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TRUNK RESERVATION BASED CONTROL OF CIRCUIT SWITCHED NETWORKS WITH DYNAMIC ROUTING

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ABSTRACT

We consider dynamic routing in nonhierarchical circuit switched networks. Using a lattice model, we first formulate the problem of spatially distributed control of the network by trunk reservation based on local information. We derive a set of integrodifferential equations for the time evolution of the distribution of link occupancies with the trunk reservation parameter appearing as a control variable (hydrodynamic limit). In the case of spatially homogeneous initial conditions these equations collapse to a controlled nonlinear vector ODE.

For a small example, we compare the performance of trunk reservation strategies derived from the above control problem to a natural analog of the separable routing scheme of Krishnan and Ott. The trunk reservation strategy appears to perform favorably in our simulations, but work on more realistic examples will be needed before definitive conclusions can be made.

1. INTRODUCTION

It has become practically feasible to use dynamic routing strategies in order to route calls in nonhierarchical circuit switched networks. As a consequence, such schemes have been the topic of several recent papers, for example, see [1]-[9], [11]. Dynamic routing schemes basically make better use of network capacity by using alternate routes between nodes when the preferred routes are not available due to local load imbalances.

However, alternate routes typically use up more capacity than the preferred (direct) routes. This can lead to performance degradation of dynamic routing schemes under heavy traffic. The point is that a situation where most calls are using alternate routes is likely to persist for a while because arriving calls will then find the network close to saturation and will be unable to make their direct connections. This can result in much poorer blocking performance than if alternate routing is not allowed.

A common technique to avoid this problem is trunk reservation for directly routed traffic, [2], [6], [7]. Namely, when the dynamic routing strategy attempts to route a call along a link, the call is accepted only if there are enough free circuits available for directly routed calls which might potentially use that link later. The choice of trunk reservation parameter on the basis of loading to optimize the performance of the network then becomes important.

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In this paper we discuss this problem. We first discuss a formulation of trunk reservation based control as a spatiotemporal control problem, with the trunk reservation parameter at a spatial location chosen on the basis of local information, i.e., information about the loading of neighbouring links. This formulation uses a lattice model and ideas from the area of interacting particle systems, [10]. It was discussed in [4], and the proof techniques are the same as those in [3]. It yields a set of integrodifferential equations for the distribution of link occupancies with the trunk reservation parameter appearing as a control variable (hydrodynamic limit). The formulation is discussed in Section 2.

In the case of spatially homogeneous initial conditions the above spatiotemporal control problem collapses to the problem of controlling a nonlinear vector ODE. This reduction is discussed in Section 3.

We use this ODE as the starting point for trunk reservation control schemes. Our purpose in this paper is to compare the performance of a simple trunk reservation strategy derived from the controlled ODE to a natural analog of the separable routing scheme of Krishnan and Ott, [8]. Separable routing has been shown to perform well in comparision to dynamic non-hierarchical routing and least loaded routing on realistic network examples, [8]. The trunk reservation scheme seems to perform favorably compared to the analog of separable routing in the small example we consider, but work on more realistic examples will be needed before definitive conclusions can be made.

We discuss the analog of separable routing in Section 4 and our simulation results in Section 5. Finally, we make some concluding remarks.

2. SPATIOTEMPORAL CONTROL PROBLEM

Here we describe our lattice model for circuit switched networks with dynamic routing and trunk reservation control.

$\mathbf{Z}^d/M$ denotes the lattice in $\mathbf{R}^d$ consisiting of points all of whose co-ordinates are rational with denominator dividing M. (In practice, $d = 2$). The points of $\mathbf{Z}^d/M$ are called *sites*. Let W denote $\{(k,l)\ ;\ k \geq 0,\ l \geq 0,\ k+l \leq C\}$. We consider a Markov process $(\eta_t^M, t \geq 0)$ on $W^{\mathbf{Z}^d/M}$. We use η to denote a generic element of $W^{\mathbf{Z}^d/M}$, and call $\eta(x)$ the *value* at site x. Let M^* denote $\left(\frac{(2M+1)^d-1}{2}\right)$. Fix $w > 0$ and $K \geq 1$, and let N^* denote $\left(\frac{(2\lfloor Mw \rfloor+1)^d-1}{K}\right)$. Let $\mathcal{M}_W$ denote the space of probability distributions on W. Given values $(k_1,l_1),(k_2,l_2),\ldots,(k_K,l_K)$, let $\mathcal{E}((k_1,l_1),\ldots,(k_K,l_K)) \in \mathcal{M}_W$ denote their empirical distribution. Let $r\ :\ \mathcal{M}_W \to \{0,1,\ldots,C\}$. The Markov process is described by the transitions

