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METASTABILITY AND PHASE TRANSITIONS ASSOCIATED TO DYNAMIC ROUTING IN NETWORKS

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ABSTRACT

Empirical studies of dynamic alternate routing in circuit switched networks reveal the existence of hysterisis phenomena, corresponding to the existence of multimodal stationary distributions, and hence metastable states. Such behaviour is also suggested by an analytical model due to Marbukh, and in a simple caricature of it due to Gibbens, Hunt, and Kelly. Metastability in large finite systems is often associated to phase transitions in an infinite limit, i.e. the finite systems can be thought of as embedded in a single Markov process that admits multiple invariant distributions for certain parameter values. The phase transition suggested by circuit switched networks with alternate routing is of an unlikely kind, first because the system has "positive rates", and secondly because the system appears to be ergodic for both very small and very large arrival rates, and to admit multiple equilibria only in an intermediate range of parameters.

The kinds of limits considered by Marbukh and Gibbens, Hunt and Kelly do not deal with the spatial variations of the system. In an attempt to preserve the spatial characteristics we consider a lattice caricature of the model of Marbukh. We derive a hydrodynamic equation for this lattice model. This is an integro-differential equation which describes how the spatial distribution of the network evolves in time. The hydrodynamic equation also admits multiple spatially homogenous equilibrium solutions for certain ranges of the parameters, which may be loosely thought of as different possible phases for the network. This equation may be particularly useful in understanding the exchange between the phases, i.e. questions like "for what parameter values is a hot spot of heavy loading in the system likely to take over the whole network by knock-on effects ?"

INTRODUCTION

This paper discuses circuit switched networks with dynamic alternate routing. The purpose of dynamic routing schemes is to adaptively adjust traffic patterns in the network in response to demand, so as to make better use of spare capacity, and to provide robustness to failures or overloads. Such schemes have been the topic of considerable recent interest, [1], [2], [3], [12], [16], [17], [18], [19], [21], [24], [25], [26], [27], [29], primarily because it has only recently been possible to implement them in practice, and because they offer improved performance over the traditional hierarchical routing schemes.

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A difficulty associated with dynamic routing schemes is the potential for metastable states. Empirical studies of such routing schemes, [1], [2], [3], [5], [17], [21], [27], reveal the existence of hysterisis phenomena, corresponding to the existence of multimodal stationary distributions. Thus the network may have several qualitatively different regimes of operation for the same offered traffic, spending long periods of time in one or the other regime and sometimes moving from one to the other in response to fluctuations in the demand. Such behaviour is characteristic of metastability, [4], [15], [20], [28]. Important performance characteristics of the network, such as blocking probabilities, typically differ considerably between regimes. All the same, the improvement in performance over hierarchical routing schemes is such that dynamic routing schemes are being implemented in real world networks, [17], [29], with control schemes, such as trunk reservation for directly routed traffic. These, if suitably chosen, mitigate the effects of the potential multiplicity of operating regimes, [1], [2], [3], [16], [17], [21], [29].

The focus of this paper is on understanding the nature of the empirically observed metastability by using particle system techniques, [11], [23]. One of the questions we are interested in is if the metastability in large finite systems can be studied through an explicit phase transition in an infinite limit. Metastability in large finite systems is often associated to phase transitions in an infinite limit, i.e. the finite systems can be thought of as embedded in a single Markov process that admits multiple invariant distributions for certain parameter values. In the next section we discuss an example of this in the context of the well known contact process. The phase transition suggested by circuit switched networks with alternate routing is of an unlikely kind, first because the system has "positive rates", and secondly because the system appears to be ergodic for both very small and very large arrival rates, and have multiple equilibria only in an intermediate range of parameters.

Another important question of interest is the nature of the exchange between the different operating regimes. To describe this simply, we need equations that describe how the spatially distributed network state evolves over time. Motivated by this we consider a lattice model in the last section, which is analogous to a model of Gibbens, Hunt and Kelly, [17]. We find a hydrodynamic equation for this lattice model, [6], [7], [30]. This is an integrodifferential equation describing the time evolution of the spatially distributed network state. This equation also admits multiple spatially homogenous time-invariant solutions for certain ranges of the parameters, which may be loosely thought of as different possible phases for the network. The model described in the last section is at best a crude caricature for the situation in a real network; nevertheless, the resulting equation may be of some use in understanding the behaviour of real networks with dynamic routing.

