On the Departure Process of a Leaky Bucket System with Long-Range Dependent Input Traffic †

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Abstract

Due to the strong experimental evidence that the traffic to be offered to future broadband networks will display long-range dependence, it is important to study the possible implications that such traffic models may have for the design and performance of these networks. In this paper we consider the popular leaky bucket flow control scheme with long-range dependent input traffic, which is a generalization of the model for long-range dependent traffic suggested in [6]. We prove that the departure process of the leaky bucket policing mechanism driven by such an arrival process is long-range dependent for any token buffer size and any - finite or infinite - cell buffer size.

1 Introduction - Problem Formulation

Recent experimental studies of traffic to be carried by broadband networks have pointed out the importance of analyzing the performance of communication networks using traffic models with long-range dependence. Long-range dependence in network traffic has been reported, for instance, in [5], where statistical analysis of measurements of Ethernet traffic at Bellcore demonstrated its self-similar nature; in [3] long-range dependence has been established in variable bit rate video traffic generated by a number of different codecs; and in [8] the presence of long-range dependence in TELNET and other wide area network traffic was concluded.

In the light of this strong experimental evidence it is important to examine in more detail the possible implications that long-range dependent traffic may have on the design and performance of broadband networks. In particular, an important question is whether the offered traffic preserves its long-range dependent nature after passing through a policing mechanism at the interface of the network. One of the proposed solutions for flow control in the context of the emerging ATM standard is the so-called *leaky bucket* scheme which is shown schematically in Figure 2. Cells of fixed size arrive at a buffer of size $B \geq 0$. The departure of cells from the buffer is controlled by *tokens* that arrive at a buffer of fixed size $C \geq 1$. An arriving cell can be transmitted only if it finds a token in the token buffer, in which case it is transmitted instantaneously by consuming a token. If the token buffer is empty, the cell has to wait for the generation of a new token. Time is assumed to be discrete and exactly one token is generated at the beginning of each unit of time. A stored cell is transmitted immediately upon the generation of a new token. Moreover, the outgoing capacity of the link is assumed to be at least C, so that

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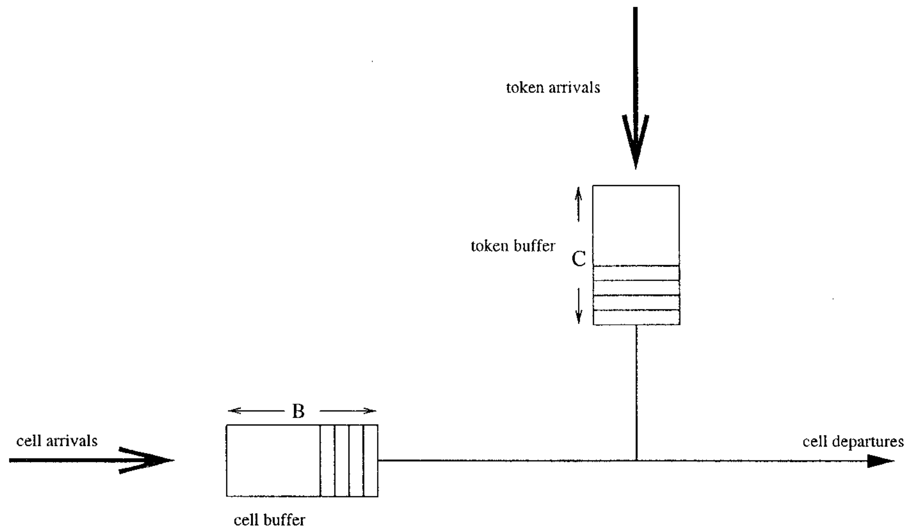

Figure 1: The leaky bucket scheme.

it imposes no limitations on the number of cells that can be transmitted instantaneously.

We assume that the cell arrival process belongs to a class of discrete-time long-range dependent traffic models which includes as a special case the one proposed in [6]. In an arrival model of this class a random number of sessions is initiated at the beginning of each unit of time, distributed as a Poisson random variable with mean λ. Each of these sessions consists of a random number τ of cells, where τ has finite mean, infinite variance, and a regularly varying tail, i.e. $P(\tau > k) \sim k^{-\alpha} L(k)$, where $1 < \alpha < 2$ and $L(\cdot)$ is a slowly varying function. Once a session is initiated, it generates one cell at the beginning of each unit of time until its termination. We will also consider the case of infinite cell buffer, $B = \infty$, in which case the stability condition $\lambda E\tau < 1$ is assumed to hold. According to the results in [7], we may consider a stationary regime in which the departure process is stationary and ergodic - this will become clear once we write the system equations later in the paper.

