EE 223: Stochastic Estimation and Control **Spring 2007**

Lecture 9 — February 13

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This lecture was not scribed. Most of the calculations done during this lecture are available in the textbook.

We studied the Riccati equation

$$P_{k+1} = A^T(P_k - P_kB(B^TP_kB + R)^{-1}B^TP_k)A + Q \ ,$$

(eqn. (4.8) on pg. 153 of the textbook). We proved Prop. 4.4.1, as stated on pp. 153 -154 of the textbook. The proof we gave was essentially identical to that in the textbook. The only difference was in the proof that $A + BL$ must be stable (where $L = -(B^TPB + R)^{-1}B^TPA$ as in eqn. (4.11) on pg. 154 of the textbook). Starting from the equation

$$P = (A + BL)^TP(A + BL) + Q + L^TRL \ ,$$

(which is eqn. (4.12) on pg. 156 of the textbook) and assuming that λ is a (possibly complex) eigenvalue of $A + BL$ with $\mid \lambda \mid \geq 1$, we let v be a nonzero (possibly complex) eigenvector corresponding to this eigenvalue, i.e. $(A + BL)v = \lambda v$. We write

$$v^*Pv = \mid \lambda \mid^2 v^*Pv + v^*Qv + (Lv)^*RLv \ ,$$

where the $*$ denotes complex conjugate transpose. Since all the terms above are real and nonnegative, we conclude that $v^*Qv = 0$ and $(Lv)^*RLv = 0$. The second of these, together with the positive definiteness of R implies that $Lv = 0$, from which we see that $Av = \lambda v$. With $Q = C^TC$ we learn from $v^*Qv = 0$ that $Cv = 0$. But then

$$\begin{bmatrix} C \\ CA \\ CA^2 \\ \vdots \\ CA^{n-1} \end{bmatrix} v = 0 \ ,$$

which contradicts our assumption that (C, A) is observable. Hence it must be that $(A+BL)$ is stable.