$$\eta(x) = (\eta_1(x), \eta_2(x)) \longrightarrow (\eta_1(x) - 1, \eta_2(x)) \text{ at rate } \eta_1(x) ,$$

$$\eta(x) = (\eta_1(x), \eta_2(x)) \longrightarrow (\eta_1(x), \eta_2(x) - 1) \text{ at rate } \eta_2(x) ,$$

$$\eta(x) = (\eta_1(x), \eta_2(x)) \longrightarrow (\eta_1(x) + 1, \eta_2(x)) \text{ at rate } \nu \text{ if } \eta_1(x) + \eta_2(x) < C ,$$

$$(\eta(x), (\eta_1(y), \eta_2(y)), (\eta_1(z), \eta_2(z)), \eta(y_1), \ldots, \eta(y_K), \eta(z_1), \ldots, \eta(z_K)) \longrightarrow (\eta(x), (\eta_1(y), \eta_2(y) + 1), (\eta_1(z), \eta_2(z) + 1), \eta(y_1), \ldots, \eta(y_K), \eta(z_1), \ldots, \eta(z_K))$$

at rate $\nu / M^*(N^*)^2$ if x, y, z are distinct sites with $y, z \in x + [-1,1]^d$, $y, y_1, \ldots, y_K$ are distinct sites with $y_1, \ldots, y_K \in y + [-w,w]^d$, $z, z_1, \ldots, z_K$ are distinct sites with $z_1, \ldots, z_K \in z + [-w,w]^d$; $\eta(x) = C$, $\eta_1(y) + \eta_2(y) < C - r(\mathcal{E}(\eta(y_1), \ldots, \eta(y_K)))$ and $\eta_1(z) + \eta_2(z) < C - r(\mathcal{E}(\eta(z_1), \ldots, \eta(z_K))$.

It can be shown that such a Markov process exists even though the number of sites is infinite, see for example Theorem 3.9 of Chapter 1 of Liggett, [10]. In our model, each site in the lattice represents a link in the network, consisting of C circuits. The value (k, l) at a site means that the link is currently supporting k directly routed calls and l alternately routed calls. At each link there is a Poisson arrival process of calls with rate ν. Each call occupies one circuit on its link if available; if the link is saturated the call randomly picks two other links which are in its $[-1,1]^d$ neighbourhood, and uses one circuit from each of these links if possible. Otherwise the call is blocked and rejected from the system. A link decides whether to accept an alternately routed call by sampling the states of K links in its $[-w,w]^d$ neighbourhood and choosing its trunk reservation parameter accordingly via the control function r. This models control based on local information.

An alternately routed call needs to be accepted by each of the two links it attempts in order to be accepted. Occupied circuits are assumed to individually become free at rate 1 whether they are occupied by directly routed calls or alternately routed calls, which is unrealistic, but seems essential to get a tractable model.

The above model captures several of the essential feature of dynamic routing which lead to the metastability of alternate routes that is the focus of trunk reservation strategies. Alternately routed calls require more bandwidth from the network and the alternate routing is done locally. Further the trunk reservation parameter is chosen on the basis of local information. On the other hand, the model obviously loses the geometric structure of the network by treating the links as independent entities.

When we take the limit as $M \to \infty$, we get a set of integrodifferential equations describing the evolution of the spatially distributed network state with the trunk reservation parameter as control. These equations are precisely what one would expect formally by law of large numbers arguments. We will not write them here. A precise limit theorem is stated in [4] and the proof technique is identical to that in [3].

