringes as the source grows from a point source to finite size.



$$d \cdot 2\theta|_{FWHM} \simeq \lambda/2$$

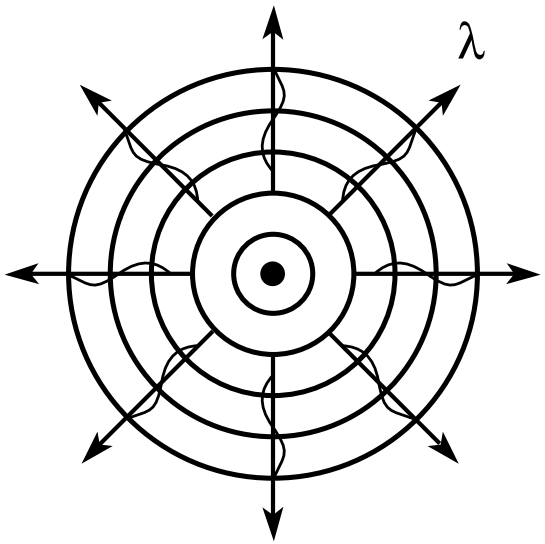
$$\lambda_{coh} = \lambda^2/2\Delta\lambda = \frac{1}{2} N_{coh}\lambda$$

Spatial and Spectral Filtering to Produce Coherent Radiation

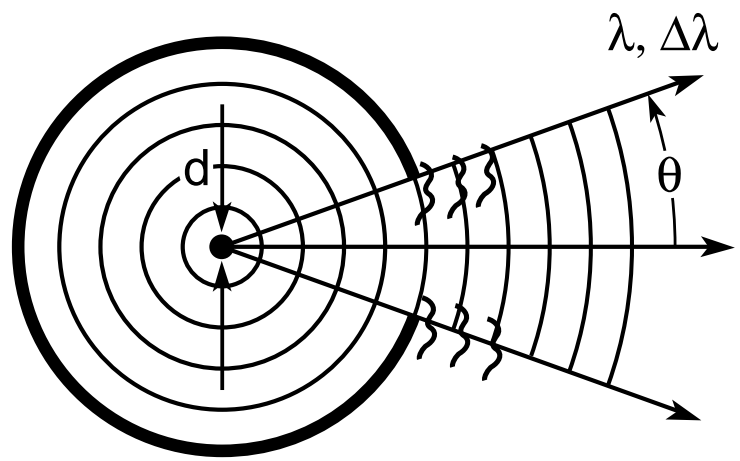


Courtesy of A. Schawlow, Stanford.

Coherence, Partial Coherence, and Incoherence



Point source oscillator
$-\infty < t < \infty$



Source of finite size,
divergence, and duration

Spatial and Temporal Coherence

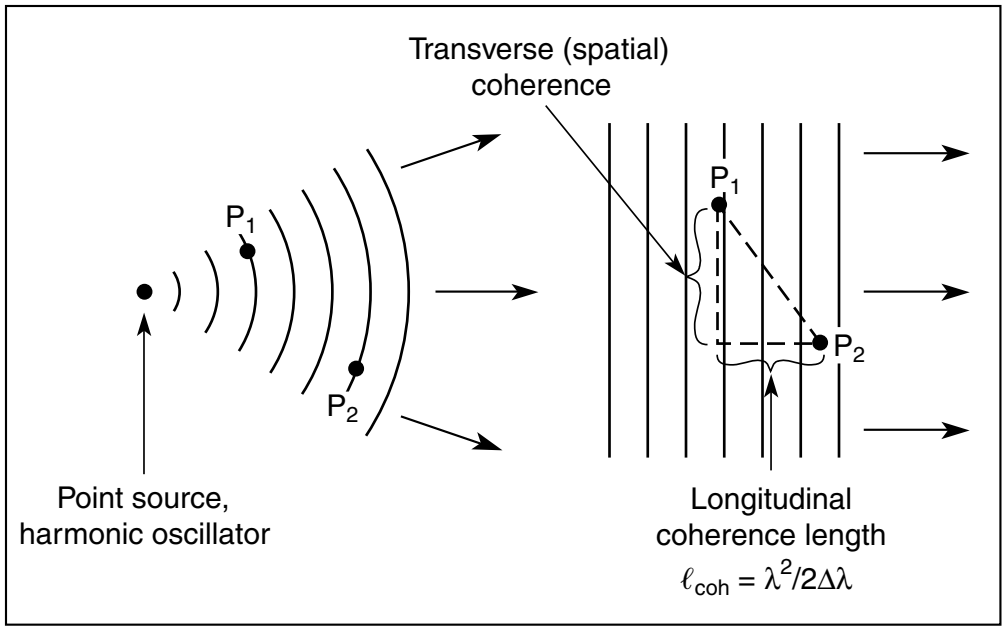
Mutual coherence factor

$$\Gamma_{12}(\tau) \equiv \langle E_1(t+\tau)E_2^*(t)\rangle \qquad (8.1)$$

Normalize degree of spatial coherence (complex coherence factor)

$$\mu_{12} = \frac{\langle E_1(t)E_2^*(t)\rangle}{\sqrt{\langle |E_1|^2\rangle}\sqrt{\langle |E_2|^2\rangle}} \qquad (8.12)$$

A high degree of coherence ($\mu \to 1$) implies an ability to form a high contrast interference (fringe) pattern. A low degree of coherence ($\mu \to 0$) implies an absence of interference, except with great care. In general radiation is partially coherent.



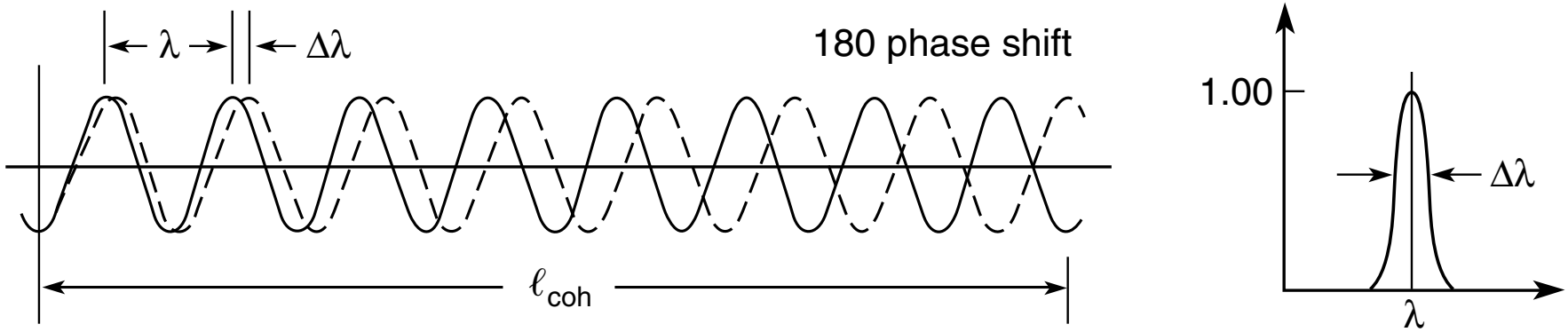
Longitudinal (temporal) coherence length

$$\ell_{\text{coh}} = \frac{\lambda^2}{2\,\Delta\lambda} \qquad (8.3)$$

Full spatial (transverse) coherence

$$d \cdot \theta = \lambda/2\pi \qquad (8.5)$$

Spectral Bandwidth and Longitudinal Coherence Length



Define a coherence length ℓ_{coh} as the distance of propagation over which radiation of spectral width $\Delta\lambda$ becomes 180° out of phase. For a wavelength λ propagating through N cycles

$$\ell_{coh} = N\lambda$$

and for a wavelength $\lambda + \Delta\lambda$, a half cycle less $(N - \frac{1}{2})$

$$\ell_{coh} = (N - \tfrac{1}{2})\,(\lambda + \Delta\lambda)$$

Equating the two

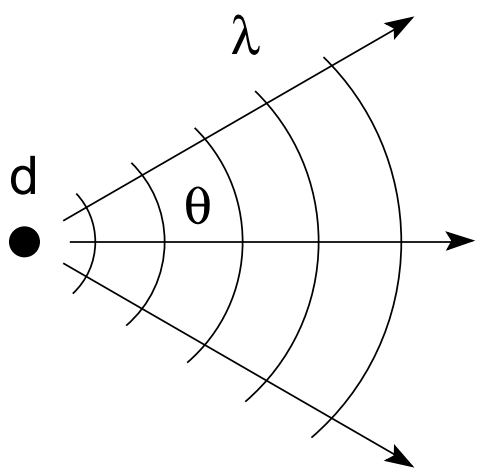
$$N = \lambda/2\Delta\lambda$$

so that

$$\boxed{\ell_{coh} = \frac{\lambda^2}{2\,\Delta\lambda}} \qquad (8.3)$$

A Practical Interpretation of Spatial Coherence

- **Associate spatial coherence with a spherical wavefront.**
- **A spherical wavefront implies a point source.**
- **How small is a "point source"?**



From Heisenberg's Uncertainty Principle ($\Delta x \cdot \Delta p \geq \frac{\hbar}{2}$), the smallest source size "d" you can resolve, with wavelength λ and half angle θ, is

$$d \cdot \theta = \frac{\lambda}{2\pi}$$

Partially Coherent Radiation Approaches Uncertainty Principle Limits

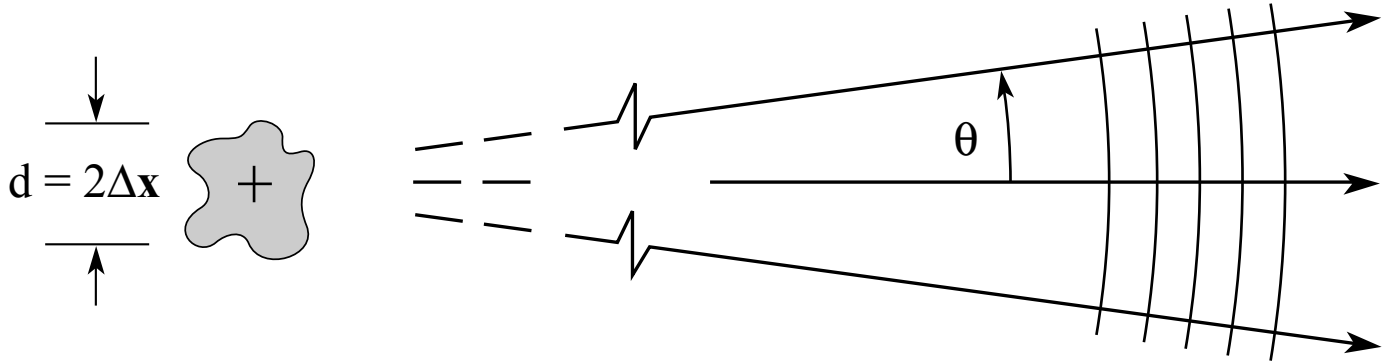
$\Delta\mathbf{x} \cdot \Delta\mathbf{p} \geq \hbar/2$ (8.4)

$\Delta\mathbf{x} \cdot \hbar\Delta\mathbf{k} \geq \hbar/2$

$\Delta\mathbf{x} \cdot \mathbf{k}\Delta\theta \geq 1/2$

$2\Delta\mathbf{x} \cdot \Delta\theta \geq \lambda/2\pi$

Standard deviations of Gaussian distributed functions (Tipler, 1978, pp. 174-189)



Spherical wavefronts occur in the limiting case

$$\boxed{d \cdot \theta = \lambda/2\pi} \quad \text{(spatially coherent)} \Big\} \frac{1}{\sqrt{e}} \text{ quantities}$$

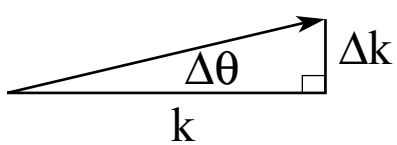
or

$$(d \cdot 2\theta)_{FWHM} \simeq \lambda/2 \Big\} \text{ FWHM quantities}$$

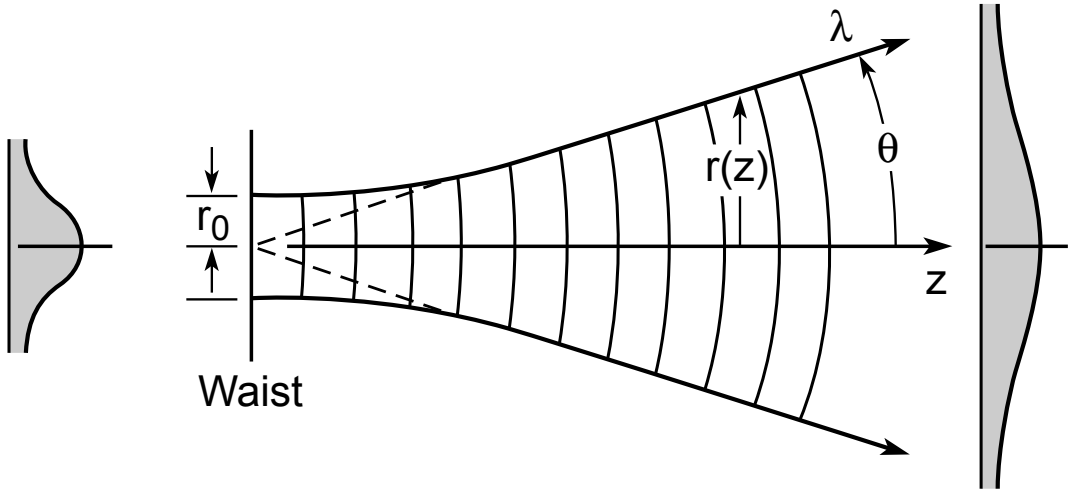
Note:

