Homework 2: Due in class on Thursday 3/1

1. Problem 4.4 on pg. 203 of the text.
2. Problem 4.13 on pg. 208 of the text.
3. Problem 4.15 on pg. 209 of the text.
4. Consider the fully observed finite horizon control problem for the scalar single input linear system
$$x_{k+1} = ax_k + bu_k + w_k, \quad k = 0, 1, \ldots, N-1 \ ,$$
with the objective (informally) being to minimize the expected quadratic cost
$$E\left[\sum_{k=0}^{N-1}\left(q(X_k - \bar{x}_k)^2 + rU_k^2\right) + q(X_N - \bar{x}_N)^2\right] \ .$$
Here a and b are nonzero real numbers, w_k is an independent sequence of real valued random variables each with zero mean and finite variance, and q and r are strictly positive real numbers. The sequence $\bar{x}_k, \quad k = 0, 1 \ldots, N$ is some fixed sequence of real numbers; the cost may be viewed as comprised of the sum of a quadratic penalty for state deviations from this desired trajectory and a quadratic cost on the control effort.

   Mimicking the discussion on pp. 148 -150 of the text, determine an optimal Markov control strategy for this problem.
5. Consider the Riccati equation (eqn. (4.8) on pg. 153 of the textbook) for the case of a scalar single input linear system, i.e. consider the equation
$$p_{k+1} = a^2 p_k\left(1 - \frac{p_k b^2}{r + p_k b^2}\right) + q \ , \quad k \geq 0 \ ,$$
defining a sequence of nonnegative real numbers starting from the nonnegative initial condition p_0. We assume the scalar analogs of the controllability and observability conditions and the scalar analogs of the conditions on the quadratic error measures hold, i.e. $a \neq 0$, $b \neq 0$, $r > 0$, and $q > 0$.

   Specializing the result of Prop 4.4.1 of pp. 153 -154 of the textbook to this scalar case, we know that, irrespective of the initial condition p_0, the sequence p_k will converge to the positive real number p that is the unique solution of the scalar algebraic Riccati equation
$$p = a^2 p\left(1 - \frac{pb^2}{r + pb^2}\right) + q \ ,$$
(which is the scalar analog of eqn. (4.9) on pg. 154 of the textbook).

   The solution p of the algebraic Riccati equation can be viewed as a function of the parameters (a, b, q, r) as they range over the appropriate set of values, identified above. Explore this function as best as you can, i.e. describe how the unique positive solution to the scalar algebraic Riccati equation varies as we the parameters defining the scalar system dynamics and the parameters defining the way we penalize state and control deviations from zero. Give intuitive explanations for all your discoveries.

This is an open ended problem. There is no “correct answer” (but there are wrong answers :-): do the best you can.