Following Cox [4], a stationary second-order stochastic process is defined to be long-range dependent if the sum of the absolute values of its covariances is infinite, i.e.

$$\sum_{m=1}^{\infty} \mid r(m) \mid = \infty, \tag{1}$$

where $r(m)$ the covariance function of the process. In this paper we analyze a leaky bucket system with the arrival process as discussed above; a straightforward calculation shows that this arrival process is long-range dependent, see e.g. Cox [4]. We examine the stationary departure process of the system and show that it too is long-range dependent.

Before going on to the statement of our results and the detailed proofs, we introduce the timing conventions and notation we will use, see Figure 2.

By convention, we assume that at the beginning of each slot a new token arrives, and that this is discarded immediately if the token buffer is full. The cells of the input process are assumed to arrive immediately after the token. The departure process is measured immediately after the maximum possible number of cells is served. After this, if there are more cells than can be stored in the cell buffer, these cells are discarded.

In what follows, we will denote by $d^{(C,B)}(k)$ the number of departures at time k of a leaky bucket system with token buffer size C and cell buffer size B. We let $X^{(C,B)}(k)$, $(Y^{(C,B)}(k))$

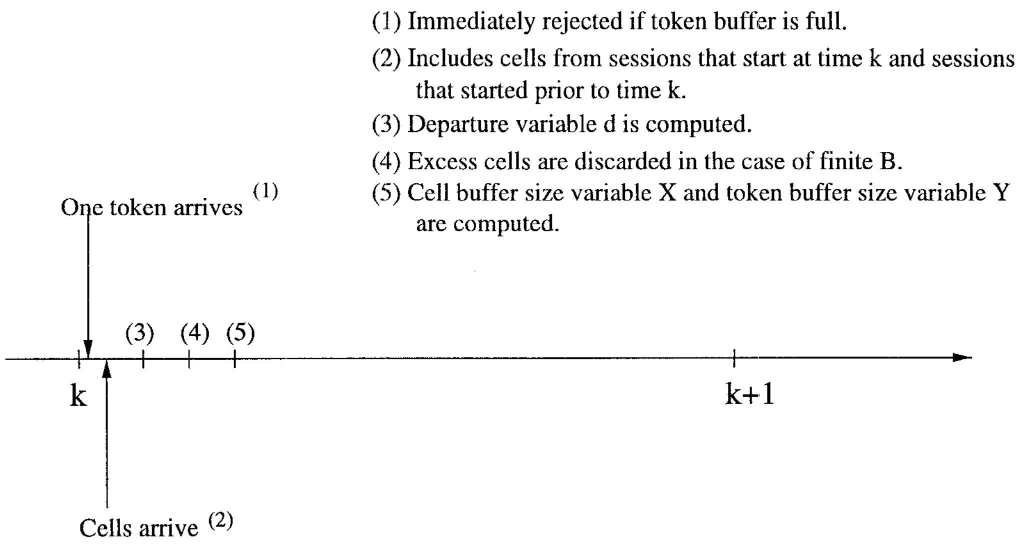

Figure 2: Timing Conventions.

denote the number of cells (tokens) in the cell (token) buffer at time k in such a system. These quantities are measured *after* the cell departures and the cell rejections at time k, see Figure 1.

In Section 2 we consider the case of where the cell buffer is infinite, i.e. $B = \infty$. In Section 3 we consider the case of finite cell buffer, i.e. $\infty > B \geq 0$. Throughout, we assume that $C \geq 1$. The explanation for this is that, with our timing conventions, in the case $C = 0$ there would never be any tokens to serve cells.