$\Delta\mathbf{p} = \hbar\Delta k$

$\Delta k = k\Delta\theta$



Propagation of a Gaussian Beam



Gaussian intensity distribution, spherically propagating wave

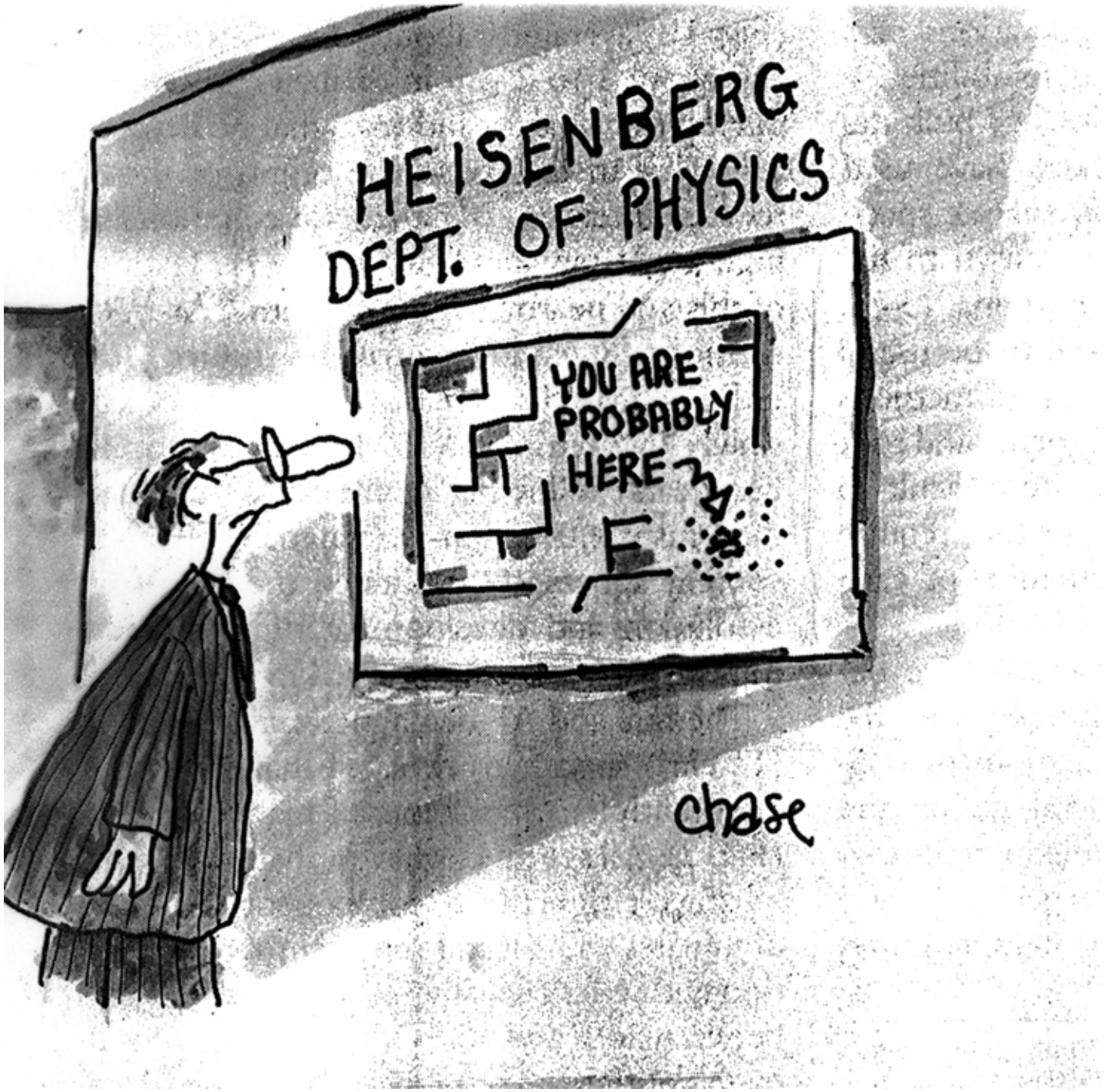
For a spherical wave propagating with a Gaussian intensity distribution, $I/I_0 = \exp(-r^2/2r_0)$, where r_0 is the $1/\sqrt{e}$ waist radius at the origin ($z = 0$), the intensity distribution grows with a $1/\sqrt{e}$ radius given by (Siegman, Lasers)

$$r(z) = r_0\sqrt{1 + \left(\frac{\lambda z}{4\pi r_0^2}\right)^2}$$

In the far field, where $z \gg 4\pi r_0^2/\lambda$, the $1/\sqrt{e}$ divergence half angle is

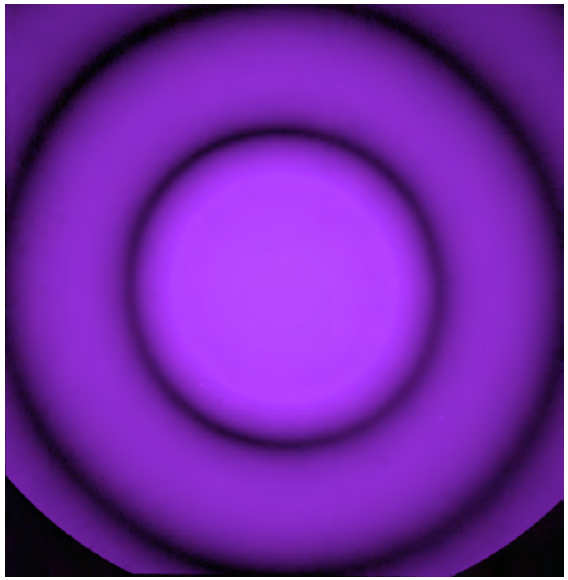
$$\theta \equiv \frac{r(z)}{z} = \frac{\lambda}{4\pi r_0}$$

with waist diameter $d = 2r_0$, we have TEM_{00} radiation with $d \cdot \theta = \lambda/2\pi$

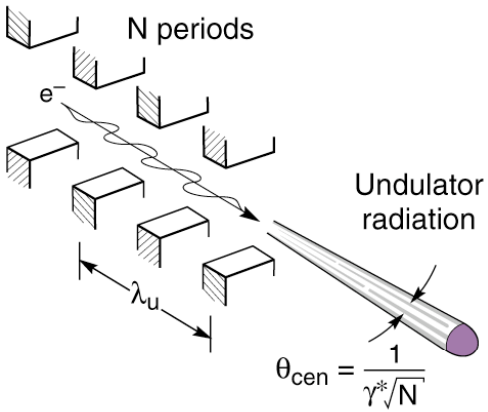

HEISENBERG
DEPT. OF PHYSICS
YOU ARE
PROBABLY
HERE
chase

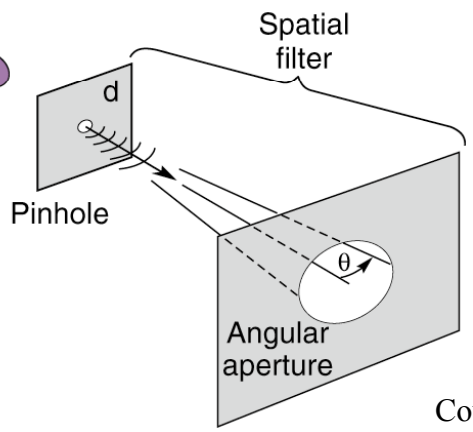
Spatially Coherent Undulator Radiation

λ = 11.2 nm



λ = 13.4 nm





1 μm^D pinhole

25 mm wide CCD at 410 mm

Courtesy of Patrick Naulleau, LBNL.

Spatial Filtering of Undulator Radiation

Undulator central radiation cone ($\frac{\lambda}{\Delta\lambda} = N$; $\theta = \frac{1}{\gamma^* \sqrt{N}}$):

$$\bar{P}_{\text{cen}} = \frac{\pi e \gamma^2 I}{\epsilon_0 \lambda_u} \frac{K^2}{(1 + K^2/2)^2} f(K) \qquad (5.41a)$$

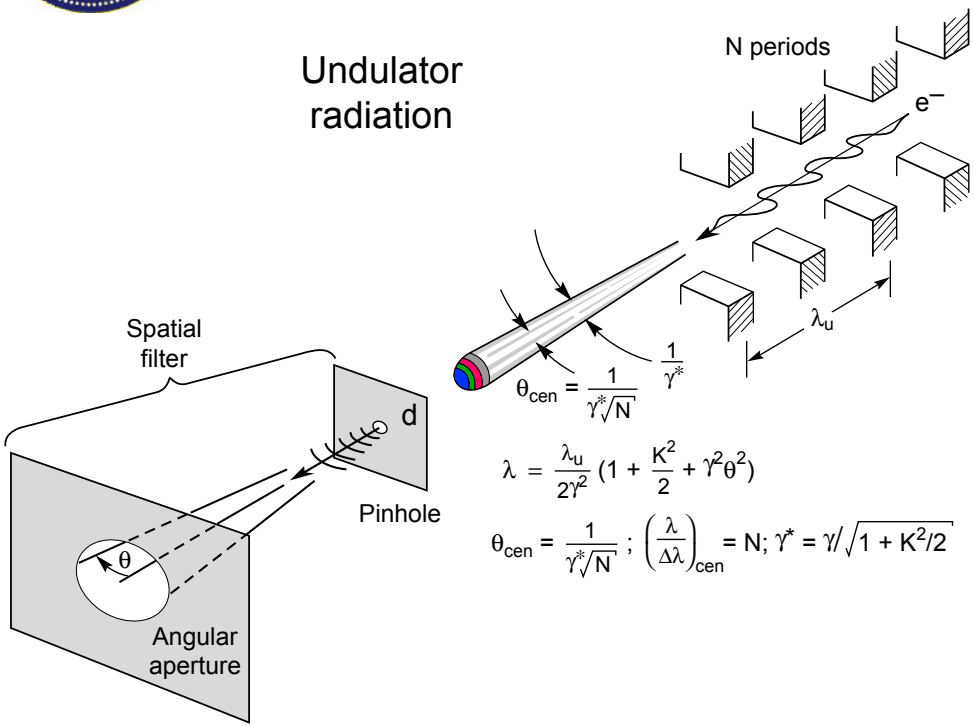
With spatial filtering (a pinhole and an angular aperture):

$$\bar{P}_{\text{coh},N} = \underbrace{\left(\frac{\lambda/2\pi}{d_x \theta_x}\right)}_{\leq 1} \underbrace{\left(\frac{\lambda/2\pi}{d_y \theta_y}\right)}_{\leq 1} \bar{P}_{\text{cen}} \qquad (8.6)$$

With eq.(5.28), $\lambda = \frac{\lambda_u}{2\gamma^2}\left(1 + \frac{K^2}{2} + \gamma^2\theta^2\right)$. Convert to photon energies where for $\theta = 0$, $\hbar\omega = \frac{\hbar\omega_0}{1 + K^2/2}$, and where $\hbar\omega_0 \equiv 4\pi\hbar c\gamma^2/\lambda_u$ corresponds to $K = 0$. For small electron beam diverence, $\sigma'^2_{x,y} << \theta^2_{\text{cen}}$, the spatially coherent power is

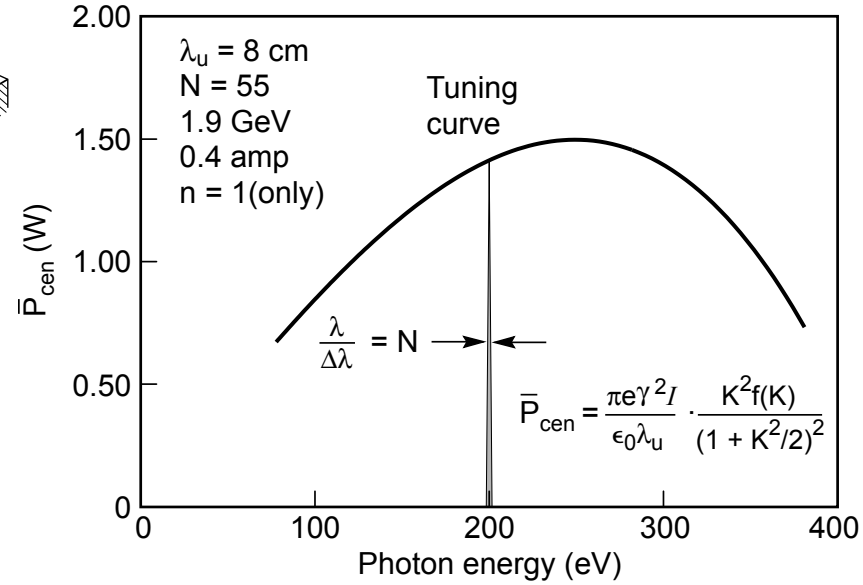
$$\boxed{\bar{P}_{\text{coh},N} = \frac{e\lambda_u I N}{8\pi\epsilon_0 d_x d_y}\left(1 - \frac{\hbar\omega}{\hbar\omega_0}\right) f(\hbar\omega/\hbar\omega_0)} \qquad (8.9)$$

