EE 223: Stochastic Estimation and Control	Spring 2007
Lecture 23 — April 12	
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This lecture was not scribed.

We first described an example of an average cost dynamic programming problem with countably infinite state space and two controls in each state where an optimal strategy does not exist in the family of stationary randomized Markov strategies, but an optimal non-Markovian strategy can be found. The discussion pertaining to this can be found in Example 4.4.2 on pp. 241 -242 of Vol. 2 of the textbook.

We then pointed out that for the average cost formulation of the linear quadratic control problem for a finite dimensional linear time invariant system in state space form, we can explicitly find solutions to the average cost Bellman equation, thereby concluding that stationary optimal linear feedback strategies exist and are given in terms of the unique positive definite solution of the algebraic Riccati equation, in the expected way, under our usual assumptions. This is discussed on pp. 242 -243 of Vol. 2 of the text.

We discussed how, from an operational point of view, state space models of linear time invariant systems are often properly viewed as secondary, the more natural models being of the input-output kind. We recalled some basic facts about the representation of causal linear time invariant discrete time input-output maps via the impulse response and the associated transfer function, noted that we are only concerned with rational transfer functions, and that we may as well restrict attention to the case where there is a delay of at least 1 between the input and the output, since we are interested in control problems.

We described one of the basic canonical realizations associated to a single-input single-output (SISO) rational transfer function describing a deterministic linear time invariant system: the controllable realization. More information about canonical realizations can be found, for instance, in Chapter 6 of "Linear System Theory and Design" by Chi-Tsong Chen.

We described the Smith canonical form of a polynomial matrix and the Smith-McMillan canonical form of a matrix whose entries are rational functions. More information about this can be found, for example, in "Linear Systems" by Thomas Kailath (look in the index for the appropriate page numbers).

We pointed out that for stochastic linear time invariant systems a treatment of realization theory can be found in Chapter 4 of "Linear Stochastic System" by Peter E. Caines. One of the most important things to know is that the samples of a stochastic process have no operational meaning, from the point of view of modeling, except in so far as their statistics are concerned. This means, for instance, that if we are interested in modeling the effects of noise processes that result from passing a white noise through a linear time invariant system, we are free to chose our representation of the filter to the extent that the statistics of the resulting process do not change. Since we are typically interested only in the second order properties (characterized by the mean and the autocorrelation function, or equivalently by the power spectral density) of the relevant random processes, we may as well choose a convenient factorization of the (rational) power spectral density so as to work with filters that are *minimum phase* (have numerator polynomials with their all roots inside the unit disk – or outside the unit disk, depending on the notation) and in addition are stable (it does not make physical sense to work with an unstable factorization of the power spectral density). Roots of the numerator polynomial on the unit disk pose a non-trivial problem in certain cases, so we just assume that away by requiring that the (rational) power spectral density of the noise process does not have any such roots in the numerator. By making the minimum phase assumption we ensure that the inverse of such a filter is also stable.

We introduced the ARMAX input-output model for SISO stochastic linear time invariant systems driven by noise having rational power spectral density, which we will work with for the next couple of lectures. The notation we use is that of equation (8.10) on pg. 121 (in Section 8 of Chapter 7) of the book of Kumar and Varaiya.