2 Infinite Cell Buffer Case

In this section we will consider the case $B = \infty$. We first examine the departure process in the case $C = 1$. In this case the leaky bucket system is equivalent to a single-server queue that is served at a constant rate equal to one cell per unit time. Thus we have one of the cases analyzed in [1], (i.e, the case $s = 1$ in the notation of [1]). We prove the following:

Lemma 1. *The departure process of the leaky bucket system in the case $C = 1$ is long-range dependent.*

Proof

Consider the stationary departure process $\{d^{(1,\infty)}(k)\}$. Since $C = 1$, we have $d^{(1,\infty)}(k) \in \{0, 1\}$, $\forall k$. Moreover, the expected value of the departure process is equal to that of the input process in stationarity, since the stability condition holds, i.e.

$$E[d^{(1,\infty)}(k)] = \lambda E\tau. \tag{2}$$

The covariance of the departure process is:

$$\begin{aligned} r^{(1,\infty)}(m) &= E[(d^{(1,\infty)}(k) - \lambda E\tau)(d^{(1,\infty)}(k+m) - \lambda E\tau)] \\ &= (\lambda E\tau)^2 P(d^{(1,\infty)}(k+m) = 0, d^{(1,\infty)}(k) = 0) \end{aligned}$$

$$
\begin{aligned}
&-(\lambda E\tau)(1-\lambda E\tau)P(d^{(1,\infty)}(k+m)=0, d^{(1,\infty)}(k)=1) \\
&-(\lambda E\tau)(1-\lambda E\tau)P(d^{(1,\infty)}(k+m)=1, d^{(1,\infty)}(k)=0) \\
&+(1-\lambda E\tau)^2 P(d^{(1,\infty)}(k+m)=1, d^{(1,\infty)}(k)=1).
\end{aligned}
$$

If we define $P^m_{i|j} \triangleq P(d^{(1,\infty)}(k+m)=i \mid d^{(1,\infty)}(k)=j)$ and take into account that because of (2) we have $P(d^{(1,\infty)}(k)=0)=1-\lambda E\tau$, $P(d^{(1,\infty)}(k)=1)=\lambda E\tau$, we get from the above relation:

$$
\begin{aligned}
r^{(1,\infty)}(m) &= (\lambda E\tau)^2(1-\lambda E\tau)P^m_{0|0} - (\lambda E\tau)^2(1-\lambda E\tau)P^m_{0|1} \\
&\quad -(\lambda E\tau)(1-\lambda E\tau)^2 P^m_{1|0} + (\lambda E\tau)(1-\lambda E\tau)^2 P^m_{1|1} \\
&= (\lambda E\tau)(1-\lambda E\tau)(P^m_{0|0}-P^m_{0|1}) \qquad (3)\\
&= (\lambda E\tau)(1-\lambda E\tau)(P^m_{1|1}-P^m_{1|0}) \qquad (4)
\end{aligned}
$$

In the above we used the fact that $P^m_{0|0}+P^m_{1|0}=P^m_{0|1}+P^m_{1|1}=1$. ¿From equation (3) one can see that $r^{(1,\infty)}(m)$ is positive, since $0<\lambda E\tau<1$ and $P^m_{0|0}>P^m_{0|1}$, which can be easily seen from the definition of $P^m_{i|j}$.

Consider now equation (4). First note that $d^{(1,\infty)}(k)=0$ means that the cell buffer at the time instant k is empty. This is because sessions continuously bring in cells, so that the absence of a departure at time k implies there are no active sessions among the sessions that arrived at or before time k, and therefore the system will never again have to handle cells corresponding to sessions that arrived at or before time k, so that in particular there are no cells left in the system at time k. We will henceforth refer to cells belonging to sessions that have been initiated at or before time k as cells of type 1, while cells belonging to sessions that arrived after time k will be referred to as cells of type 2. Thus, if $d^{(1,\infty)}(k)=0$ there are no cells of type 1 in the system from time k onwards. Assume that beginning at time $k+1$ priority is given to cells of type 2 over cells of type 1; this does not change the departure process, which only counts the total number of cells departing at any time. If at some point there are no cells of type 2 in the system, then the service of any remaining cells of type 1 will be resumed. Then we can easily see the following :

Proposition 1. For any $m \geq 1$ we have

$$
P^m_{1|1} - P^m_{1|0} = P\{\ A\ cell\ of\ type\ 1\ departs\ at\ time\ k+m \mid d^{(1,\infty)}(k)=1\} \qquad (5)
$$

For a rigorous proof of this proposition, the reader is referred to [9].

