

Lecture 16 — March 13

*Lecturer: Venkat Anantharam**Scribe: none*

This lecture was not scribed. Most of the calculations done during this lecture are available in the textbook.

We discussed some basic properties of jointly Gaussian random vectors. Apart from those mentioned in the last lecture, we also recalled the following facts:

- If Z_0, Z_1, Z_2 are jointly Gaussian random vectors with respective means m_0, m_1, m_2 , and if Z_1 and Z_2 are uncorrelated (hence independent) then we have

$$E[Z_0 | Z_1, Z_2] = m_0 + A_1(Z_1 - m_1) + A_2(Z_2 - m_2) ,$$

where the matrices A_1 and A_2 satisfy

$$E[Z_0 | Z_1] = m_0 + A_1(Z_1 - m_1) \quad \text{and} \quad E[Z_0 | Z_2] = m_0 + A_2(Z_2 - m_2) .$$

- If Z_1, \dots, Z_L are jointly Gaussian random vectors and each of Y_1, \dots, Y_M is an affine vector function of (Z_1, \dots, Z_L) , then $(Z_1, \dots, Z_L, Y_1, \dots, Y_M)$ are jointly Gaussian.

A corollary of this worth noting is that if Z_1 and Z_2 are jointly Gaussian, then so are $Z_1 - E[Z_1 | Z_2]$ and Z_2 .

We then considered the partially observed LQ problem with jointly Gaussian assumptions on the system noise, the observation noise, and the initial condition. All the conditional laws that showed up in our general discussion will turn out in this case to be Gaussian. We developed the formulas for the Kalman filter that allows us to propagate the conditional mean of the state given the observations and also to propagate the covariance matrix of the the conditional law of the state given the observations. The notation used for this derivation was identical to that in Appendix E of the text book. The formulas we wrote down were also identical to the formulas you can find there.