PARTIAL SURVEY OF PREVIOUS WORK

Marbukh

Marbukh, analyzes several dynamic routing strategies in large completely connected networks, [24, [25]. In the simplest version of the model of Marbukh, we are given a completely connected network on n nodes, with a two way communication link between each pair of nodes, consisting of C circuits. Call requests between any pair of nodes a and b arrive according to independent Poisson processes of rate ν. If link (a,b) is not saturated, the call occupies one circuit in the link. If link (a,b) is saturated, the call randomly chooses a third node c such that each of the channels (a,c) and (c,b) has at least one circuit free, and simultaneously occupies one circuit on each of these links. If there is no such c, the call is lost. Each call holds the circuits it occupies for an exponential time of mean 1, after which it simultaneously releases them.

Let N denote $\binom{n}{2}$. Let $X^n_{ab}(t)$, $1 \leq a,b \leq n$, denote the number of circuits occupied on link (a,b) at time t in the network with n nodes. (Note that $(X^n_{ab}, 1 \leq a,b \leq n)$ are not enough to specify the state of the system, because two link calls simultaneously release the circuits they occupy). In Marbukh, [24], the following (as yet unproved) assumptions are made :

1) For any $a_1, b_1, \ldots a_l, b_l$,

$$\lim_{n\to\infty} P(X^n_{a_1b_1}(t) = k_1, \ldots X^n_{a_lb_l}(t) = k_l) = \prod_{j=1}^{l} \lim_{n\to\infty} P(X^n_{a_jb_j}(t) = k_j) \ ,$$

the convergence to the limits being uniform over t in any finite interval $[0,T]$. (In particular, the initial conditions are assumed to be of product type).

2) $\lim_{n\to\infty} P(X^n_{ab}(t) = k) = \gamma_k(t)$ is independent of a, b.

Under these assumptions one finds that $(\gamma_0, \gamma_1, \ldots, \gamma_C)$ satisfy a differential equation on the C-dimensional simplex. This differential equation takes the form

$$\begin{aligned}\dot\gamma_0 &= \gamma_1 - (\nu + 2\nu\gamma_C(1-\gamma_C)^{-1})\gamma_0 \ ,\\ \dot\gamma_k &= (k+1)\gamma_{k+1} + (\nu + 2\nu\gamma_C(1-\gamma_C)^{-1})\gamma_{k-1}\\ &\quad - (k + \nu + 2\nu\gamma_C(1-\gamma_C)^{-1})\gamma_k \ , 0 < k < C \ ,\\ \dot\gamma_C &= -C\gamma_C + (\nu + 2\nu\gamma_C(1-\gamma_C)^{-1})\gamma_{C-1} \ . \qquad (M)\end{aligned}$$

When one looks for equilibrium points of eqns. (M), one finds the following : If $\nu > C/2$, then $(\gamma_0, \ldots, \gamma_C) = (0, 0, \ldots, 1)$ is a stable equilibrium. Further, for $C \geq 3$, there is a value $\nu^* > C/2$ such that for all $\nu < \nu^*$ there is another stable equilibrium $(\gamma_0^*, \ldots, \gamma_C^*)$ given by

$$\begin{aligned}\gamma_k^* &= \frac{(\nu + 2\nu\gamma_C^*(1-\gamma_C^*)^{-1})^k C!}{(\nu + 2\nu\gamma_C^*(1-\gamma_C^*)^{-1})^C k!}\gamma_C^* \ ,\\ \gamma_C^* &= E(\nu + 2\nu\gamma_C^*(1-\gamma_C^*)^{-1}, C) \ , \qquad (M_{eq})\end{aligned}$$

where $E(\nu, C) = \frac{\nu^C/C!}{\sum_{l=0}^{C}\nu^l/l!}$. In particular, for $C/2 < \nu < \nu^*$, eqns. (M) have more than one stable equilibrium. This is a consequence of the multimodality of the stationary distribution of the Markov process describing the network for large n.