Let Z^*_k denote the remaining number of cells just prior to time k, that it is already known will have to be handled by the system. Thus Z^*_k consists of the cells that are in the cell buffer immediately before time k as well as all the future cells of the sessions that are still active immediately before time k that have not yet arrived in the system. Moreover, let ϕ_k be the total number of cells that are brought in by the sessions that are initiated at time k, i.e. the cells that they bring in at time k and all future cell that they will bring in. Then the number of cells of type 1 that the system will have to handle is seen to be equal to

$$
N_1 = (Z^*_k + \phi_k - 1)^+, \qquad (6)
$$

where we take into account that in the time slot $[k, k+1)$ at most one cell of type 1 leaves the system. (In the preceding equation $x^+ = max\{0, x\}$.) The random variables Z^*_k and ϕ_k are assumed to have their stationary distribution in the analysis that follows.

According to the discussion so far, in order to show that the departure process is long-range dependent it suffices to show that

$$\sum_{m=1}^{\infty} P(\Delta_1(k+m)) = \infty \ . \tag{7}$$

where $\Delta_1(k)$ denotes the event that the departure of a cell of type 1 occurs at time k. Indeed, if we moreover define $\Delta(k)$ to be the event that a cell of any type departs at time k, then:

$$\begin{aligned} &P\{\ A\ cell\ of\ type\ 1\ departs\ at\ time\ k+m \mid d^{(1,\infty)}(k) = 1\} \\ &= \frac{P(\Delta_1(k+m), \Delta(k))}{P(\Delta(k))} = \frac{P(\Delta_1(k+m))}{P(\Delta(k))} \propto P(\Delta_1(k+m)) \end{aligned}$$

since $\Delta_1(k+m)$ is an event contained in $\Delta(k)$ for all m.

We now have:

$$\begin{aligned} \sum_{m=1}^{\infty} P(\Delta_1(k+m)) &= \sum_{m=1}^{\infty}\sum_{i=1}^{\infty} P(\Delta_1(k+m), N_1 = i) \\ &= \sum_{i=1}^{\infty}\sum_{m=1}^{\infty} P(\Delta_1(k+m), N_1 = i) \\ &= \sum_{i=1}^{\infty} P(N_1 = i) \sum_{m=1}^{\infty} P(\Delta_1(k+m) \mid N_1 = i) \\ &= \sum_{i=1}^{\infty} iP(N_1 = i) = E[N_1] \ , \end{aligned} \tag{8}$$

where we can justify the interchange in the order of summation in going from the first to the second line because the summands are nonnegative, and we can justify the last step because all cells of type 1 have to eventually leave the system.

But from Lemma 3.2 of [1] we have that

$$P(Z_k^* > x) \sim x^{1-\alpha} L(x) \tag{9}$$

where $1 < \alpha < 2$ is the parameter of the regularly varying tail of the session duration and $L(\cdot)$ is a slowly varying function. ¿From equations (6), (8), and (9), it follows immediately that equation (7) holds, showing that the stationary departure process of the leaky bucket system with infinite cell buffer in the case $C = 1$ is indeed long-range dependent, and thus completing the proof of Lemma 1. □

We will now consider the case of $C > 1$. In keeping with the notation used above we denote by $\{d^{(C,\infty)}(k)\}$ the stationary departure process of the leaky bucket system in the case that the token buffer size is C and the cell buffer size is infinite. In what follows we will refer to the leaky bucket system with token buffer size equal to C (1) as the C-system (1-system). The key observation we need is the following lemma:

Lemma 2. *It is possible to construct stationary versions of the C-system and the 1-system on the same sample space such that for every sample path of the arrival process the following inequality holds:*

$$\begin{aligned} &d^{(1,\infty)}(k) + d^{(1,\infty)}(k+1) + \ldots + d^{(1,\infty)}(k+m) \\ &\quad \le d^{(C,\infty)}(k) + d^{(C,\infty)}(k+1) + \ldots + d^{(C,\infty)}(k+m) + (C-1), \end{aligned} \tag{10}$$

for any k and any $m \geq 0$.