Spatially Filtered Undulator Radiation



$$\lambda = \frac{\lambda_u}{2\gamma^2}\left(1 + \frac{K^2}{2} + \gamma^2\theta^2\right)$$

$$\theta_{cen} = \frac{1}{\gamma^*\sqrt{N}}\ ;\ \left(\frac{\lambda}{\Delta\lambda}\right)_{cen} = N;\ \gamma^* = \gamma/\sqrt{1 + K^2/2}$$

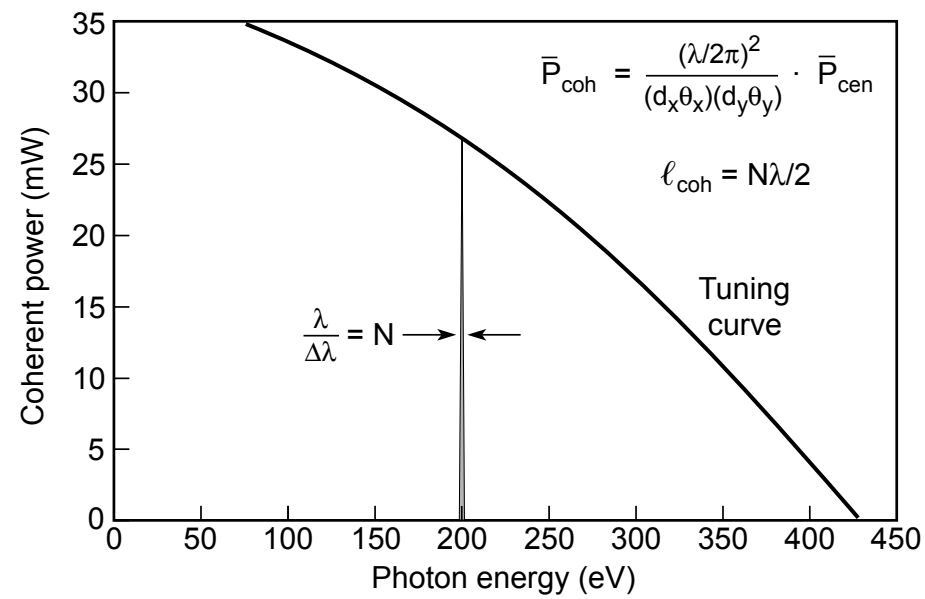


Using a pinhole-aperture spatial filter, passing only radiation that satisfies $d \cdot \theta = \lambda/2\pi$

$$\bar{P}_{\mathrm{coh},N} = \left(\frac{\lambda/2\pi}{d_x\theta_x}\right)\left(\frac{\lambda/2\pi}{d_y\theta_y}\right)\bar{P}_{\mathrm{cen}} \qquad (8.6)$$

$$\bar{P}_{\mathrm{coh},N} = \frac{e\lambda_u I N}{8\pi\epsilon_0 d_x d_y}\left(1 - \frac{\hbar\omega}{\hbar\omega_0}\right) f(\hbar\omega/\hbar\omega_0) \qquad (8.9)$$

for $d_x = 2\sigma_x$, $d_y = 2\sigma_y$, $\theta_{Tx} \rightarrow \theta_x$, $\theta_{Ty} \rightarrow \theta_y$, and $\sigma'^2 << \theta_{cen}^2$.



Spatial and Spectral Filtering of Undulater Radiation

In addition to the pinhole – angular aperture for spatial filtering and spatial coherence, add a monochromator for narrowed bandwidth and increased temporal coherence:

$$\bar{P}_{\mathrm{coh},\lambda/\Delta\lambda} = \underbrace{\eta}_{\substack{\text{beamline}\\ \text{efficiency}}} \underbrace{\frac{(\lambda/2\pi)^2}{(d_x\theta_x)(d_y,\theta_y)}}_{\substack{\text{spatial}\\ \text{filtering}}} \cdot \underbrace{N\frac{\Delta\lambda}{\lambda}}_{\substack{\text{spectral}\\ \text{filtering}}} \cdot \bar{P}_{\mathrm{cen}} \qquad (8.10a)$$

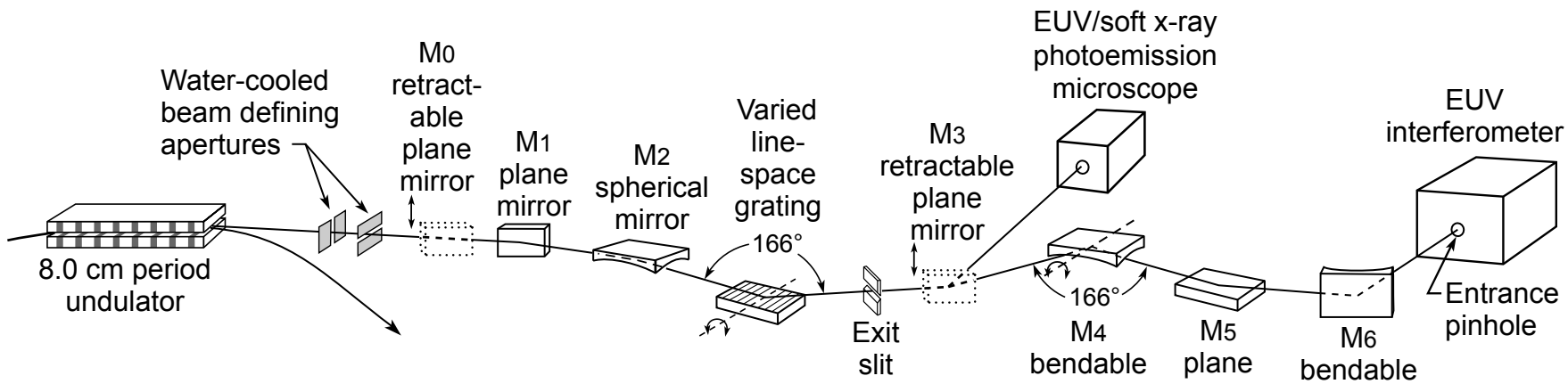
which for $\sigma'^2_{x,y} << \theta^2_{cen}$ (the undulator condition) gives the spatially and temporally coherent power ($d \cdot \theta = \lambda/2\pi$; $l_{\mathrm{coh}} = \dfrac{\lambda^2}{2\,\Delta\lambda}$)

$$\boxed{\bar{P}_{\mathrm{coh},\lambda/\Delta\lambda} = \frac{e\lambda_u I\eta(\Delta\lambda/\lambda)N^2}{8\pi\epsilon_0 d_x d_y} \cdot \left(1 - \frac{\hbar\omega}{\hbar\omega_0}\right) f(\hbar\omega/\hbar\omega_0)} \qquad (8.10c)$$

which we note scales as N^2.

Spatially and Spectrally Filtered Undulator Radiation

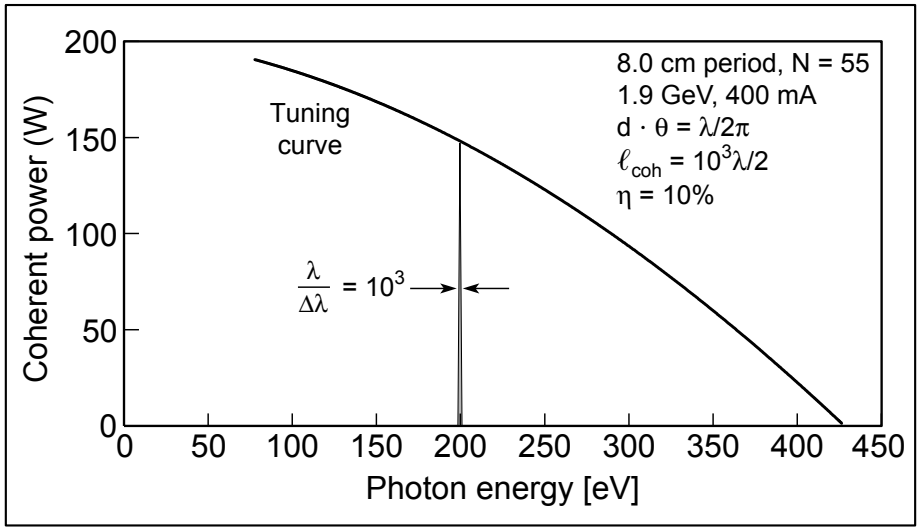
- Pinhole filtering for full spatial coherence
- Monochromator for spectral filtering to $\lambda/\Delta\lambda > N$



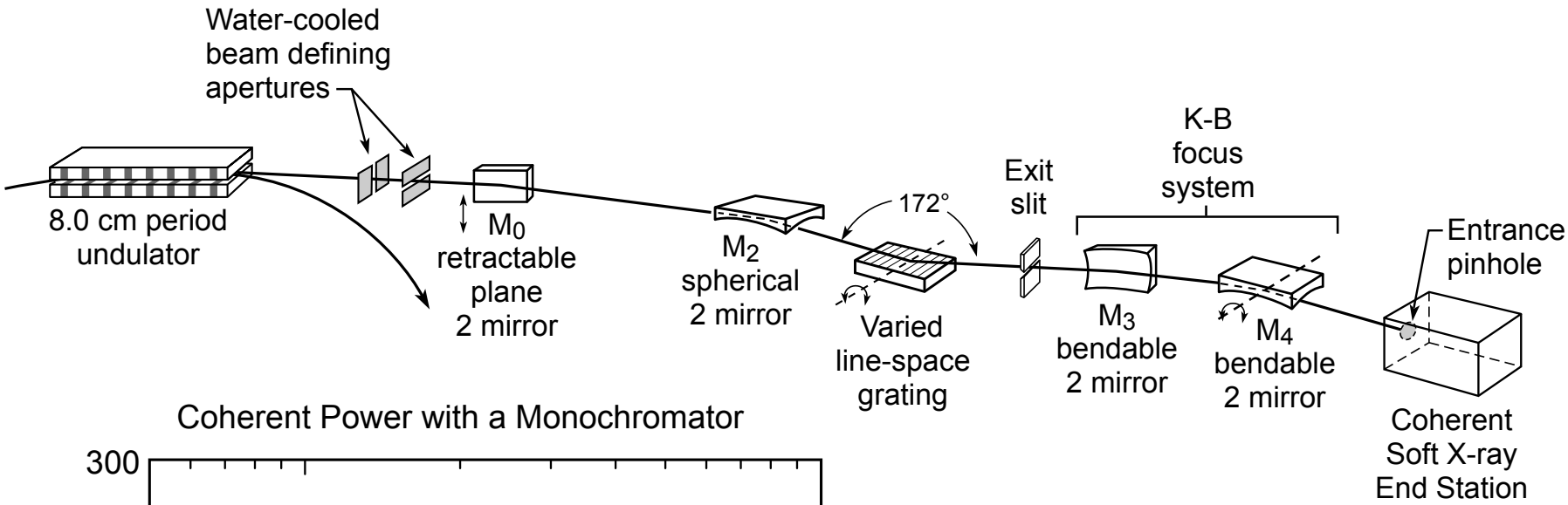
$$\bar{P}_{\mathrm{coh},\lambda/\Delta\lambda} = \underbrace{\eta}_{\text{beamline efficiency}} \underbrace{\frac{(\lambda/2\pi)^2}{(d_x\theta_x)(d_y\theta_y)}}_{\text{spatial filtering}} \cdot \underbrace{N\frac{\Delta\lambda}{\lambda}}_{\text{spectral filtering}} \cdot \bar{P}_{\mathrm{cen}} \qquad (8.10a)$$