The assumptions (1) and (2) of Marbukh are related to the hypothesis of propagation of chaos in statistical mechanics, [31], [34]. They are quite likely true.

Gibbens, Hunt and Kelly

Gibbens, Hunt and Kelly, [17] consider a caricature of the model of Marbukh, which bypasses the spatial features of the model. Consider a collection of N links, each of which consists of C circuits. At each link, calls arrive according to a Poisson process of rate ν. If its link is not saturated, the call occupies one circuit on the link. If its link is saturated, the call chooses two distinct links at random from the remaining $N-1$ links, and if neither is saturated, the call occupies one circuit from each of these two links. Otherwise the call is blocked and rejected from the system. Each occupied circuit is held for an independent exponential time of mean 1. (Note that when a call occupies two circuits after making a succesful choice of alternate route, it is assumed that these circuits are released independently).

Let $\gamma_k^N(t)$, $0 \leq k \leq C$, denote the fraction of the N links that have k occupied circuits at time t. $(\gamma_0^N, \gamma_1^N, \ldots, \gamma_C^N)$ evolves as a Markov process on the C-dimensional simplex. In [17], an ODE limit is found for the evolution as $N \to \infty$. Namely, if the initial conditions $(\gamma_0^N(0), \gamma_1^N(0), \ldots, \gamma_C^N(0))$ converge weakly to a limit $(\gamma_0(0), \gamma_1(0), \ldots, \gamma_C(0))$, then the process converges to the deterministic process given by the equations

$$\begin{aligned}\dot\gamma_0 &= -\gamma_1 + (\nu + 2\nu\gamma_C(1-\gamma_C))\gamma_0 \ ,\\ \dot\gamma_k &= (k+1)\gamma_{k+1} + (\nu + 2\nu\gamma_C(1-\gamma_C))\gamma_{k-1}\\ &\quad - (k + \nu + 2\nu\gamma_C(1-\gamma_C))\gamma_k \ , 0 < k < C \ ,\\ \dot\gamma_C &= -C\gamma_C + (\nu + 2\nu\gamma_C(1-\gamma_C))\gamma_{C-1} \ , \qquad (GHK)\end{aligned}$$

with the appropriate initial conditions.

When one looks for equilibrium points of eqns. (GHK), one finds the following : The equilibrium points are given by the solutions of

$$\begin{aligned}\gamma_k^* &= \frac{(\nu + 2\nu\gamma_C^*(1-\gamma_C^*))^k C!}{(\nu + 2\nu\gamma_C^*(1-\gamma_C^*))^C k!}\gamma_C^* \ .\\ \gamma_C^* &= E(\nu + 2\nu\gamma_C^*(1-\gamma_C^*), C) \ . \qquad (GHK_{eq})\end{aligned}$$

When C is large enough, one finds that there is a range of ν in which eqns. (GHK_{eq}) admit three solutions, two of which are stable. To the left and right of this range there is a unique stable solution.

Contact Process

The existence of metastability in large finite systems is often associated with phase transitions in an infinite system which is a natural limit of the finite systems. A nice example of this connection is in the well known contact process, whose finite time behaviour has been analysed by several authors, [4], [9], [10], [28]. The (nearest neighbour) contact process on $\mathbf{Z}$ a Markov process $(\eta_t, t \geq 0)$ with state space $\{0,1\}^{\mathbf{Z}}$. The points of $\mathbf{Z}$ are called *sites*. If η denotes a generic element of $\{0,1\}^{\mathbf{Z}}$, we say x is *infected* under η if $\eta(x) = 1$, and *healthy* if $\eta(x) = 0$. The process evolves according to the following rules : Infected sites become healthy at rate 1. At rate ν an infected site picks one of its two neighbouring sites at random and infects it (if this site is already infected, nothing happens).

