A Schur-Saddle Function Property in CDMA

Pramod Viswanath
Flarion Technologies,
135, Bedminster One,
Rt.202/206S, Bedminster, NJ
07921

Venkat Anantharam [1]
Department of EECS
University of California, Berkeley
Berkeley, CA 94720 USA
Email ananth@eecs.berkeley.edu

Abstract — **Consider Code Division MultiAccess (DS-CDMA) with colored additive Gaussian noise. The best "performance" (by an appropriate choice of powers and signature sequences of the users) of this multiple access scheme is a function of the constraints/requirements of the individual users and of the structure of the additive colored noise. The thesis of this paper is that this function has a *saddle* property: it is *convex* in the covariance of the additive noise and *concave* in the user constraints/requirements. By working on a partial order on probability measures (the Schur-order or the order of dilation), we strengthen this thesis by showing that the performance of CDMA is Schur-order preserving. In other words, the more skewed the user constraints/requirements are, the performance decreases. On the other hand, the more skewed the covariance of the additive noise is, the performance increases. In contrast, we show that this saddle function property breaks down if the signature sequences cannot be specifically designed and are instead chosen randomly.**

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I. Introduction

Consider DS-CDMA with colored additive Gaussian noise. The baseband uplink DS-CDMA model is a vector multiple access channel and models communication from users within a cell to the base station exploiting multiple degrees of freedom. Interference from those users communicating with neighboring base stations is modeled by additive colored noise. In this work we focus on the linear minimum mean square error (LMMSE) receiver and use the signal to noise ratio (SIR) of the estimate of the symbol transmitted as our performance measure. Our focus is on the characterization of the tuple of signal to noise ratios that can be met subject to a transmit power constraint on the users. We consider this question in the context of two scenarios: one, when the signature sequences of the users can be designed specifically to meet the target SIRs, and second, when the signature sequences are chosen randomly (but fixed throughout).

II. Main Results

Our main result is a complete characterization of the admissible performance measure in both the scenarios. Furthermore, our characterization is constructive and we derive combinatorial algorithms to construct the optimal allocations (powers and signature sequences) that achieve such performance. A central contribution of this characterization is the derivation of a qualitative feature of the optimal performance measure. We show that the minimum average power required (with the choice of design of signature sequences) to meet a set of target performance requirements of the users is a *saddle* function: it is *convex* in the additive noise covariances and *concave* in the set of performance requirements. By working on a partial order on probability measures (the Schur-order or the order of dilation), we strengthen this thesis by showing that the performance of CDMA is Schur-order preserving. As a corollary of our analytic methods in deriving these results, we derive a *combinatorial* algorithm that solves a class of convex optimization problems.

On the other hand, with random signature sequences, the performance measure of the SIR of a unit received power user (in a large system, large number of users and large processing gain) is *not* a saddle function: it is *convex* in the received power distribution of the users and and is a *convex* function of the additive noise covariance. We strengthen this result by showing that the convexity is Schur-order preserving.

These results can also be viewed as colored noise extensions of those in [1] and [2]. Precise and detailed version of this summary is available in [3].

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References

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