3. CONTROLLED ODE

We may derive qualitative insights into the nature of good trunk reservation strategies by studying the control problem formulated above. Suppose that the initial condition is spatially homogeneous and the the aim is to choose the trunk reservation control so as to maximize a per link long term average revenue criterion. A natural revenue criterion is a weighted sum $Ak + Bl$ when there are k directly routed calls and l alternately routed calls being handled by the link. Let $\underline{\gamma} = (\gamma_{kl}, (k,l) \in W) \in \mathcal{M}_W$ evolve according to the controlled nonlinear ODE

$$\begin{aligned}
\dot{\gamma}_{kl} = & -(k+l)\gamma_{kl} - \nu\gamma_{kl} 1(k+l<C) \\
& - 2\nu\delta_C\Big(\sum_{i=0}^{C-1} \sigma_{C-i-1} 1(r(\underline{\gamma}) = i)\Big)\gamma_{kl}\, 1(r(\underline{\gamma}) < C-(k+l)) \\
& + ((k+1)\gamma_{k+1,l} + (l+1)\gamma_{k,l+1}) 1(k+l<C) \\
& + \nu\gamma_{k-1,l} 1(k>0) \\
& + 2\nu\delta_C\Big(\sum_{i=0}^{C-1} \sigma_{C-i-1} 1(r(\underline{\gamma}) = i)\Big)\gamma_{k,l-1}\, 1(r(\underline{\gamma}) \le C-(k+l))
\end{aligned} \tag{1}$$

where $\delta_i = \sum_{k+l=i} \gamma_{kl}$, $0 \le i \le C$, and $\sigma_i = \sum_{j=0}^{i} \delta_j$, the aim being to maximize

$$J^* = \lim_{T\to\infty} \int_0^T \sum_{k,l} (Ak + Bl)\gamma_{kl}(t)dt$$

by the time invariant choice of control $r(\underline{\gamma})$. For spatially homogeneous initial conditions and taking the limit as $K \to \infty$, the spatiotemporal control problem discussed in Section 2 reduces to the above. Again, these equations have a clear formal interpretation.

4. AN ANALOG OF SEPARABLE ROUTING

Separable routing is a scheme proposed by Krishnan and Ott, [8], for the problem of minimizing network blocking. The idea is to adopt a dynamic programming framework with average cost criterion and to approximate the relative costs of states by the relative costs in an appropriately optimized separable problem which comes from assuming that the overflow traffic is Poisson. For details, see Krishnan and Ott, [8]. In this section, we will develop an analogous idea in our problem.

Consider a network with n links each having C circuits. At each link there is a Poisson(ν) arrival process of calls between the nodes that define the link. Each arriving call can be connected along either the one link direct route or along any two-link alternate route that connects the nodes involved in the call. A call that is connected on its direct route generates a reward of A units per unit time it is connected and a call that is connected as a two-link call generates a reward of B units per unit time for each link along its route. Each connected call has a holding time which is exponential with rate 1 and holds one circuit on each link along its route.

We consider the problem of maximizing the long term average reward by a routing strategy that is allowed knowledge of the state of the network. This is a Markov decision problem.

The dynamic programming equation reads

$$J^* = \max_u \{r(\mathbf{x}) + \sum_{\mathbf{y}} q_{\mathbf{xy}}(u)(w(\mathbf{y}) - w(\mathbf{x}))\} .$$

Here J^* is the optimal long term average reward, $r(\mathbf{x})$ is the rate at which reward is generated in state $\mathbf{x}$, $q_{\mathbf{xy}}(u)$ is the rate of transitions from $\mathbf{x}$ to $\mathbf{y}$ when the control is u, and $w(\mathbf{x})$ is the so-called "relative reward" of being in state $\mathbf{x}$. The optimal control strategy $u^*(\mathbf{x})$ is the one maximizing the right hand side.

Thus, if the relative rewards $w(\mathbf{x})$ can be computed, the optimal routing strategy can be determined. The idea of separable routing is to choose a control strategy based on a separable approximation to $w(\mathbf{x})$. The prescription we use, following Krishnan and Ott, [8], is the following :

For each link, list the corresponding two-link alternate routes, and fix a probability distribution over the direct route and these two-link alternate routes. Now consider the probabilistic routing scheme which routes calls arriving at a link according to these probability distributions. This results in an effective arrival rate of direct and indirect calls at each link (ignoring blocking).

Now consider a single link with C circuits in isolation with a Poisson (λ_d) arrival process of "direct" calls and a Poisson (λ_r) arrival process of "indirect" calls. Each call is accepted if there is a circuit free, holds one circuit, and releases it after an exponential time of rate 1. Accepted direct calls generate reward at rate A and accepted indirect calls generate reward at rate B.

For this single link, we may easily compute the "relative rewards" of the different states for the link. The relative reward $\tilde{w}(x) - \tilde{w}(y)$ of state x over state y is the long term expected difference in overall reward when the link is started from x, resp. y.