There is no difficulty in constructing a unique Markov process on $\{0,1\}^{\mathbf{Z}}$ with these rates, even though the number of sites is infinite. See Liggett, [23], Chapter 6, or Durrett, [11], Chapter 4, for the construction. What is interesting is the existence of a critical value $0 < \nu_c < \infty$ with the following features : If $\nu < \nu_c$, then the process is *ergodic*, i.e. it admits a unique invariant distribution (all sites healthy) to which the distribution of the process converges (weakly) starting from any initial condition. However, if $\nu > \nu_c$, then the process is non-ergodic. It admits a one-parameter family of invariant distributions, each of which is a mixture of the "all healthy" distribution and the *upper invariant distribution*. The latter is the distribution to which the process converges starting with all sites infected (it exists because the process is *attractive*, see [23], Chapter 3, Section 2, for defn.) and is different from the "all healthy" distribution whenever $\nu > \nu_c$.

The phase transition in the contact process on $\mathbf{Z}$ betrays itself in the contact process on $[N] = \{1, 2, \ldots, N\}$ for N sufficiently large. This is the Markov process $(\eta_t^N, t \geq 0)$ on $\{0,1\}^{[N]}$ with dynamics as above, except that an infected boundary site (1 or N) attempts to infect its (unique) neighbour at rate $\nu/2$. Clearly this process has a unique invariant distribution, which is the one with all sites healthy. Let σ_N have the distribution of the time to reach the "all healthy" state from the "all infected" state. Then one can prove, [4], [9], [10], [28] :

$$\frac{\sigma_N}{\log N} \longrightarrow a_1(\nu) \quad \text{if} \quad \nu < \nu_c \ ,$$

$$\frac{\log \sigma_N}{N} \longrightarrow a_2(\nu) \quad \text{if} \quad \nu > \nu_c \ ,$$

the convergence being in probability.

Note the enormous qualitative change in the settling behaviour of the process for large N as ν crosses ν_c. The change in behaviour of the large finite system directly corresponds to phase transitions in the infinite limit.

<u>Remarks on the possibility of phase transitions</u>

The existence of metastability in large circuit switched networks with dynamic alternate routing suggests that there might be a natural infinite limit which exhibits phase transitions. A difficulty with formulating such a limit is that there is no natural analog of the uniform choice of alternate routes. One idea is to replace this dynamic with a long range interaction which favours links that are close to the blocked link. Alternately, although it is less likely, one might investigate the possibility that phase transitions show up in finite range models.

A simple finite range model for dynamic alternate routing is the following : Let $\mathbf{Z}^2$ denote the integer lattice in $\mathbf{R}^2$. We call the points of $\mathbf{Z}^2$ *sites*. We think of link consisiting of C circuits as sitting at each site. Each link has a Poisson process of calls of rate ν. A call that finds its first-choice link not saturated occupies one circuit on the link. If the call finds its link saturated, it pick two other links in its $[-L, L]^2$ neighbourhood and, if possible, it occupies one circuit on each link. Otherwise, it is blocked and rejected from the system.

We may expect that for C large enough so that the equations (GHK_{eq}) have multiple equilibria, if L turns out to be large enough, the above system will exhibit the following behaviour : (1) it is ergodic for all small enough ν. (2) It has an intermediate range of ν in which it has multiple invariant distributions. (3) It is ergodic for all sufficiently large ν. Apart from being able to prove (1), which is easy, and being morally convinced that (3) is true, we are unable to resolve this question at the present writing. The difficulty with studying phase transitions in systems of the type at hand appears to be fairly deep. The system has "positive rates", i.e. for any site x, whatever the configuration η, there is a positive lower bound on the rate at which the value at x changes. There are very few examples of finite range systems with positive rates which exhibit phase transitions. These appear to be limited to the family of Stochastic Ising models, Liggett, [23], Chapter 4, a family of discrete time examples due to Toom, [32], [33], and a continuous time process due to Durrett and Gray, [8], which is based on Toom's "northeast corner model", [22]. See also Gacs, [13], and Gacs, et. al., [14].

