Particle Simulations in Plasma

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Abstract

Computer simulations are efficient tools that are not only used for theory predictions but also to predict performance in plasma physics applications to fusion reactors and other devices. Because of the complexity of plasma physics, live computer simulation is far more instructive than viewing periodic phase plots.

Many theories have been set in the past about ion and electron behavior in plasma. This project dedicates itself in proving some of these theories. We use XES1, a particle simulation program, to simulate particle behavior. One of the theories that we are proving is Landau damping, which predicts that in collision-less plasma, there is a decrease in electrostatic energy and a gain in kinetic energy resulting from the transfer from an electrostatic wave to the particles. With XES1, we have been able to prove that this damping in electrostatic energy occurs by applying a small sinusoidal perturbation (excitation) in x.

The cold and warm two-stream instability have been other theories we have been working on. It occurs when two opposing streams (\(v_{\text{thermal}} < 0.7 v_{\text{drift}}\)) are unstable and this instability grows in time. After running the simulation for a long time we have seen that the overall velocity distribution is not Maxwellian.
1. Introduction

This project is about plasma simulations using XES1, a particle simulation program, in order to find the behavior of electrons and ions, in plasma. We also use these simulations to prove theory predictions done in the past like Landau damping and the two-stream instability. The Landau damping was run with small excitation, linear and with large excitation non-linear, with particle trapping. The two-stream grew linearly from noise excitation to fully non-linear behavior, saturation [1].

1.1 XES1 is an Electrostatic 1 dimensional code. This code simulates periodic plasma, and runs different diagnostics like:
- Vx Phase Space
- Electric Field
- Potential
- Density
- Kinetic Energy
- Total Energy
- Field Energy
- Fourier Modes
- Velocity Space

1.2 XES1 in order to work needs an input file, (name.inp) which has the characteristics of the global parameters, and the parameters describing each species of particles.

1.2.1 Global Parameters

nsp The number of particle species to simulate
l The length of the system.
dt The time step.
nt The total number of steps to be run.
mmax Maximum number of electrostatic energy modes to view.
ng The total number of grid points (power of 2).
iw Weighting to be used:
0 for zero order(NGP)
1 for first order(CIC, PIC)
2 for second order(quadratic spline)
3 for third order (cubic spline)
ec Momentum conserving/Energy conserving flag
0 Momentum conserving scheme (recommended)
1 Energy conserving scheme
epsi 1/epsilon o (usually 1)
a1 Compensation factor (a1=0 means no compensation)
a2 Smoothing factor (a2=0 means no smoothing)
E0 Magnitude of an applied electric field.
w0 Frequency of the applied electric field
accum  Velocity diagnostic parameter. 0 turns them off

1.2.2 Species Parameters

n  Number of particles
nv2  Exponent of quiet start distribution f(v), usually zero
nlg  Number of sub groups to be given the same velocity distribution, usually one
mode  Number of mode to be given an initial perturbation in x, vx
wp  wp(positive) plasma frequency
wc  wc(signed) cyclotron frequency
qm  q/m(signed)
v1  Provides Gaussian velocity distribution of thermal velocity vt1 centered on
    vx=vo, vy=0, using random number routine; maximum number velocity is 6vt1
vt2  Provides Gaussian (or other) velocity distribution of thermal velocity vt2 using
    inverse distribution functions, giving ordered velocities
v0  Drift velocity in x direction (signed)
x1  Magnitude of perturbation in x, generally less than half the uniform particle
    spacing, n/l; used as x1 cos(2pi x mode/l + thetax)
v1  Magnitude of perturbation in v; used as v1 as sin(2pi x mode/l +thetav)

theta  θ
x  θx
v  θv

nbins  The number of bins to use when accumulating the velocity distribution
diagnostics.

1.3 Plasmas

Before I go any deeper into particle simulation we need to be able to understand
what plasma really is. Plasma is the fourth state of matter. We have solid to liquid to gas,
and to plasma as the temperature increases. In the plasma state we find electrons, ions,
and neutral particles. For our purposes “the simulated plasmas have the following
characteristics (1) they are driven electrically; (2) charged particles collisions with neutral
gas molecules are important; (3) there are boundaries at which surface looses are
important; (4) ionization of neutrals sustains the plasma in the steady state and (5) the
electrons are not in thermal equilibrium with the ions.”[2] page 6.

2. Landau Damping

Landau Damping is often referred as “collision-less damping” which results from
the resonant energy exchange between waves and plasma particles [3].