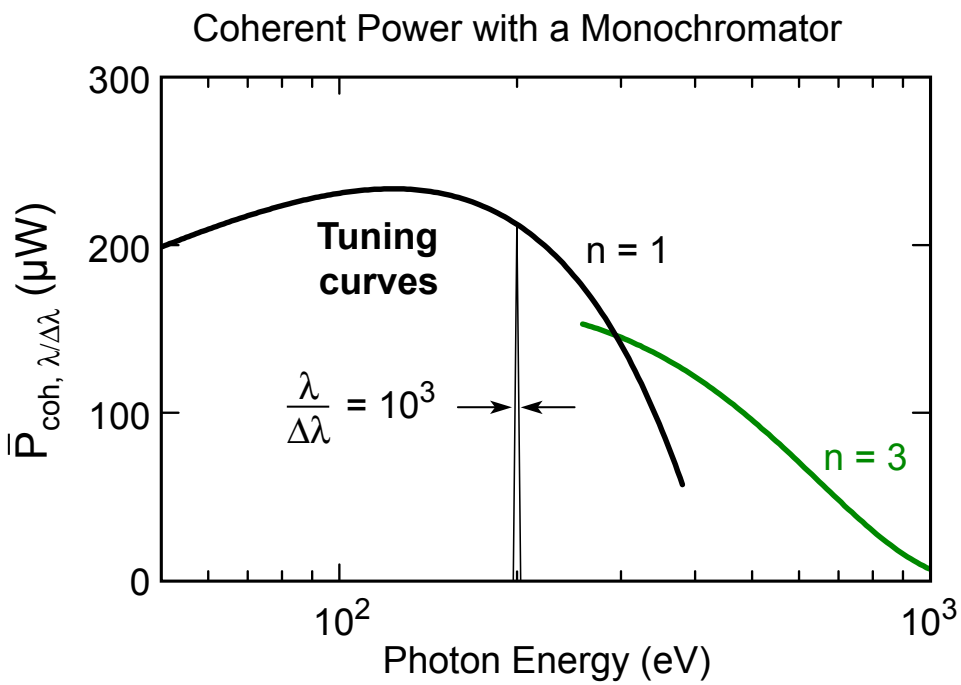
$$\bar{P}_{\mathrm{coh},\lambda/\Delta\lambda} = \frac{e\lambda_u I\eta(\Delta\lambda/\lambda)N^2}{8\pi\epsilon_0 d_x d_y} \cdot \left(1 - \frac{\hbar\omega}{\hbar\omega_0}\right) f(\hbar\omega/\hbar\omega_0)$$

$$\left(\sigma'^2 \ll \theta_{\mathrm{cen}}^2\right) \qquad (8.10c)$$



Coherent Soft X-Ray Beamline: Use of a Higher Harmonic (n = 3) to AccessShorter Wavelengths

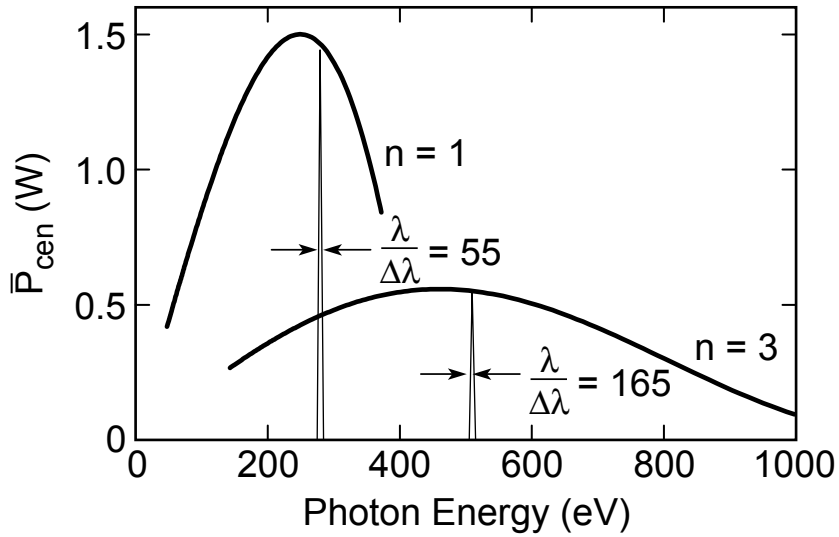




8.0 cm period, N = 55
1.9 GeV, 400 mA
$d \cdot \theta = \lambda/2\pi$
$\ell_{coh} = 1000\ \lambda/2$
$\eta_{euv} = 10\%$, $\eta_{sxr} = 10\%$