<u>LATTICE CARICATURE</u>

In this section we analyze a lattice caricature of the model of Marbukh which has the virtue of preserving spatial features of the system. Let $\mathbf{Z}^d/M$ denote the lattice in $\mathbf{R}^d$ consisiting of points all of whose co-ordinates are rational with denominator dividing M. The points of $\mathbf{Z}^d/M$ are called *sites*. Let W denote $\{0, 1, \ldots, C\}$. We consider a Markov process $(\eta_t^M, t \geq 0)$ on $W^{\mathbf{Z}^d/M}$ which caricatures a circuit switched network with dynamic routing (the statements below are true for any d, but the situations $d = 1$, and $d = 2$ are likely to be of most interest). We use η to denote a generic element of $W^{\mathbf{Z}^d/M}$, and call $\eta(x)$ the *value* at site x. Let M^* denote $\binom{(2M+1)^d - 1}{2}$. The Markov process is described by the transitions

$$\eta(x) \ \rightarrow \eta(x) + 1 \ \text{ at rate 1 if } \eta(x) \neq 0 \ ,$$

$$\eta(x) \ \rightarrow \eta(x) - 1 \ \text{ at rate } \nu \text{ if } \eta(x) \neq C \ ,$$

$$(\eta(x), \eta(y), \eta(z)) \ \rightarrow (\eta(x), \eta(y) + 1, \eta(z) + 1) \ \text{ at rate } \nu/M^* \ \text{ if } x, y, z \text{ are distinct sites with } \eta(x) = C, \eta(y) < C, \eta(z) < C \text{ and } y, z \in x + [-1, 1]^d \ .$$

There is no difficulty constructing such a Markov process even though the number of sites is infinite. See Liggett, [23], Chapter 1, Section 3, for details; Theorem 3.9 of that section applies directly.

We think of each site in the lattice as representing a link in our network, which consists of C circuits. We think of the value at a site as giving the number of occupied circuits in the corresponding link. Occupied circuits become free at rate 1. At each link there is a Poisson process of calls with rate ν. Each call occupies one circuit on its link if available; if the link is saturated the call randomly picks two other links which are in its $[-1,1]^d$ neighbourhood, and uses one circuit from each of these links if possible. Otherwise the call is blocked and rejected from the system. Note that because we have a compressed lattice, the interaction actually has range M on the scale of links.

For $x \in \mathbf{Z}^d/M$, let $u_M(t,x,k)$ denote $P(\eta_t^M(x) = k)$, $0 \le k \le C$. We extend the definition of $u_M(t,\cdot,k)$ to $\mathbf{R}^d$ by setting $u_M(t,x,k) = u_M(t,[x]_M,k)$ for $x \in \mathbf{R}^d$, where $[x]_M$ denotes the minimum element in $\mathbf{Z}^d/M$ which dominates x in the usual partial order on $\mathbf{R}^d$. Let $u(0,x,k)$, $0 \le k \le C$, be continuous functions with bounded derivative and with $\sum_{k=0}^{C} u(0,x,k) = 1$. Let $u(t,x,k)$, $0 \le k \le C$ denote the solution of the integrodifferential equations

$$\frac{\partial u(t,x,0)}{\partial t} = u(t,x,1) - \nu(1+2^{1-d} \iint_{q,r\in[-1,1]^d} u(t,x+q,C)(1-u(t,x+q+r,C))dqdr)u(t,x,0) ,$$

$$\frac{\partial u(t,x,k)}{\partial t} = (k+1)u(t,x,k+1) + \nu(1+2^{1-d} \iint_{q,r\in[-1,1]^d} u(t,x+q,C)(1-u(t,x+q+r,C))dqdr)u(t,x,k-1) - (k+\nu(1+2^{1-d} \iint_{q,r\in[-1,1]^d} u(t,x+q,C)(1-u(t,x+q+r,C))dqdr)u(t,x,k) ,\quad 0<k<C ,$$

$$\frac{\partial u(t,x,C)}{\partial t} = -Cu(t,x,C) + \nu(1+2^{1-d} \iint_{q,r\in[-1,1]^d} u(t,x+q,C)(1-u(t,x+q+r,C))dqdr)u(t,x,C-1) . \qquad (Hyd)$$

