Homework 3: Due in class on Thursday 3/15

1. Problem 5.1 on pp. 271 -272 of the text.
2. Problem 5.2 on pg. 272 of the text.
3. Problem 5.10 on pg. 277 of the text.
4. Problem 5.13 on pg. 278 of the text.
5. Consider the single input-single output, two dimensional linear Gaussian stochastic system over the finite horizon $\{0, 1, \ldots N\}$, described by the equations

$$\begin{bmatrix} x_{k+1}(1) \\ x_{k+1}(2) \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x_k(1) \\ x_k(2) \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u_k + \begin{bmatrix} 1 \\ 1 \end{bmatrix} w_k \ ,$$

$$y_k = [1 \quad 1] \begin{bmatrix} x_k(1) \\ x_k(2) \end{bmatrix} + v_k \ ,$$

where (w_k) and (v_k) are i.i.d. $N(0,1)$ and $\begin{bmatrix} x_0(1) \\ x_0(2) \end{bmatrix} \sim N(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, I)$, and $(x_0, w_0, \ldots, w_{N-1}, v_0, \ldots, v_N)$ are mutually independent.

(a) Sketch the Kalman state estimator for this system in block diagram form.

(b) Verify that the system is observable (assume N is large enough).

(c) Determine the asymptotic covariance $\overline{\Sigma}$, of the Kalman state estimate, and verify that it satisfies the appropriate algebraic Riccati equation.

(d) Determine the asymptotic gain of the Kalman filter. (This is the asymptotic form of the matrix that is applied to the innovations terms in the one step state estimate update, see equation (E.48) on pg. 496 of the text).