Coherent Power at the ALS

U8



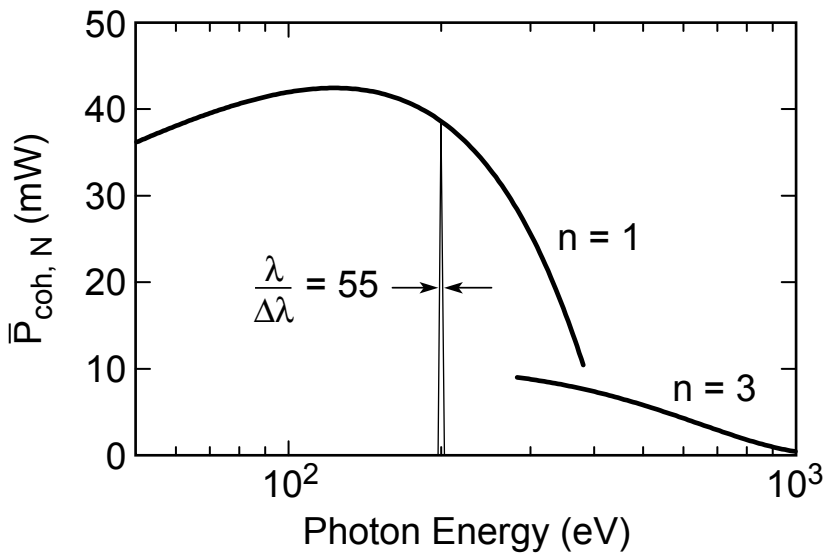
1.9 GeV, 400 mA

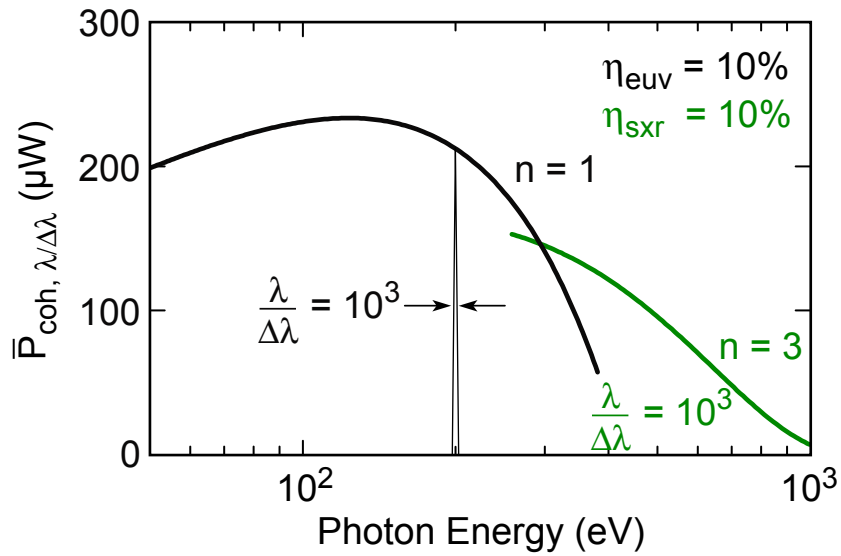
λ_u = 80 mm, N = 55

$0.5 \le K \le 4.0$

σ_x = 260 μm, σ_x' = 23 μr

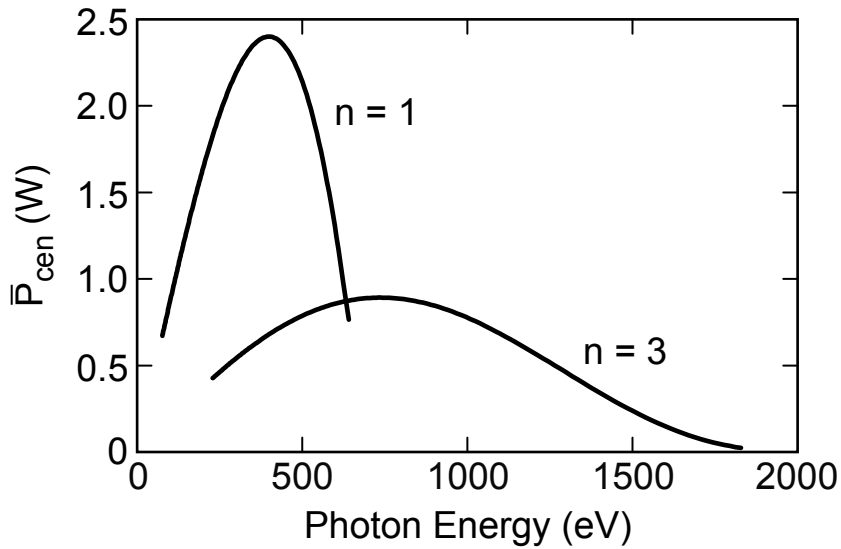
σ_y = 16 μm, σ_y' = 3.9 μr





Coherent Power at the ALS

U5



1.9 GeV, 400 mA

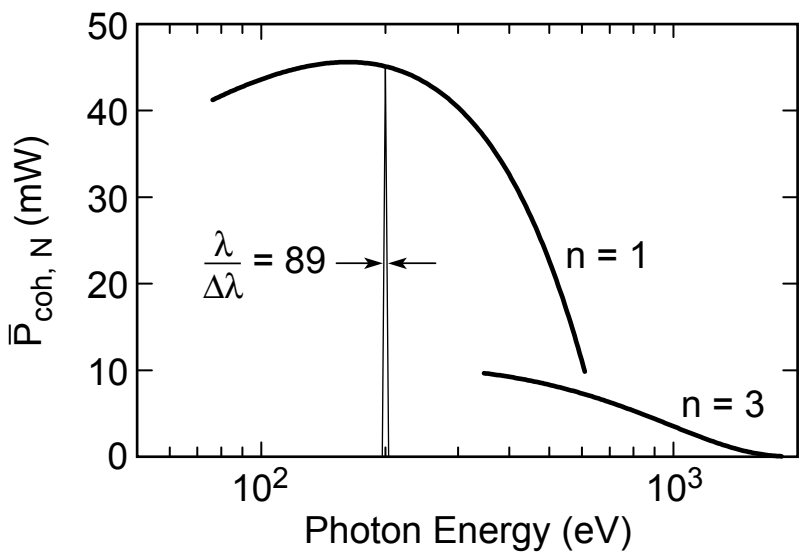
λ_u = 50 mm, N = 89

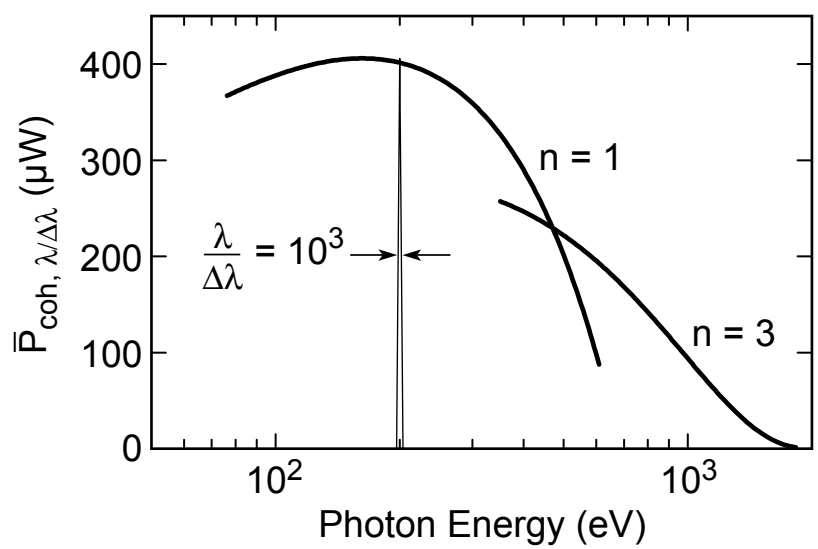
$0.5 \le K \le 4.0$

σ_x = 260 μm, σ_x' = 23 μr

σ_y = 16 μm, σ_y' = 3.9 μr

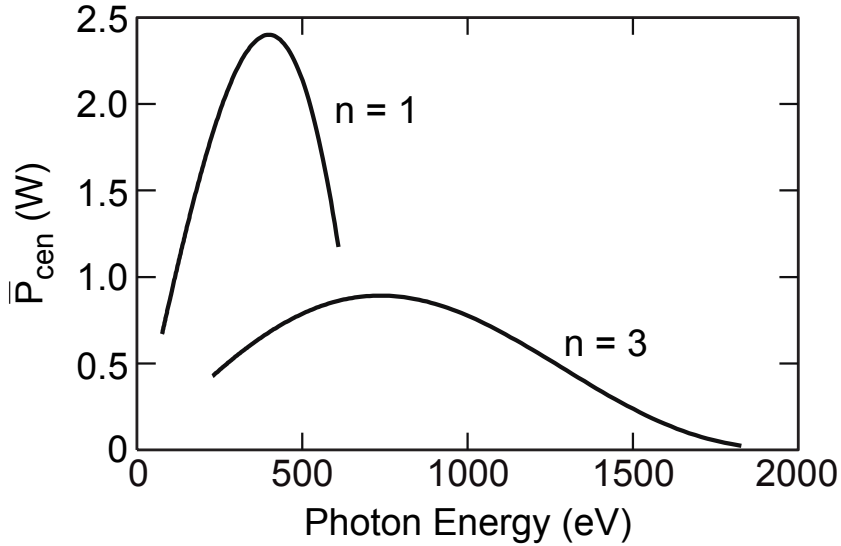
η = 10%





Coherent Power for an EPU at the ALS

U5 EPU



1.9 GeV, 400 mA

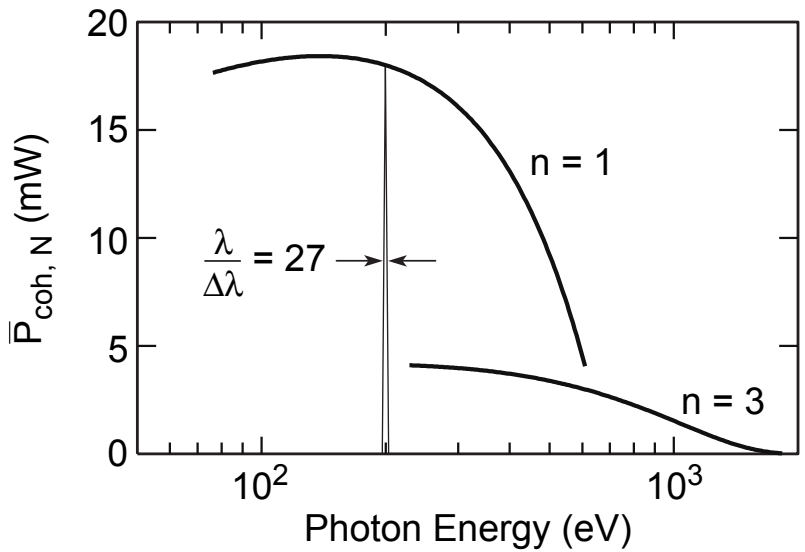
λ_u = 50 mm, N = 27

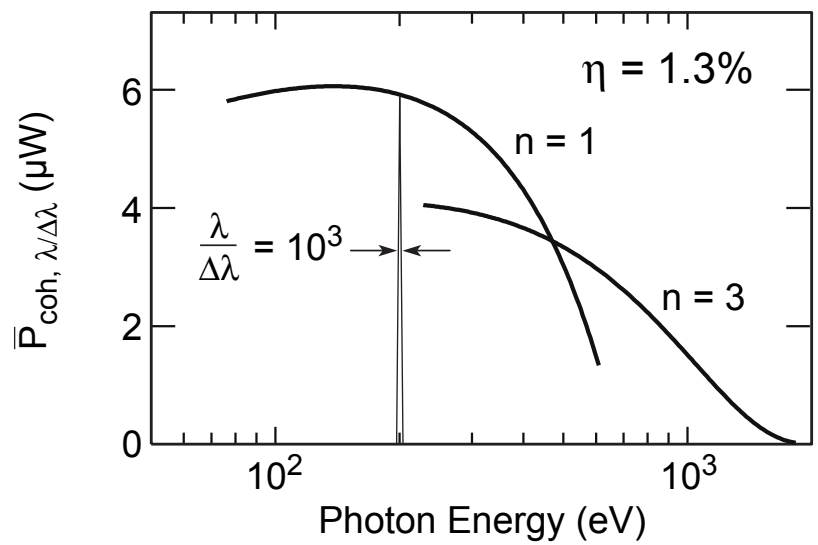
$0.5 \le K \le 4.0$

σ_x = 260 μm, σ_x' = 23 μr

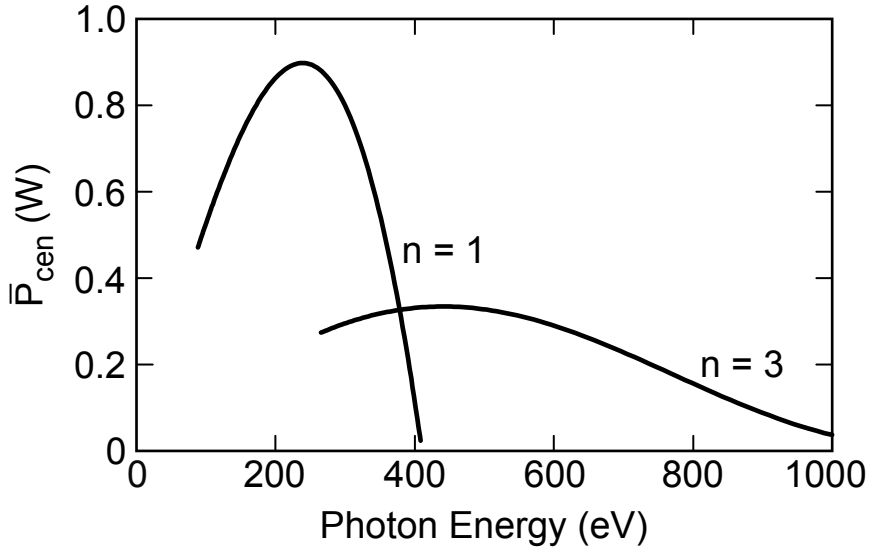
σ_y = 16 μm, σ_y' = 3.9 μr

θ_{cen} = 61μr @ K = 0.87 (500 eV)





