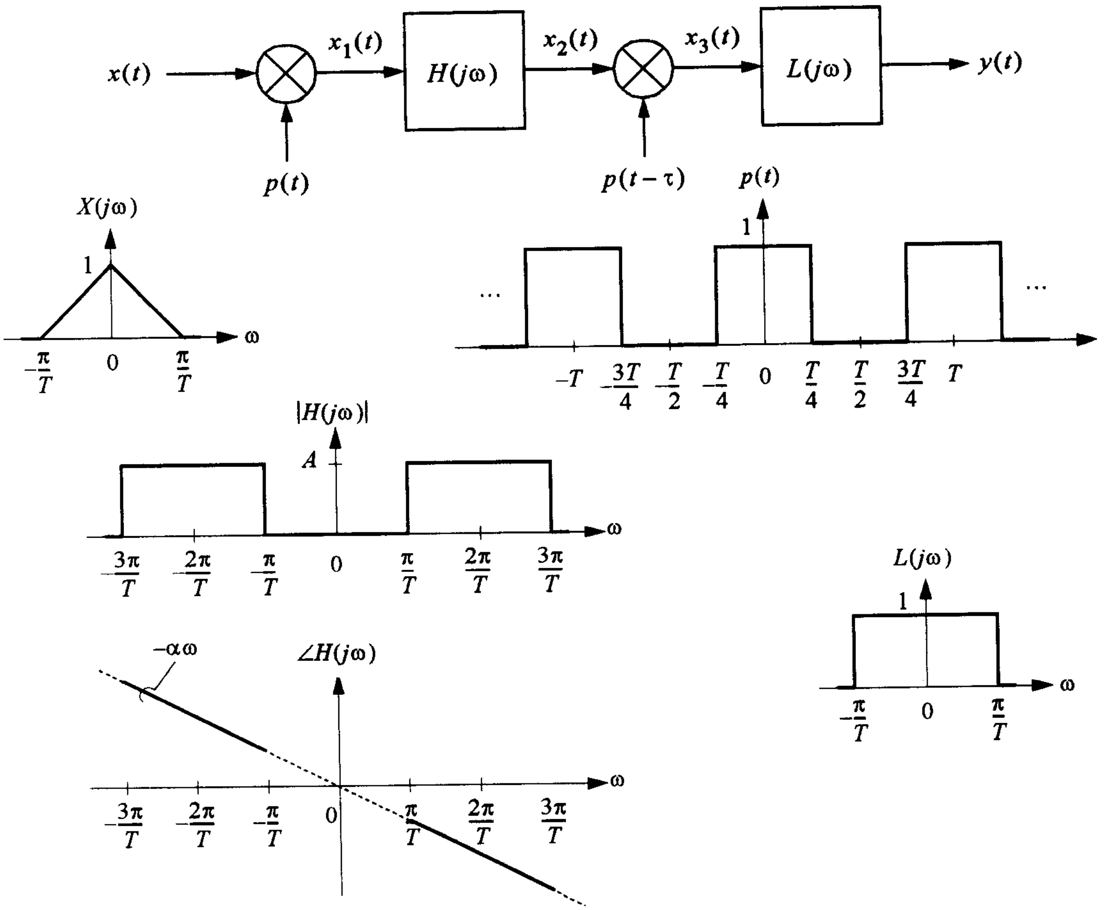
This problem concerns a scheme for implementing high-gain, d.c.-coupled amplifiers. A continuous-time signal $x(t)$, having Fourier transform $X(j\omega)$, is bandlimited as shown. A periodic pulse train $p(t)$ is used to modulate $x(t)$ onto a carrier. A bandpass amplifier $H(j\omega)$ amplifies the resulting signal. A delayed copy of the pulse train, $p(t-\tau)$, modulates the signal back to baseband, and the lowpass filter $L(j\omega)$ removes the undesired passband components.



- Write down the Fourier transforms of $p(t)$ and $p(t-\tau)$.
- Write down an expression for $X_1(j\omega)$ in terms of $X(j\omega)$.
- Sketch $X_1(j\omega)$ over the range $\frac{-7\pi}{T} \le \omega \le \frac{7\pi}{T}$.
- Find an expression for $X_2(j\omega)$ in terms of $X(j\omega)$.
- Find an expression for $X_3(j\omega)$ in terms of $X(j\omega)$.
- Find an expression for $Y(j\omega)$ in terms of $X(j\omega)$.
- Specify a value of τ such that the overall system [with input $x(t)$ and output $y(t)$] achieves the largest possible positive gain. For this choice of τ, find an expression for $y(t)$.

Consider a bin of capacity 1. It is initially empty : $Y_0 = 0$.

At time $n = 1, 2, ...$ a packet of size X_n arrives. Packet sizes are i.i.d. and uniform on $[0, 1]$.

An arriving packet, at the time it arrives, is placed in the bin if there is enough empty space in the bin (i.e. not already occupied by other packets). Otherwise the packet is rejected.

Let Y_n denote the total size of all the packets in the bin after the arrival (and placement) of the n-th packet. Let a_{nk} denote the k-th moment of Y_n, $k = 1, 2, ...$

Determine a_{nk}, $n, k = 1, 2, ...$

What is $\lim_{n \to \infty} a_{nk}$?

③ Communications Sp00

Communications Preliminary Examinations – January 12, 2000
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Let $\{X(n), n \geq 0\}$ be an irreducible discrete Markov chain on the state space $\{0, 2\}$.
Define $\{Y(n), n \geq 0\}$ as follows:

a. $Y(0)$ is a random variable that takes values in $\{0, 1, 2, \ldots\}$ and is independent of $\{X(n), n \geq 0\}$;
b. For $n \geq 0$, $Y(n+1) = Y(n) + X(n) - 1\{Y(n) > 0\}$.

1. Is $\{Y(n), n \geq 0\}$ Markov? Explain.

2. Under what conditions does $\{Y(n), n \geq 0\}$ admit a stationary version? Under these conditions, how would you compute the invariant distribution of $Y(n)$.

3. Can $\{Y(n), n \geq 0\}$ be time-reversible?

4. How would you calculate $E[\tau_N | Y(0) = 0]$ where $\tau_N := \min\{n \geq 0 \mid Y(n) = N\}$?