In this experiment I applied a small initial perturbation (or excitation) in order to
observe the decaying rate of the electrostatic energy, gain in kinetic energy, and changes
of the distribution function, f(v). My first observations were that the damping rate was
larger than that predicted by linear theory.
Figure 1. The input file for Landau damping with 100,000 particles and initial velocity $v_0 = 0.00$

```
Landau.inp

nsp----------|--------dt------nt-----mmax-----l/a-----accum
1 6.283185307 0.2 300 3 0 1
ng--------iw------ec------epsi------a1------a2------E0------w0
512 2 0 1.00 0.00 1000 0 0

SPECIES 1: Warm Electron Plasma

n----------nv2------nlg------mode
100000 0 1 1
wp------wc------qm------vt1------vt2------v0
1.00 0.00 -1.00 0.0 0.40 0.00
xl------vl------thetax------thetav
0.1 0.0 0.00 0.00
nbins------vlower------vupper
50 -3.00 3.00
```

Figure 2. The input file for Landau damping with 100,000 particles and initial velocity $v_0 = -1.2$

```
Landau.p.inp

nsp----------|--------dt------nt-----mmax-----l/a-----accum
1 6.283185307 0.2 300 3 0 1
ng--------iw------ec------epsi------a1------a2------E0------w0
512 2 0 1.00 0.00 1000 0 0

SPECIES 1: Warm Electron Plasma

n----------nv2------nlg------mode
100000 0 1 1
wp------wc------qm------vt1------vt2------v0
1.00 0.00 -1.00 0.0 0.40 -1.2
xl------vl------thetax------thetav
0.1 0.0 0.00 0.00
nbins------vlower------vupper
50 -3.00 3.00
```
2.1 Experiment 1 (Mode 1 ESE)
This simulation is done using Landau.inp (figure 1) and changing the perturbation x1; shown is the electrostatic energy of the plasma on a few (plasma) periods. It is done in a log scale; the red line added is the decay (\(w_i\)). The \(w_i\) gets closer to the theorized value as the perturbation is smaller.

\[
x1=0.1 \quad w_i=0.10708135
\]

\[
x1=0.01 \quad w_i=0.10020312097
\]

\[
x1=0.001 \quad w_i=0.088342
\]

The equations used to find the slope of the damp are:
\[
ESE = A \times (\exp (w_i t - iw_i t))^2
\]
\[
ESE = A \times \cos (2w_i t) \times \exp(-2w_i t)
\]
\[
ESE(t) = ESE(0) \times \exp(-2w_i t)
\]
\[
\ln(ESE(t)) - \ln(ESE(0)) = \ln(\exp(-2w_i t))
\]
\[
\ln(ESE(t)) - \ln(ESE(0)) = -2w_i t
\]
2.1.1 Trapping (Vx-X Phase Space)
Considering the velocity distribution, we find out that some of the electrons move faster and slower than the wave phase velocity $w/k$. Transforming to the frame moving with the phase velocity of the wave, we see easily that the electrons in a velocity band $w/k \pm v_{tr}$ are trapped by the wave, oscillating about velocity $w/k$ [1].
Figure 3. The input file for Landau damping with 1,000,000 particles and initial velocity $v_0 = 0.0$.

Figure 4. The $w_1$ theoretical value is 0.06 from Jackson [4].
2.2 Experiment 2 (Mode 1 ESE)

The reason for using more particles is that we can find a more accurate result for the damping rate. And to get more resolution I change $\Delta t = 0.1$ and the number of grids to $NG = 8192$.

ESE1 with 1000000 particles

- $x_1 = 0.1$, $wi = 0.075246$
- $x_1 = 0.01$, $wi = 0.0704603$
- $x_1 = 0.001$, $wi = 0.06924564$
- $x_1 = 0.0001$, $wi = 0.06901563$
- $x_1 = 0.00001$, $wi = 0.0650748$
These changes made it possible to run a better simulation, although the simulation was running slow (in the 5-10 min are) the results were good. In the previous run I found out that they were run at ng = 512, but noticing that adding the number of grids gives you a way better result. Before if you wanted to run a x1 = 0.00001 perturbation you would not get a good result. With 1,000,000 particles we find that the get a wi closer to the theoretical value.

Advantages of running with more particles:
1- If the perturbation was x1=0.00001, the damping in the plasma continues over many more plasma oscillations, than with fewer particles.
2- The wi was a lot closer to the theorized value. At x1=0.00001 we can see that wi=0.0650748 and the theorized value is wi=0.06 as shown in figure 4.

In order to obtain a well-defined damped oscillation we found it necessary to use many more computational cells.

