Asymptotically Optimal Waterfilling in Multiple Antenna Multiple Access Channels

Pramod Viswanath,[1] Department of EECS, University of California at Berkeley, Berkeley, CA94720-1772. e-mail:pvi@eecs.berkeley.edu

David N. C. Tse, Department of EECS, University of California Berkeley, Berkeley, CA94720-1772. e-mail:dtse@eecs.berkeley.edu

Venkat Anantharam, Department of EECS, University of California Berkeley, Berkeley, CA94720-1772. e-mail:ananth@eecs.berkeley.edu

Abstract — **This paper considers "vector multiple access channels" (VMAC) where each user has multiple "degrees of freedom" and studies the effect of power allocation as a function of the channel state on the "sum capacity" defined as the maximum sum of rates of users per unit degree of freedom at which the users can jointly reliably transmit, in an information theoretic sense. A concrete example of a VMAC is a MAC with multiple antennas at the receiver where the antennas provide spatial degrees of freedom. Our main result is the identification of a simple dynamic power allocation scheme that is optimal in a large system, i.e., in the regime of a large number of users and a correspondingly large number of antennas. A key feature of this policy is that, for any user, it depends only on the instantaneous amplitude of the slow fading component of the vector channel of that user alone and the structure of the policy is "waterfilling".**

I. Introduction and Problem Statement

A discrete time baseband frequency flat channel fading model for the multiple antenna, multiple access channel is the following:

$$\mathbf{y}(n) = \sum_{i=1}^{K} x_i(n) h_i^s(n) \mathbf{h}_i^f(n) + \mathbf{w}(n) \quad .$$

Here K denotes the number of users and n the channel use instant. The user symbols are denoted by x_i and $\mathbf{y}(n)$ is the received signal (thought of as a N dimensional vector, N being the number of antennas at the receiver) at the antenna array at time instant n. Here $\mathbf{w}(n)$ is an additive white Gaussian noise. The channel (a vector with N components) from user i to the antenna array at time instant n is written as $h_i^s(n)\mathbf{h}_i^f(n)$. Here h_i^s is a scalar that varies *slowly* in time and captures the distance loss and the shadowing effects and thus depends only on the user. The fast fading component which is changing due to the destructive and constructive additions of the signals from multiple paths is represented by the vector $\mathbf{h}_i^f$ which depends on the individual antenna elements. For the purpose of this summary, we will assume that $\{h_i^s(n)\}_n$ and $\{\mathbf{h}_i^f(n)\}_n$ are independent stationary and ergodic processes. We are interested in the scenario of coherent communication, the scenario when the receiver is able to track the channel variations reliably.

Our performance measure is the long term sum capacity: sum of rates at which users jointly reliably communicate. These rates are time averaged with a power constraint on the users which is also averaged in time. We are interested in the characterizing sum capacity with and without feedback of channel states to the users. If there is no feedback to the users, then a coding theorem shows that the users transmit at constant power. When there is feedback information of the channel state, users can modulate their power based on this knowledge. The problem addressed here is the characterization of the power allocation policies that are optimal in the sense of maximizing sum capacity of the system.

II. Main Result

In the one antenna scenario, there is a simple characterization of the optimal power policy (only the user with the best channel is allowed to transmit and this user uses a waterfilling power policy) and the gap between sum capacity by using this optimal policy and the sum capacity with no channel state feedback is very large (unbounded in the number of users). However, in the general case of multiple antennas, there is no closed form solution to the optimal power policy which for any user is some function of the paths from all the users to the antenna array. Our main result below identifies a simple waterfilling power allocation policy that is optimal in the regime of large number of users and antennas: consider the power policy that for any user depends only on the slow fading component of that user alone and the structure is that of "waterfilling". Observe that in the regime when slow fading is constant over the time scale of communication, this policy simply allocates constant power. Our main result is that this is a very good approximation to the complicated optimal power policy. In particular, this means that feeding back only the slow fading component is asymptotically sufficient in the multiple antenna scenario. Denoting the ratio of users to antennas by $\lfloor \alpha N \rfloor$, we have our main result below.

Theorem 1 *For all α, for all SNR levels, for all fading distributions,*

$$\limsup_{N\to\infty} \sqrt{N} \,(\textit{Sum Capacity with optimal power policy} - \textit{Sum Capacity with waterfilling policy}) < \infty$$

The details of this summary are available in [1].

References

[1] P. Viswanath, D. Tse and V. Anantharam, "Asymptotically Optimal Waterfilling in Vector Multiple Access Channels", accepted in *IEEE Transactions on Information Theory* subject to minor revisions, Feb 2000. Also available as UCB/ERL Memorandum. M99/54.

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