Proof

We will use the pathwise construction of the departure process of a leaky bucket system presented in [2]. Recall that $X^{(C,\infty)}(k)$, $(Y^{(C,\infty)}(k))$ denote the number of cells (tokens) in the cell (token) buffer at time k in the system with token buffer size C. Obviously, $0 \leq X^{(C,\infty)}(k) < \infty$ and $0 \leq Y^{(C,\infty)}(k) \leq C$. We further note that $X^{(C,\infty)}(k)Y^{(C,\infty)}(k) = 0$ for all k, so that the system can be described by the parameter $Z^{(C,\infty)}(k) = X^{(C,\infty)}(k) - Y^{(C,\infty)}(k)$. If $Z^{(C,\infty)}(k) > 0$ there are $Z^{(C,\infty)}(k)$ cells and no tokens, if $-C \leq Z^{(C,\infty)}(k) < 0$ there are $-Z^{(C,\infty)}(k)$ tokens and no cells, and if $Z^{(C,\infty)}(k) = 0$ then there are neither cells nor tokens. Let $a(k)$ denote the number of cells arriving into the leaky bucket at time k. Then $Z^{(C,\infty)}(k)$ is given by the recursion

$$Z^{(C,\infty)}(k+1) = Z^{(C,\infty)}(k) + a(k+1) - I\{Z^{(C,\infty)}(k) \geq -C+1\} \tag{11}$$

where $I(A)$ denotes the indicator random variable of the event A. If we now define the quantity $W(k) = Z^{(C,\infty)}(k) + C = X^{(C,\infty)}(k) - Y^{(C,\infty)}(k) + C$, we get the following recursion for $W(k)$ from (11):

$$W(k+1) = W(k) + a(k+1) - I\{W(k) > 0\} = (W(k) - 1)^{+} + a(k+1) \tag{12}$$

Note that $W(k) \geq 0$, $\forall k$. Relation (12) is nothing more than a special case of the familiar Lindley recursion from basic queueing theory [10] and from [7] we know that, since the stability condition $\lambda E\tau < 1$ holds, there is a unique stationary and ergodic process that satisfies the recursion (12). Note that the distribution of W does not depend on the choice of the token buffer size C. Nevertheless, the distributions of $Z^{(C,\infty)} = W - C$, $X^{(C,\infty)} = max\{W - C, 0\}$ and $Y^{(C,\infty)} = max\{C - W, 0\}$ do depend on C, as they should.

The stationary departure process of the leaky bucket system for any C can be constructed from the stationary sample path of the W process as shown in Figure 3. In order to determine the number of cells $d^{(C,\infty)}(k)$ that leave the C-system at time k we consider the following cases in terms of the transition $W(k-1) \to W(k)$:
a) $W(k-1) = 0$: In that case the number of departing cells is $d^{(C,\infty)}(k) = min\{W(k), C\}$.
b) $0 < W(k-1) \leq C$: In that case the number of departing cells is $d^{(C,\infty)}(k) = min\{W(k), C\} - W(k-1) + 1$.
c) $W(k-1) > C$: In that case there is exactly one departure at time k due to the arriving token, i.e. $d^{(C,\infty)}(k) = 1$.
The above considerations hold for any token buffer size and therefore also for $C = 1$. This means that the departure processes for a whole range of systems, corresponding to different values of C, can be read off directly from the sample path of W. Note that for $W(k-1) > C$ we can only have one departure at any time instant.

Let $d_{k,m}^{(C,\infty)} = d^{(C,\infty)}(k) + \ldots + d^{(C,\infty)}(k+m)$ be the number of departing cells from the C-system during the interval $[k, k+m]$, $m > 0$. Also let $\Delta d_{k,m} = d_{k,m}^{(1,\infty)} - d_{k,m}^{(C,\infty)}$ be the excess of the number of departing cells from the 1-system over the C-system over this interval. We are interested in an upper bound for the quantity $\Delta d_{k,m}$. Using the above results (a) - (c) we have the following cases:
- $W(k-1) \in \{0,1\}$ and $W(k+m) \in \{0,1\}$: Then $\Delta d_{k,m} = 0$.
- $W(k-1) \in \{0,1\}$ and $W(k+m) \in \{2,\ldots,C\}$: Then $\Delta d_{k,m} = 1 - W(k+m)$.
- $W(k-1) \in \{0,1\}$ and $W(k+m) > C$: Then $\Delta d_{k,m} = 1 - C$.
- $W(k-1) \in \{2,\ldots,C\}$ and $W(k+m) \in \{0,1\}$: Then $\Delta d_{k,m} = W(k-1) - 1$.