Coherent Power at MAX II



1.5 GeV, 250 mA

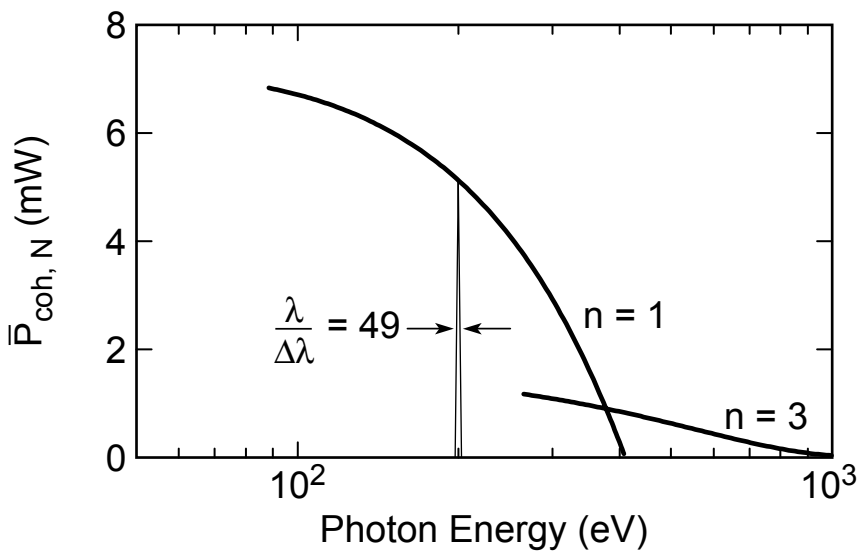
λ_u = 52 mm, N = 49

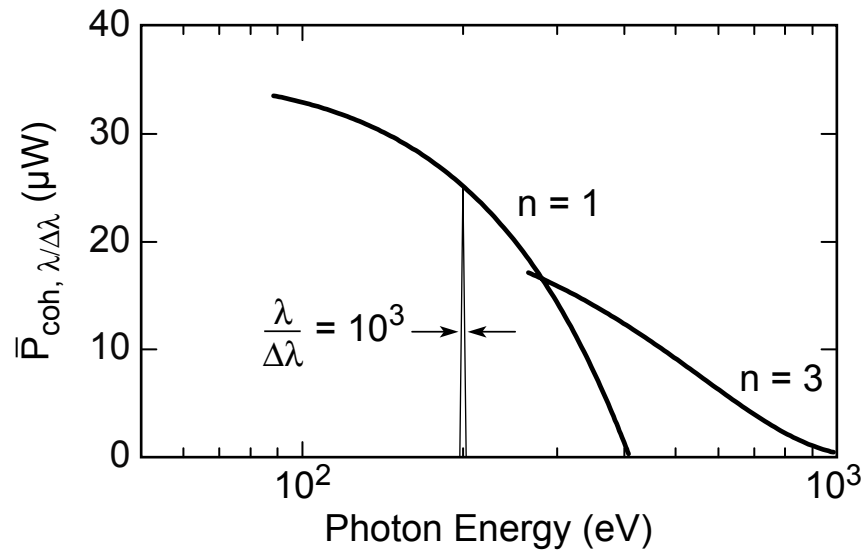
$0.1 \le K \le 2.7$

σ_x = 300 μm, σ_x' = 26 μr

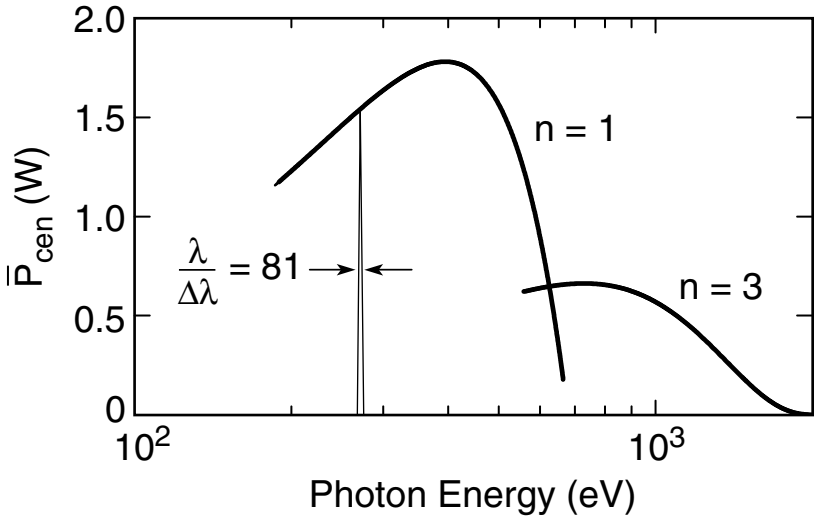
σ_y = 45 μm, σ_y' = 20 μr

η = 10%





Coherent Power at Elettra



2.0 GeV, 300 mA

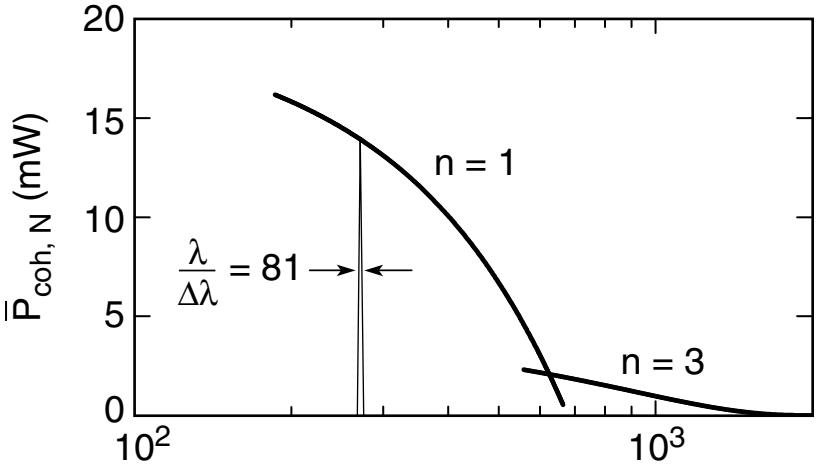
$\lambda_u = 56$ mm, N = 81

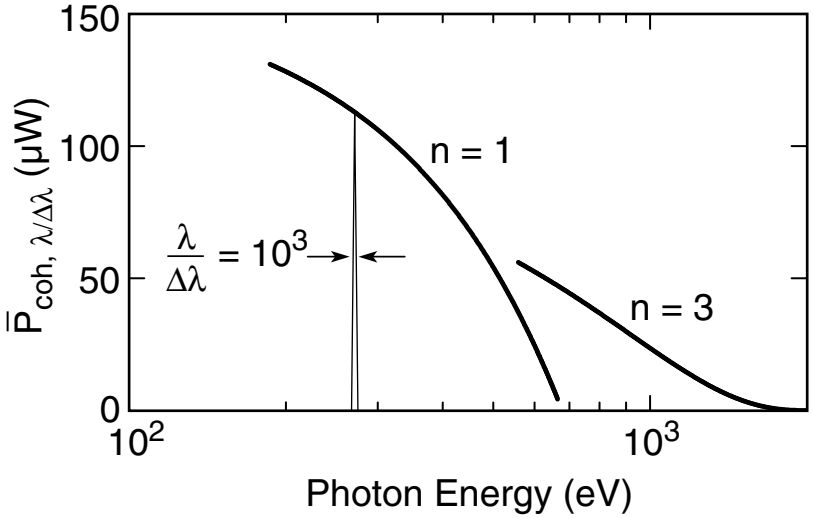
$0.5 \leq K \leq 2.3$

$\sigma_x = 255$ μm, $\sigma_x' = 23$ μr

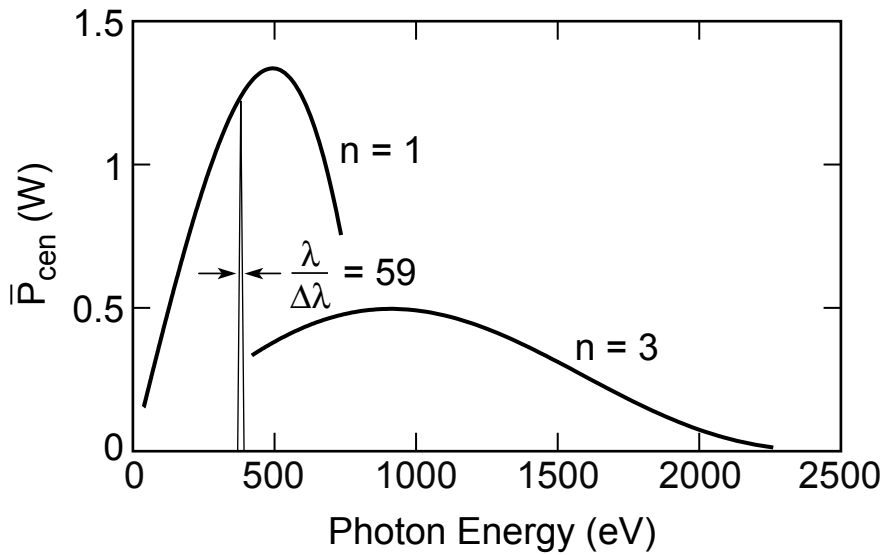
$\sigma_y = 31$ μm, $\sigma_y' = 9$ μr

$\eta = 10\%$





Coherent Power at the Pohang Light Source



2.5 GeV, 180 mA

λ_u = 70 mm, N = 59

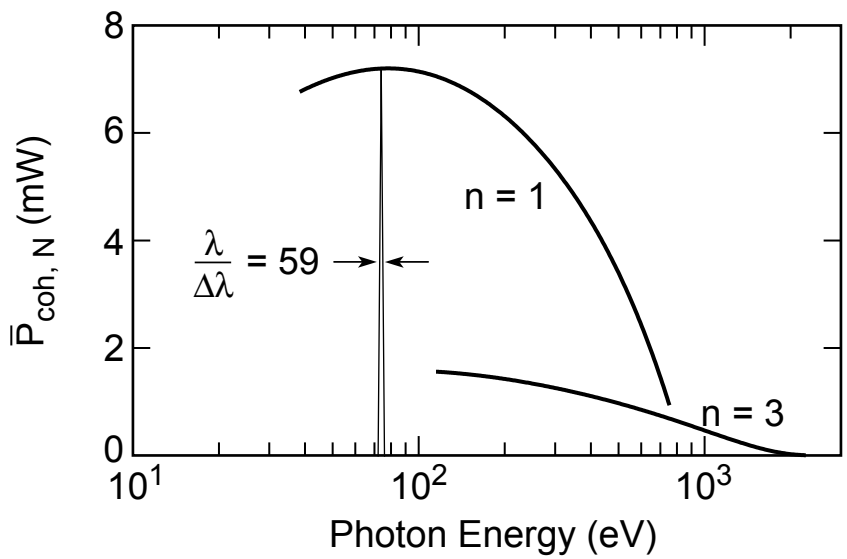
$0 \le K \le 6.5$

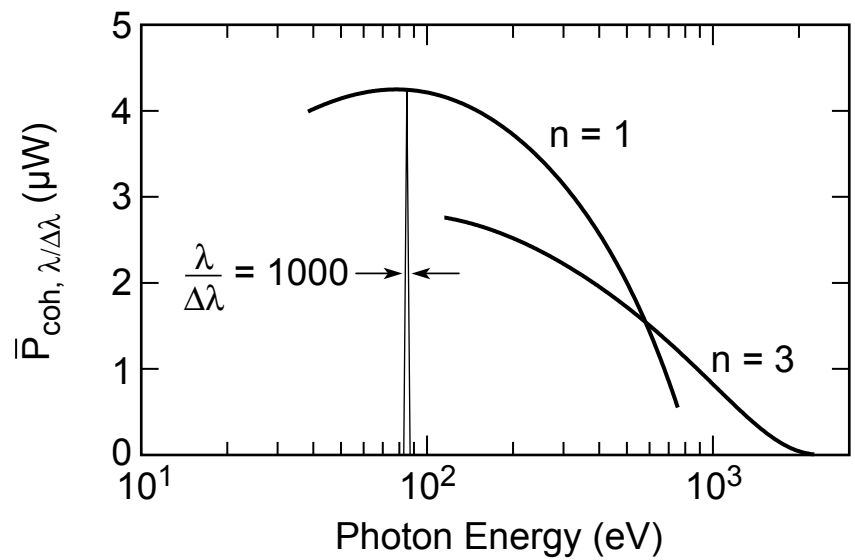
σ_x = 433 μm, σ_x' = 43 μr

σ_y = 27 μm, σ_y' = 6.8 μr

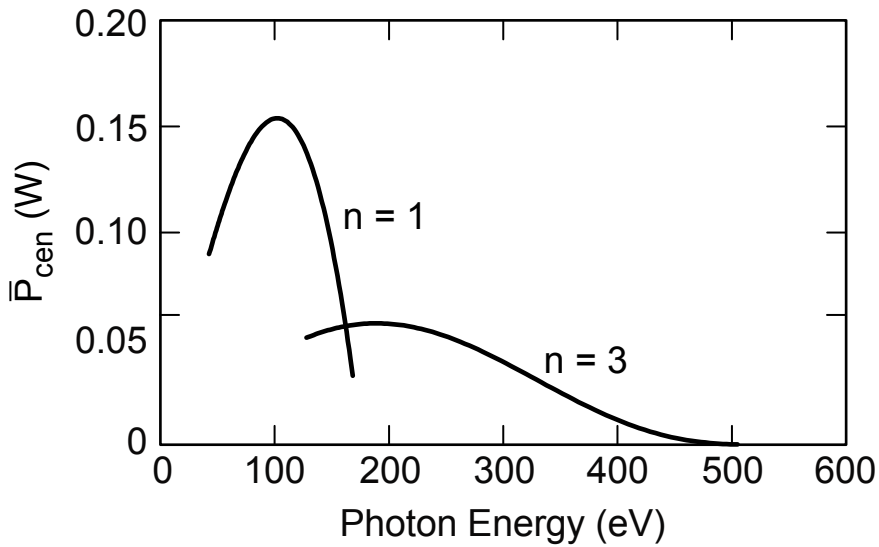
θ_{cen} = 33 μr @ K = 1

η = 10%





Coherent Power at New Subaru



1.0 GeV, 100 mA

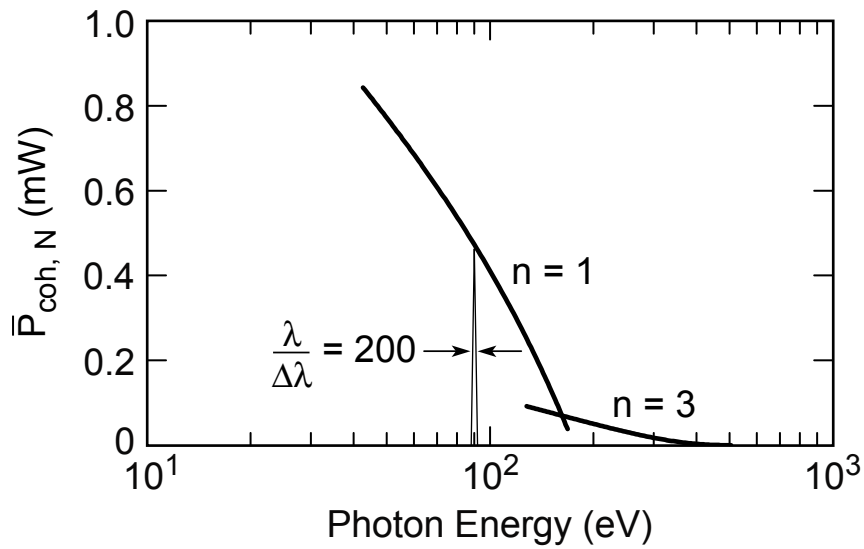
λ_u = 54 mm, N = 200

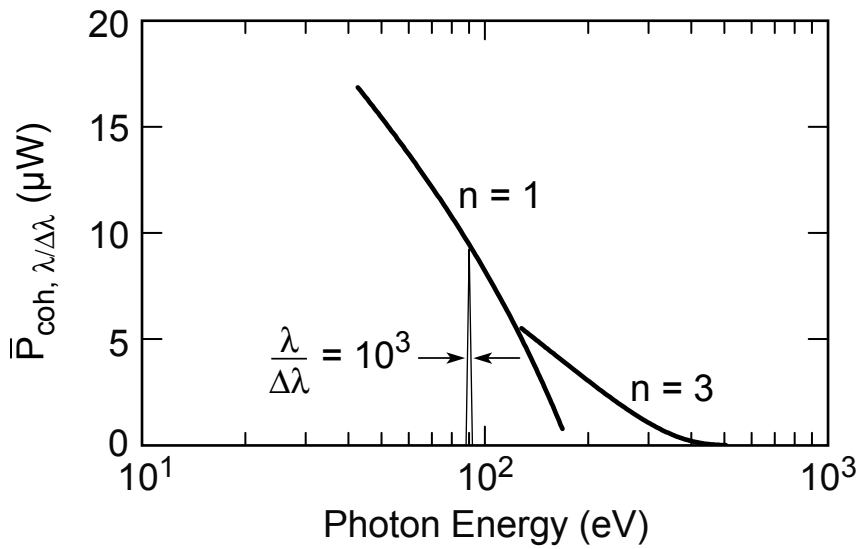
$0.3 \le K \le 2.5$

σ_x = 450 μm, σ_x' = 89 μr