Many things can go wrong running a simulation on of them is that the program is not loading all the particles, we can see this behavior when checking f(v). In one of my previous experiment I tried to run the program with 1,000,000 particles but there were some kind of errors in the Electrostatic field behavior. F(v) was checked and we found out the program only loaded half the particles. You can see this behavior in figure 6.
Figure 6. $f(v)$ All species vs velocity. Behavior when loading 1,000,000 without the right amount of grids.

2stream.inp

nsp----------l----------dt----nt--mmax--l/a--accum
2  31.415926538 0.1 300 5 0 1
ng--------iw------ec------epsilon------a1------a2------E0------w0
512 1 0 1.00 0.00 0.00 0.00

SPECIES 1: Cold Electron Plasma

n--------nv2------nlg-----mode
2048 0 1 1
wp------wc------qm------vt1------vt2------v0
1.00 0.00 -1.00 0.0 0.001 1.00
x1--------v1------thetax------thetav
0.0001 0.0 0.00 0.00
nbins-----vlower-----vupper
100 -2.00 2.00

SPECIES 2: Cold Electron Plasma

n--------nv2------nlg-----mode
2048 0 1 1
wp------wc------qm------vt1------vt2------v0
1.00 0.00 -1.00 0.0 0.00 -1.00
x1--------v1------thetax------thetav
-0.0001 0.0 0.00 0.00
nbins-----vlower-----vupper
100 -2.00 2.00

Figure 7. The input file for Cold two-stream with 100,000 particles.
3. Two-stream instability

Two opposing electron streams ($v_{\text{thermal}} < 0.7 \ v_{\text{drift}}$) are unstable and grow in time, in a fixed ion background, in a periodic system. [1] Cold stream growth of $w_{\text{imag}} = wp/2$, largest known, violently spreads out $f(v)$, including making $f(v=0)$ larger than the Maxwellian value, than oscillating about that, but still trapping in phase space, not reaching a Maxwellian $f(v)$ for a long time. Initially warm stream growth is smaller and has a much smaller $f(v=0)$, Maxwellian, and approaches a stable Penrose double-humped $f(v)$. 

Vx-X Phase Space | f(v) | rho(x)  
--- | --- | ---  
$t = 20$ | ![Graph](image1.png) | ![Graph](image2.png)  
$t = 27$ | ![Graph](image3.png) | ![Graph](image4.png)  
$t = 126.4$ | ![Graph](image5.png) | ![Graph](image6.png)  

Figure 8. The input file for Warm two-stream with 100,000 particles.

```
w2stream.inp

nsp--------l---------dt-----nt---mmax---l/a---accum
2  31.415926535  0.1  300  9  0  1
ng-------iw------ec------epsil------al------a2------E0------w0
512  2  0  1.00  0.00  0.00  0.00  0.00

SPECIES 1: Cold Electron Plasma

n--------nv2------nlg------mode
2048  0  1  1
wp------wc------qm------vt1------vt2------v0
1.00  0.00  -1.00  0.0  0.4  1.00
x1--------vl------thetax------thetav
0.0001  0.0  0.00  0.00
nbins------vlower------vupper
100  -3.00  3.00

SPECIES 2: Cold Electron Plasma

n--------nv2------nlg------mode
2048  0  1  1
wp------wc------qm------vt1------vt2------v0
1.00  0.00  -1.00  0.0  0.4  -1.00
x1--------vl------thetax------thetav
-0.0001  0.0  0.00  0.00
nbins------vlower------vupper
100  -3.00  3.00
```
4. Warm two-stream instability

When two warm streams interact with \( v_t/v_0 = 0.4 \) mode 4 has the fastest growing rate in time. We can see this at \( t=30 \), mode 4 dominates, in the \( v_x-x \) phase space and in the charge density, \( \rho \). By time 63.3, the four phase space vortices have merged leaving two vortices. By time 244.8, all vortices in phase space have become just one vortex, which is very long lived.

Vx-X Phase Space   \( f(v) \)   mode4
\( t = 30 \)
\( t = 63.3 \)  mode2
\( t = 244.8 \)  mode1

5. Conclusion

Proving theories specially Landau damping has been one of the greatest accomplishment in plasma physics. We have been able to simulate that the behavior of Landau damping was indeed as predicted by analytical theory. We found out that when using 1,000,000 particles to simulate Landau we found a closer \( \omega_i \) value to the predicted linear theory. And doing more investigation on particle simulation has been really insightful, finding out the behavior of the two-stream stability, going from fastest growing rate mode 4 and after a long time going to fastest growing rate mode1. Computer simulation really helps to make the complex world of plasma physics more
understandable and easier to approach. And it has many advantages in the studies of fusion reactor behavior and other devices.

6. References