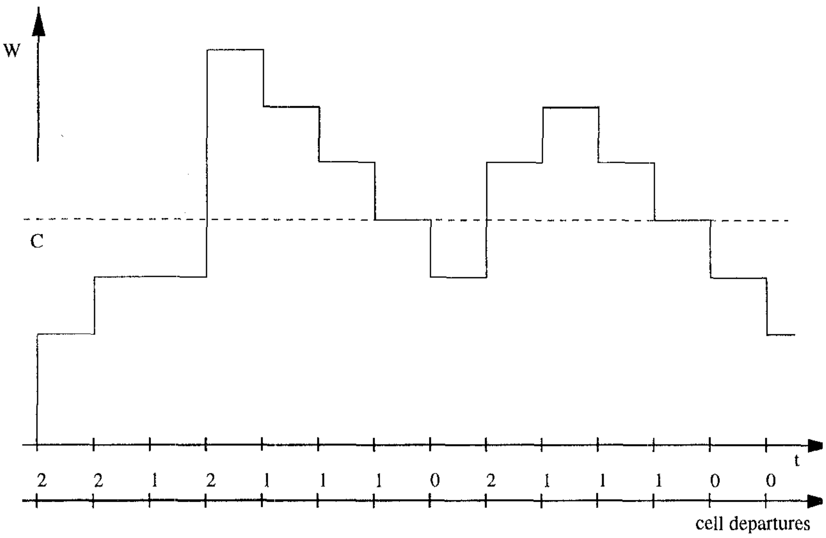

Figure 3: Construction of the departure process. The numbers on the axis labeled 'cell departures' give the number of cells leaving the system at each time instant for the specific realization of the process W.

- $W(k-1) \in \{2, \ldots, C\}$ and $W(k+m) \in \{2, \ldots, C\}$: Then $\Delta d_{k,m} = W(k-1) - W(k+m)$.
- $W(k-1) \in \{2, \ldots, C\}$ and $W(k+m) > C$: Then $\Delta d_{k,m} = W(k-1) - C$.
- $W(k-1) > C$ and $W(k+m) \in \{0, 1\}$: Then $\Delta d_{k,m} = C - 1$.
- $W(k-1) > C$ and $W(k+m) \in \{2, \ldots, C\}$: Then $\Delta d_{k,m} = C - W(k+m)$.
- $W(k-1) > C$ and $W(k+m) > C$: Then $\Delta d_{k,m} = 0$.

We see that the greatest value of $\Delta d_{k,m}$ is achieved for any interval $[k, k+m]$ with $W(k-1) \geq C$ and $W(k+m) \leq 1$ and for such an interval we get $\Delta d_{k,m} = C - 1$. Hence, $d_{k,m}^{(1,\infty)} - d_{k,m}^{(C,\infty)} \leq C - 1$, for any k and for all $m > 0$, and the proof is complete. □

Now we can prove the following:

Theorem1. *The stationary departure process of the leaky bucket system is long-range dependent for any token buffer size C.*

Proof

We have shown the result for $C = 1$. To prove the result in the case $C > 1$ we proceed as follows: Both sides of (10) are nonnegative, so we may square both sides and the inequality will still hold. If we also take expectations on both sides and subtract the quantity $E[d^{(1,\infty)}(k)]^2 = E[d^{(C,\infty)}(k)]^2 = (\lambda E\tau)^2$ from each expectation term on both sides we get the following relation:

$$(m+1)r^{(1,\infty)}(0) + 2\sum_{i=1}^{m}(m-i+1)r^{(1,\infty)}(i) \leq$$
$$(m+1)r^{(C,\infty)}(0) + 2\sum_{i=1}^{m}(m-i+1)r^{(C,\infty)}(i) + 2(m+1)C\lambda E\tau + C^2 \qquad (13)$$

where in the above we took into account that the output process is stationary. Let $\Sigma_0^{(C,\infty)}(l) \triangleq$

$\sum_{i=0}^{l}\left|r^{(C,\infty)}(i)\right|$ and $\Sigma_1^{(1,\infty)}(l) \stackrel{\triangle}{=} \sum_{i=1}^{l} r^{(1,\infty)}(i)$. Also let $d \stackrel{\triangle}{=} C\lambda E\tau$ and $g \stackrel{\triangle}{=} 2C\lambda E\tau + C^2$. Since $r^{(1,\infty)}(0), r^{(C,\infty)}(0) \geq 0$ we get from (13) after a few simple algebraic manipulations.