Then we have the following :

Theorem 1 : Fix $T < \infty$. Suppose that we start $(\eta_t^M)_t$ with initial configuration the product measure having $P(\eta_0^M(x) = k) = u_M(0,x,k)$, $0 \le k \le C$. If $u_M(0,x,k) \to u(0,x,k)$ unformly on compact sets, $0 \le k \le C$, then $u_M(t,x,k) \to u(t,x,k)$ for all $t \in [0,T]$, $x \in \mathbf{R}^d$ and $0 \le k \le C$.

Theorem 1 is a statement about pointwise convergence of probabilities. There is a corresponding functional limit theorem. Let $\mathcal{S}(\mathbf{R}^d)$ denote the space of Schwarz functions on $\mathbf{R}^d$ and $\mathcal{S}^*(\mathbf{R}^d)$ its topological dual (see, e.g., Zemanian, [35]). Then we have :

Theorem 2 : Given $\phi_k \in \mathcal{S}(\mathbf{R}^d)$, $0 \le k \le C$, let

$$X_t^M(\phi) = \frac{1}{(2M)^d} \sum_{x\in\mathbf{Z}^d/M} \sum_{k=0}^{C} \phi_k(x) 1(\eta_t^M(x) = k) .$$

We may view X^M as an element of $D([0,T],(\mathcal{S}^*(\mathbf{R}^d))^{C+1})$. Then $X^M \Rightarrow X_.$, where $\Rightarrow$ denotes weak convergence, and

$$X_.(\phi) = \sum_{k=0}^{C} \int_{x\in\mathbf{R}^d} \phi_k(x) u(\cdot,x,k)\, dx .$$

When we look for spatially homogenous solutions of eqns. (Hyd) which are time invariant, we are led to the same equations (GHK_{eq}) found by Gibbens, Hunt and Kelly. Thus we see that for large enough C, there is a range of ν over which eqns. (Hyd) admit three spatially homogenous solutions. These may be loosely thought of as different phases associated to the network. The exchange between the phases can be studied by numerically integrating the eqns. from the appropriate initial conditions.

Sketch of the proof of Theorems 1 and 2 : The key idea is to consider a branching tree running backwards in time, which is in some sense a dual to the forward time process. We start from $x \in \mathbf{R}^d$ at time t, with a single "particle" alive at x. At time $t-s$ (reversed time s), there is a certain set $\mathcal{P}_s$ of "particles" which are alive. A particle alive at reversed time s stays alive on reversed time $[s,t]$. Each live particle p is at a point $x(p) \in \mathbf{R}^d$ and does not move. Further, at each reversed time s, we are given a map

$$F_s : W \to 2^{W^{\mathcal{P}_s}} .$$

If $C \in W^{\mathcal{P}_s}$ is such that $C \in F_s(k)$, this means that if the point $x(p)$ has value $C(p)$ for each $p \in \mathcal{P}_s$, then tracing the forward time process from time $t-s$ to t, the resulting value at x is k. Each particle alive at reversed time s has associated with it Poisson processes of rates 1 and ν at the times of which it autonomously reevaluates its "feasible set" for each $k \in W$, and a Poisson process of rate 2ν, at the times of which it picks random points $y \in x(p)+[-1,1]^d$ and $z \in y+[-1,1]^d$, generating one new particle at each of these points, and reevaluating joint feasible sets for each $k \in W$, with y thought of as the potential seeding point, and $x(p)$ and z as the points checked by y.

From the branching tree on $\mathbf{R}^d$, we construct a branching tree on $\mathbf{Z}^d/M$ by moving particles to the nearest lattice point that dominates their assigned point. Now, more than one particle may be alive at a site. We need to show that this happens with vanishingly small probability as $M \to \infty$. We also need to show that if we construct branching trees starting from two distinct sites $x, y \in \mathbf{Z}^d/M$, the probability that they intersect, (i.e. that some site is occupied by particles from each process) has vanishingly small probability as $M \to \infty$.

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