σ_y = 220 μm, σ_y' = 18 μr

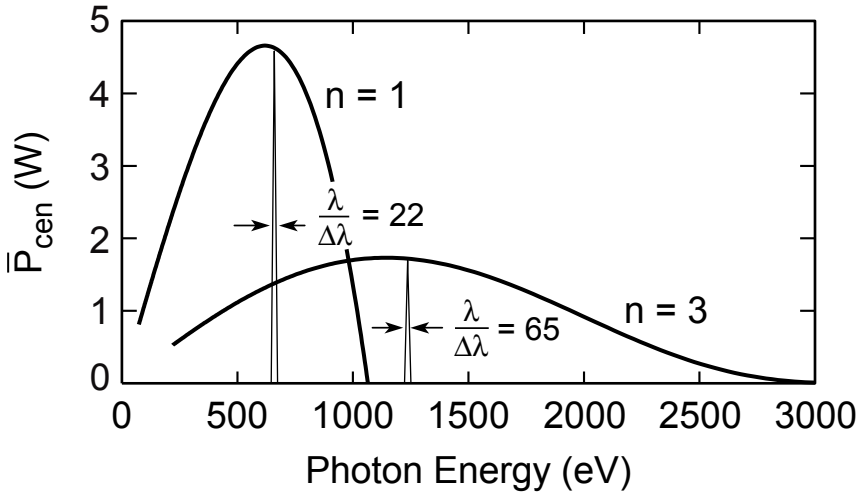
η = 10%





Coherent Power at the Canadian Light Source

CLS, 75 mm EPU



2.9 GeV, 500 mA

λ_u = 75 mm, N = 21.5

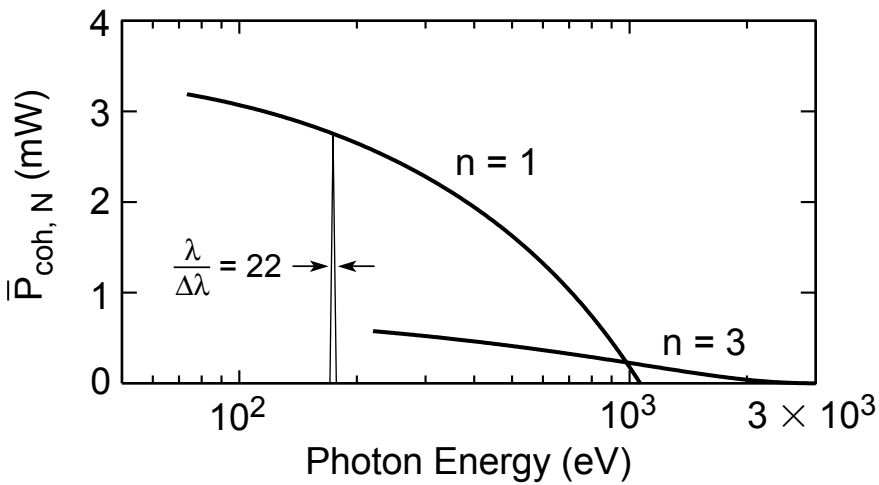
$0 \le K \le 5.2$

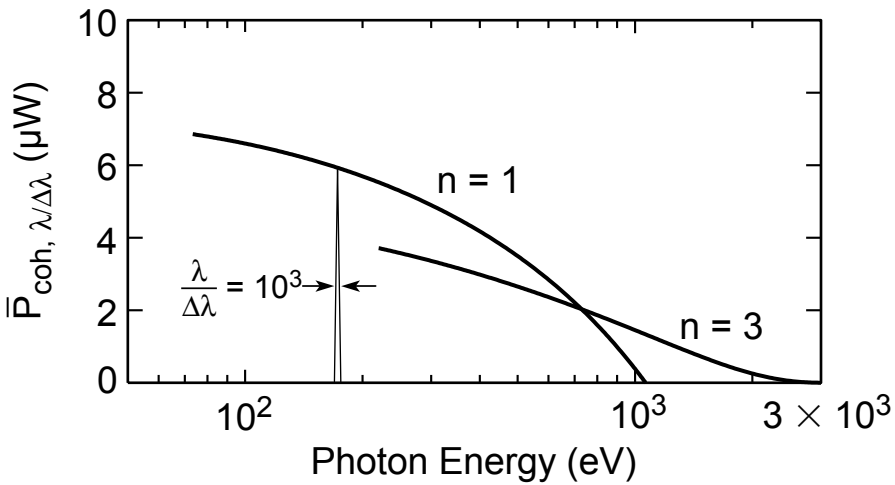
σ_x = 440 μm, σ_x' = 47 μr

σ_y = 88 μm, σ_y' = 21 μr

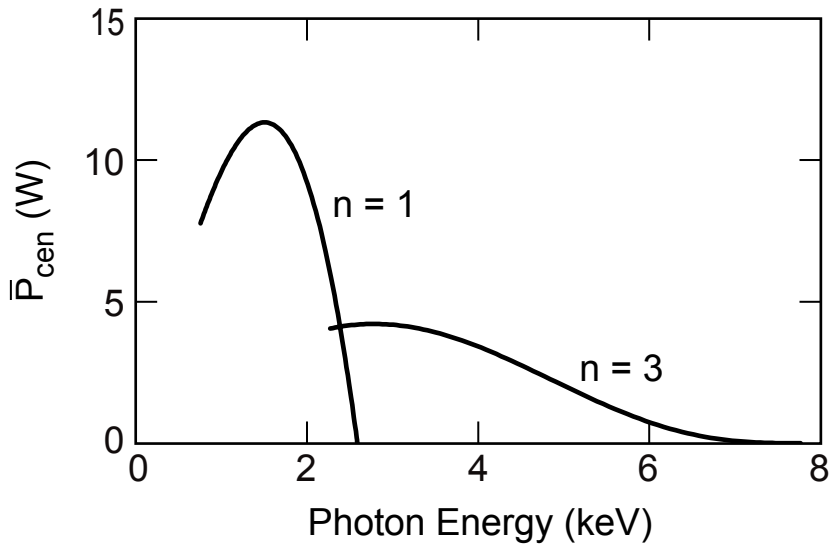
θ_{cen} = 47 μr @ K = 1

η = 10%





Coherent Power Predicted with SPEAR 3 at SSRL



3.0 GeV, 500 mA

λ_u = 3.3 cm, N = 105

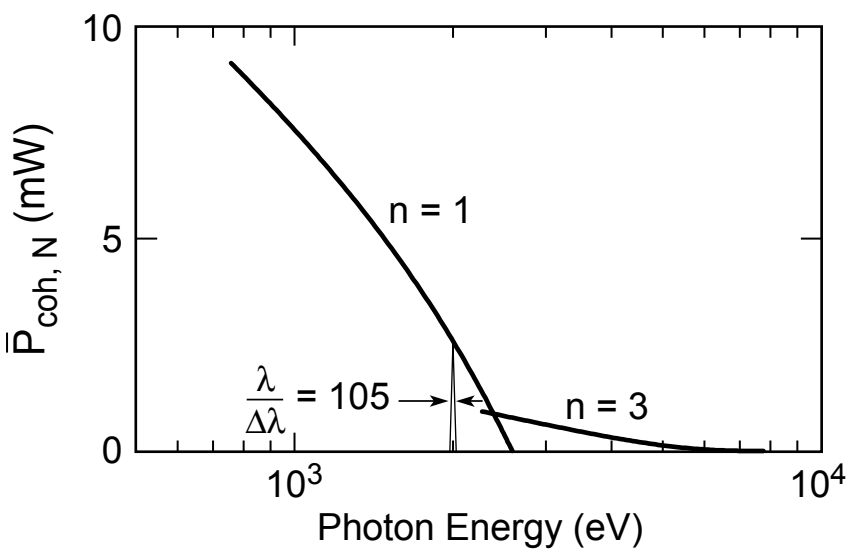
$0 \le K \le 2.2$

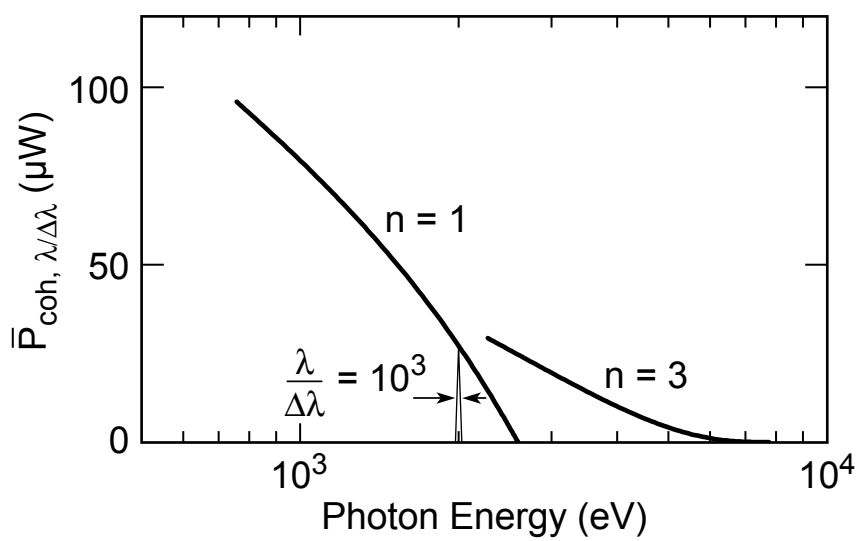
σ_x = 436 μm, σ_x' = 43 μrad

σ_y = 30 μm, σ_y' = 6.3 μrad

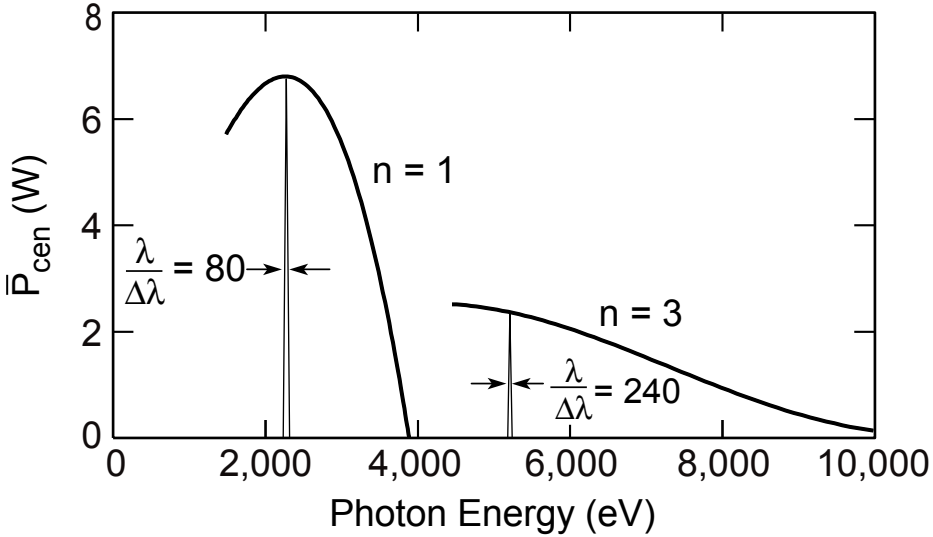
θ_{cen} = 17 μrad @ K = 1

η = 10%





Coherent Power at the Australian Synchrotron



3.0 GeV, 200 mA

λ_u = 22 mm, N = 80

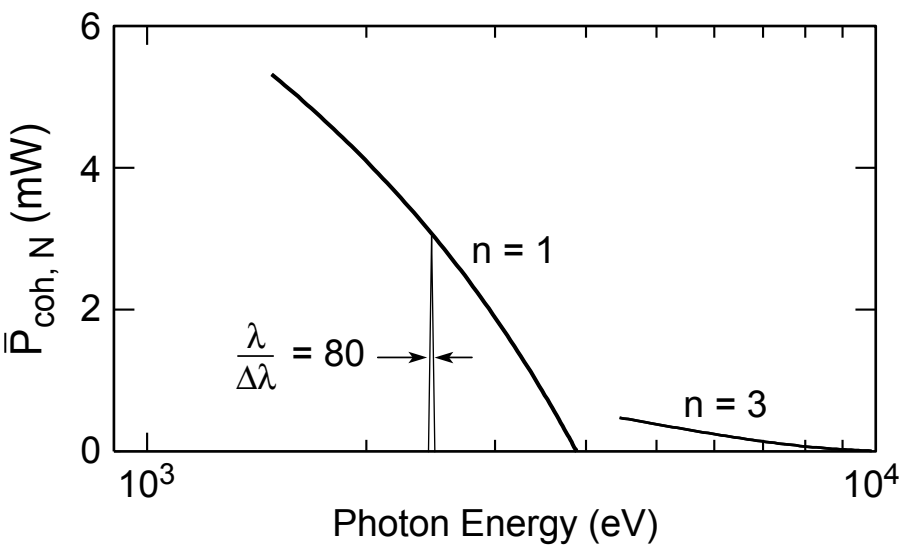
$0 \le K \le 1.8$

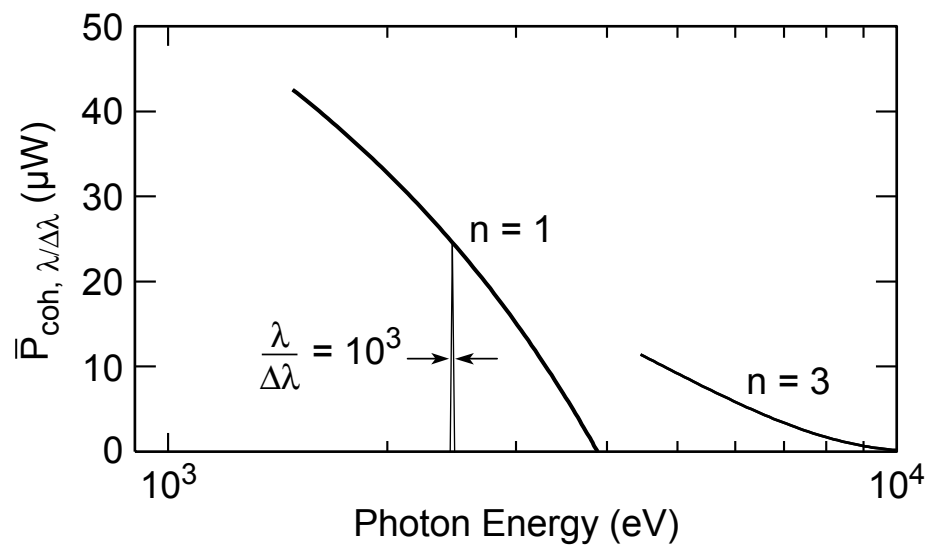
σ_x = 320 μm, σ_x' = 34 μrad

σ_y = 16 μm, σ_y' = 6 μrad

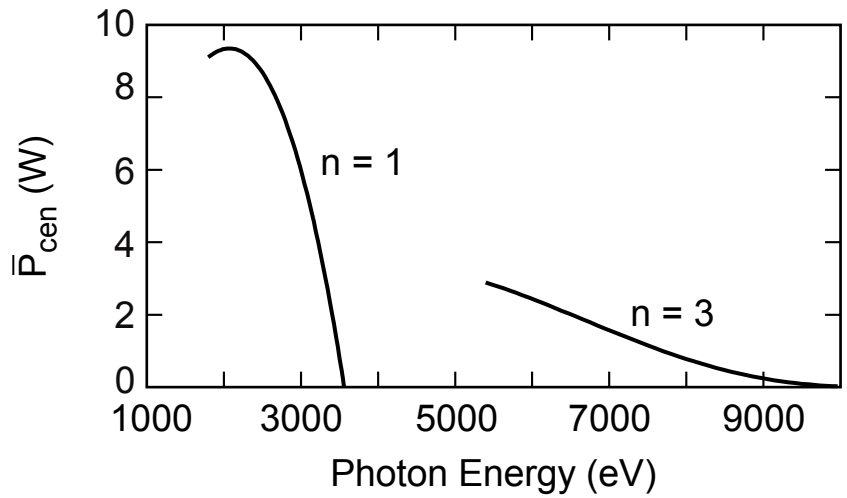
θ_{cen} = 23 μrad @ K = 1

η = 10%





Coherent Power at the UK's Diamond Synchrotron Facility