The idea is to approximate the relative reward function $w(\mathbf{x})$ by $\sum \tilde{w}(x_i)$, the summation being over the links of the network. The probability distributions on which the approximation is based are the ones for which the long term average reward under the above probabilistic non-alternate routing is maximized.

5. NUMERICAL COMPARISION OF STRATEGIES

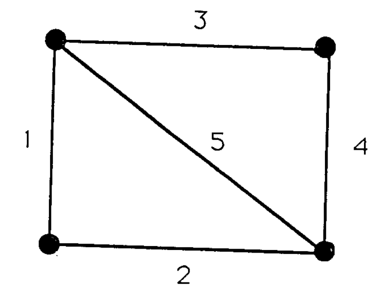
We carried out a comparision of the analog of separable routing and a simple trunk reservation based strategy motivated by the control problem (1) in the simple network shown in Fig. 1. The results of this comparision are shown in Table 1. As the table shows, the trunk reservation strategy performs quite favorably, particularly under heavy loading. In this section we describe how this comparision was carried out.

The analog to separable routing was constructed as follows : Probabilistic non-alternate routing in the network of Fig. 1 is characterized by probabilities $p_1, \ldots, p_4$, p_{51}, p_{52}, where

$p_i = P($ a call on link i is routed along the direct path $)$,

$p_{51} = P($ a call on link 5 is routed along path $(1,2))$,

$p_{52} = P($ a call on link 5 is routed along path $(3,4))$,



C = 3

Fig. 1

For each ν, A and B, we generated 10,000 random vectors of probabilities and chose from them the one which maximizes the total long term average reward over the five links with Poisson arrival processes of the corresponding effective service rates (an expression for this overall reward is easily written down). Using the approximate optimal control based on the separable relative reward associated with these optimal probabilities, a discrete event simulation was carried out to generate the numbers in Table 1 in the "Separable routing" column.

The control problem (1) was used to derive a simple trunk reservation based strategy as follows : Starting empty, we ran the differential equations starting with control $u = 0$ and deciding when to switch to $u = 1$, $u = 2$ and $u = 3$ so as to maximize the overall reward over a time of 1000 units. We used the fraction of empty links at the optimal transition points so generated (which turned out to be monotone ordered) to define a control strategy which reserves no trunks when the fraction of empty links is above a threshold, reserves exactly one trunk when the fraction is above another lower threshold, and so on. Subsequently, we ran a discrete event simulation where each link implemented this trunk reservation strategy based on complete knowledge of the empirical occupation distribution of the network.

A/B	Analog of Separable Routing	Trunk Reservation
1.0	0.19	0.10
1.2	0.18	0.12
1.5	0.18	0.15
1.8	0.18	0.18
2.0	0.19	0.20
2.5	0.22	0.25
3.0	0.23	0.30
4.0	0.26	0.41
5.0	0.28	0.51
10.0	0.38	1.03

$\nu = 0.1$

A/B	Analog of Separable Routing	Trunk Reservation
1.0	1.83	2.26
1.2	2.02	2.63
1.5	2.80	3.18
1.8	3.62	3.61
2.0	3.93	4.01
2.5	4.71	4.50
3.0	5.47	5.94
4.0	7.03	7.92
5.0	8.57	9.91
10.0	16.27	19.81

$\nu = 3.0$

A/B	Analog of Separable Routing	Trunk Reservation
1.0	2.08	2.75
1.2	2.31	3.22
1.5	2.63	4.01
1.8	2.96	4.81
2.0	3.18	5.35
2.5	3.72	6.68
3.0	4.26	8.02
4.0	8.18	10.70
5.0	11.71	13.37
10.0	22.88	26.75

$\nu = 10.0$

Numbers are accurate to roughly within $\pm$ 0.03 in repeated runs

TABLE 1

6. CONCLUDING REMARKS

The hydrodynamic limit assumes that the links in the network all have the same number of circuits C. This was necessary for the rigorous derivation of hydrodynamic limit from which equations (1) were derived by taking the formal limit as $K \to \infty$. Separable routing, however, can be considered for quite general networks. To rigorously get an analog of the hydrodynamic limit for general networks is not easy ; however, one can formally write down equations analogous to (1) for each link in the network to choose a trunk reservation parameter on the basis of the local link occupancy profile. Any attempt to apply this idea to practical situations should probably proceed from such formal equations, rather than (1).

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