$$\sum_{l=1}^{m}(\Sigma_1^{(1,\infty)}(l) - d) \leq \sum_{l=0}^{m}\Sigma_0^{(C,\infty)}(l) \ + g \tag{14}$$

In order to show that $\{d^{(C,\infty)}(k)\}$ is long-range dependent, it suffices to show that for every $M > 0$ there exists some integer L, such that

$$\Sigma_0^{(C,\infty)}(l) > M, \quad \forall l \geq L \tag{15}$$

Note that since $\Sigma_0^{(C,\infty)}(l)$ is monotone increasing in l, then if (15) holds for some index L, it will definitely hold for all indices $l \geq L$. To show (15) we may argue by contradiction. Suppose that for some M_0 there is no integer L, such that $\Sigma_0^{(C,\infty)}(L) > M_0$. But, since $\lim_{l\to\infty}\Sigma_1^{(1,\infty)}(l) = \infty$, this means that there exists some integer K, such that $\Sigma_1^{(1,\infty)}(l) - d > M_0 > \Sigma_0^{(C,\infty)}(l)$, $\forall l \geq K$. This contradicts the inequality in (14) and therefore we must have that (15) holds or equivalently that $\{d^{(C,\infty)}(k)\}$ is long-range dependent. □

3 Finite Cell Buffer Case

We now consider the case where the cell buffer is finite, $0 \leq B < \infty$. The results in this case follow along the basic ideas of the analysis for the infinite cell buffer case, and detailed proofs are, therefore, omitted. For the proofs of the results in this section the reader is referred to [9]. Again, we first consider the case of $C = 1$ for any finite cell buffer size B. The following result is shown:

Lemma 3. *The departure process of the leaky bucket system in the case $C = 1$ is long-range dependent for any cell buffer size B.*

Next, the general case of $C > 1$ is considered. We denote by $\{d^{(C,B)}(k)\}$ the stationary departure process of the leaky bucket system in the case that the token buffer size is C and the cell buffer size is B. In what follows, we will call the corresponding system the (C, B)-system. The relevant range of the parameters is $C \geq 1$ and $0 \leq B < \infty$. The following lemma is proved:

Lemma 4. *For every (C_1, B_1) and (C_2, B_2) such that $C_1 + B_1 = B_2 + C_2$ and $C_1 \leq C_2$, it is possible to construct the (C_1, B_1)-system and the (C_2, B_2)-system on the same sample space such that, for every sample path of the arrival process the following inequalities hold:*

$$\begin{aligned} &d^{(C_1,B_1)}(k) + d^{(C_1,B_1)}(k+1) + \ldots + d^{(C_1,B_1)}(k+m) \\ &\quad \leq d^{(C_2,B_2)}(k) + d^{(C_2,B_2)}(k+1) + \ldots + d^{(C_2,B_2)}(k+m) + (C_2 - C_1), \end{aligned} \tag{16}$$

$$\begin{aligned} &d^{(C_1,B_1)}(k) + d^{(C_1,B_1)}(k+1) + \ldots + d^{(C_1,B_1)}(k+m) \\ &\quad \geq d^{(C_2,B_2)}(k) + d^{(C_2,B_2)}(k+1) + \ldots + d^{(C_2,B_2)}(k+m) - (C_2 - C_1), \end{aligned} \tag{17}$$

for any k and any $m \geq 0$.

Finally, using the above lemmas the main result can be shown:

Theorem2. *The departure process of the leaky bucket system is long-range dependent for any token buffer size $C \geq 1$ and any finite cell buffer size $0 \leq B < \infty$.*

4 Concluding Remarks

We have studied the departure process of a leaky bucket system in an ATM network fed by a class of proposed models for long-range dependent input traffic. We established the fact that the departure process is long-range dependent for any token buffer size and any - finite or infinite - cell buffer size. This result provides additional evidence supporting the importance of the empirical observations that there is long-range dependence in the offered traffic that broadband networks are expected to carry. Indeed, what it demonstrates is that this long-range dependence cannot be removed by the kinds of flow control schemes that are currently being envisioned for these networks.

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