3.0 GeV, 300 mA

λ_u = 2.4 cm, N = 82

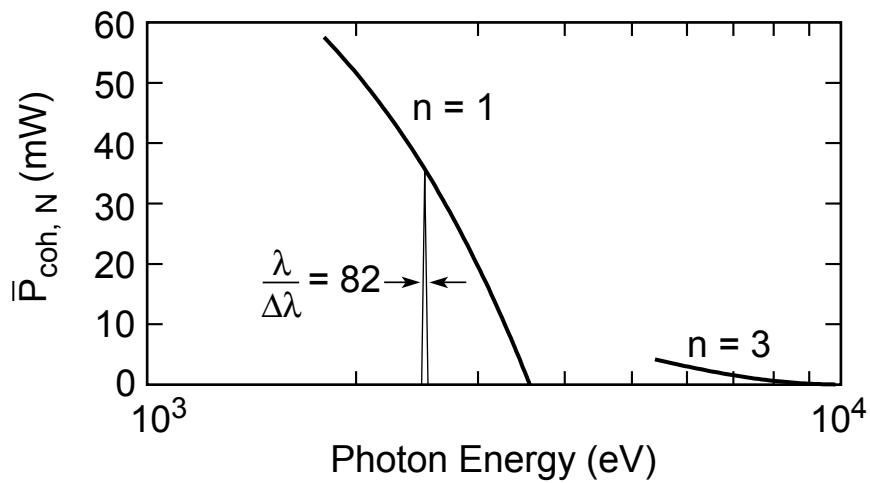
$0 \le K \le 1.4$

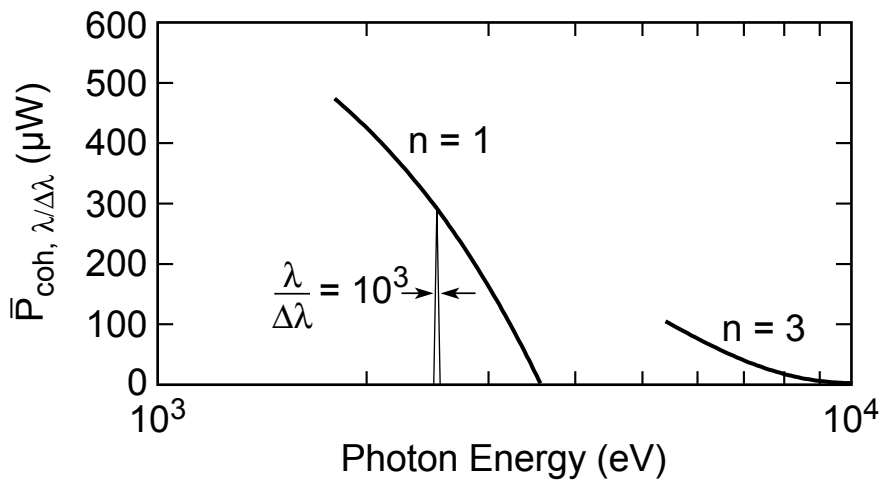
σ_x = 123 μm, σ_x' = 24 μr

σ_y = 6.4 μm, σ_y' = 4.2 μr

θ_{cen} = 23 μr @ K = 1

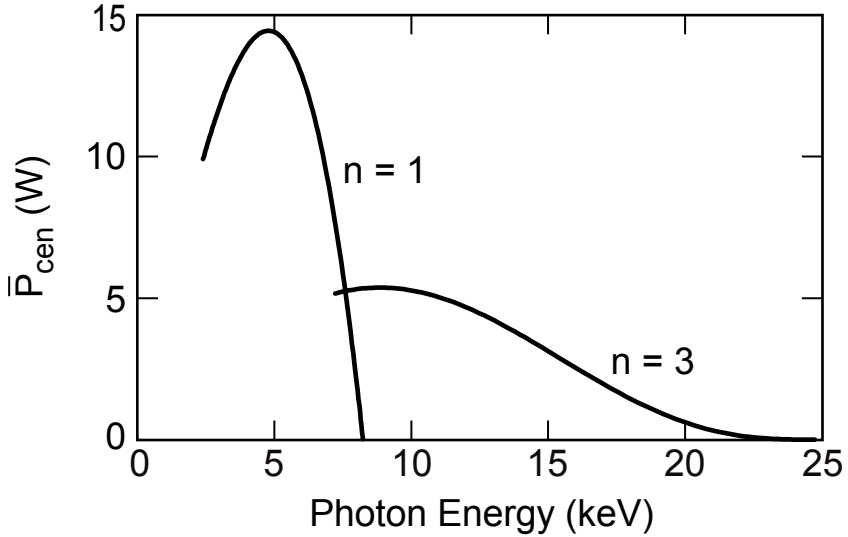
η = 10%





Courtesy of Brian Kennedy (King's College London), Susan Smith (Daresbury), and Yanwei Liu (LBNL)

Coherent Power at ESRF



6.0 GeV, 200 mA

λ_u = 42 mm, N = 38

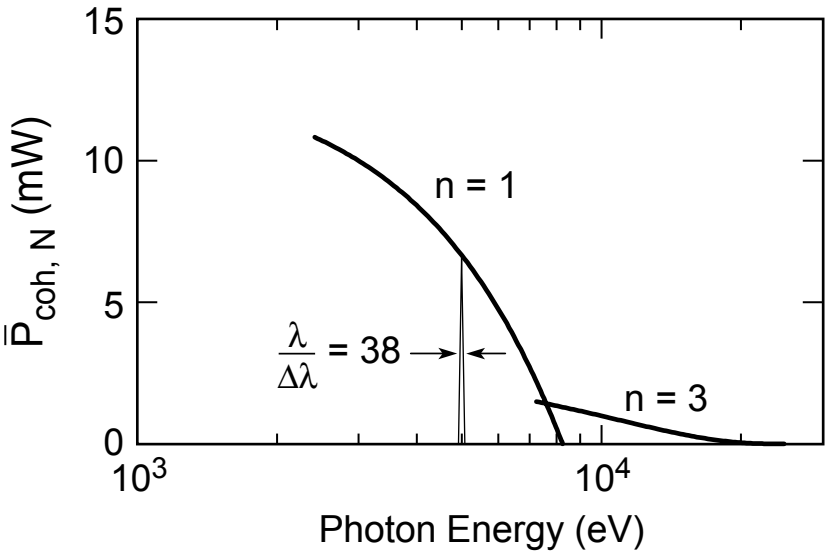
$0 \leq K \leq 2.1$

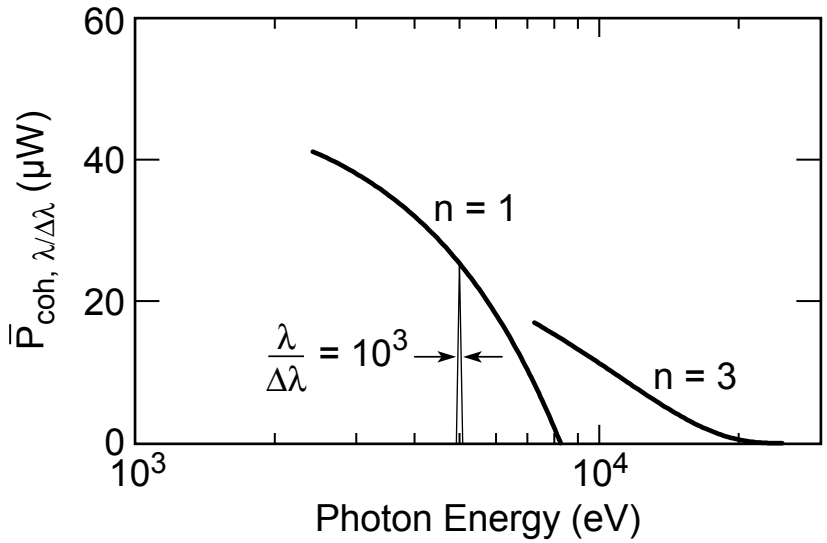
σ_x = 395 μm, σ_x' = 11 μr

σ_y = 9.9 μm, σ_y' = 3.9 μr

η + 10%

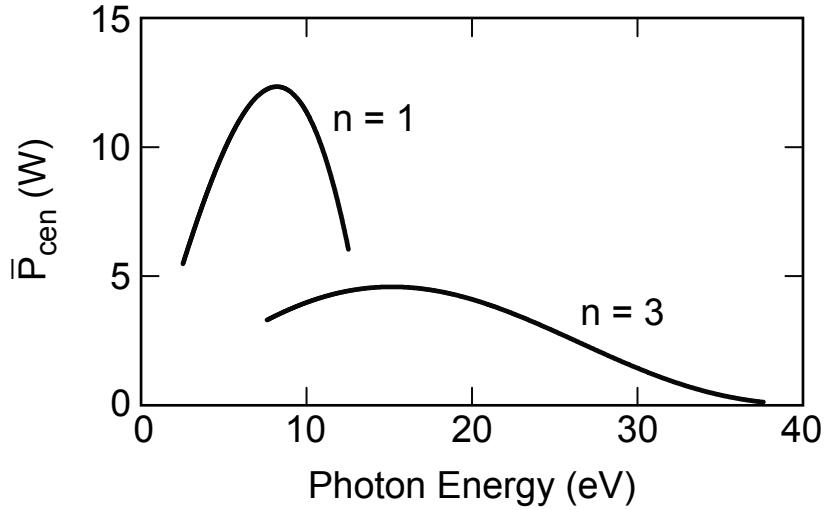
(high beta)





Coherent Power at the APS

APS



7.00 GeV, 100 mA

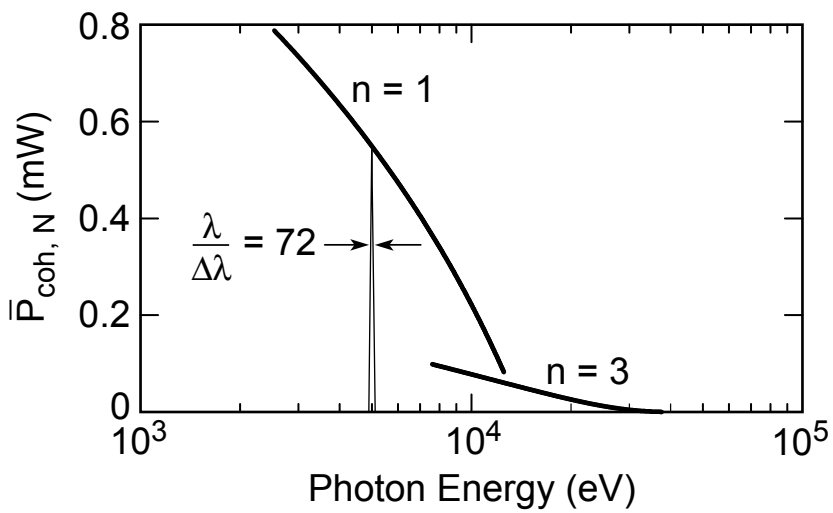
λ_u = 33 mm, N = 72

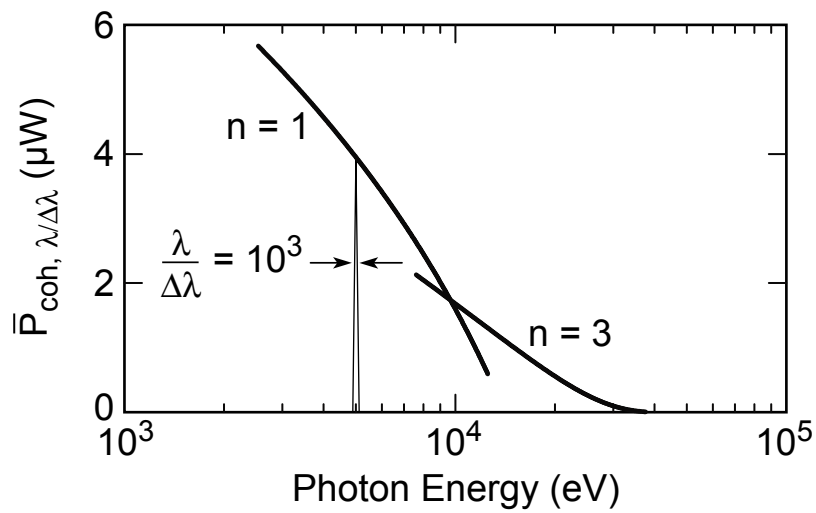
$0.5 \leq K \leq 3.0$

σ_x = 320 μm, σ_x' = 23 μr

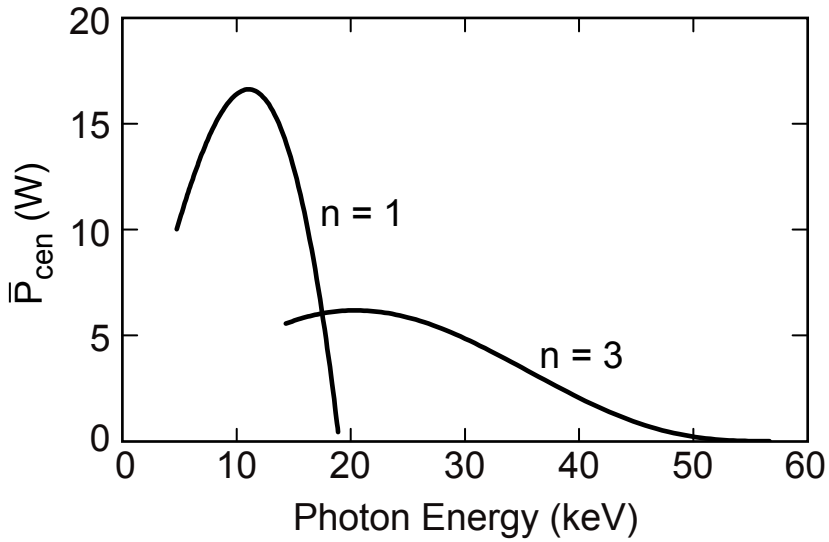
σ_y = 50 μm, σ_y' = 7 μr

η = 10%





Coherent Power at SPring-8



8 GeV, 100 mA

λ_u = 32 mm, N = 140

$0 \leq K \leq 2.46$

σ_x = 393 μm, σ_x' = 15.7 μr

σ_y = 4.98 μm, σ_y' = 1.24 μr

η = 10%