

# C280, Computer Vision

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Lecture 18: Multiview and Photometric  
Stereo

# Today

- Multiview stereo revisited
- Shape from large image collections
- Voxel Coloring
- Digital Forensics
- Photometric Stereo

# Multi-view Stereo

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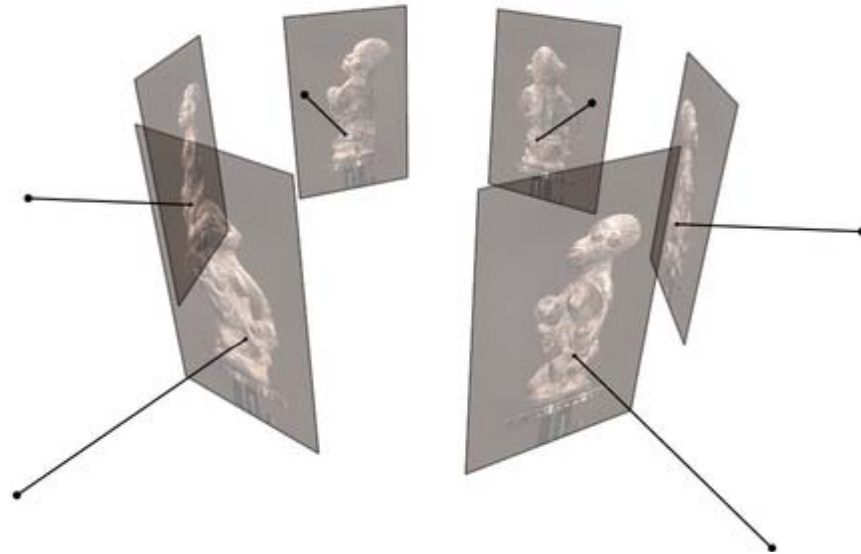


# Multi-view Stereo

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Input: calibrated images from several viewpoints

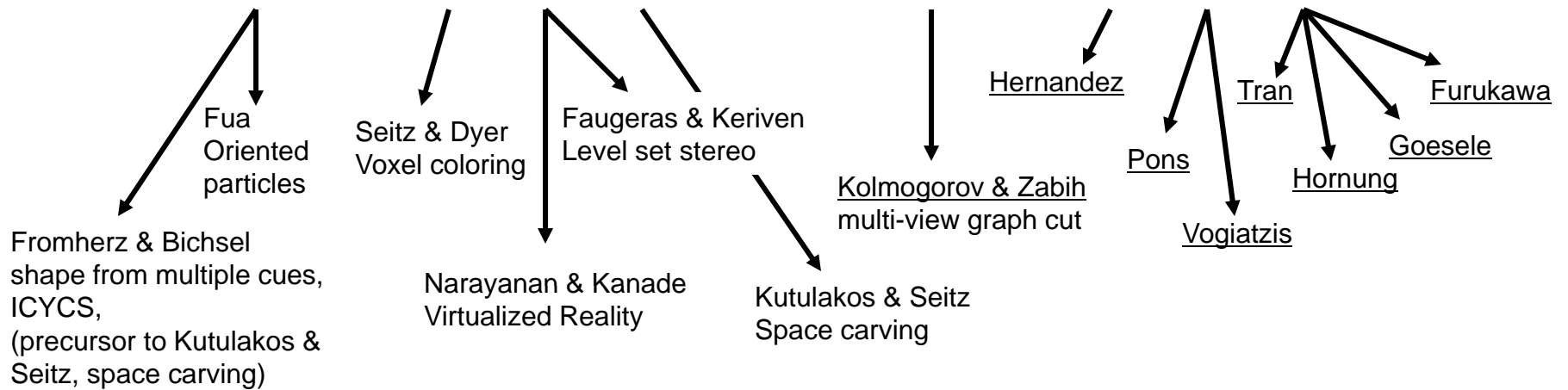
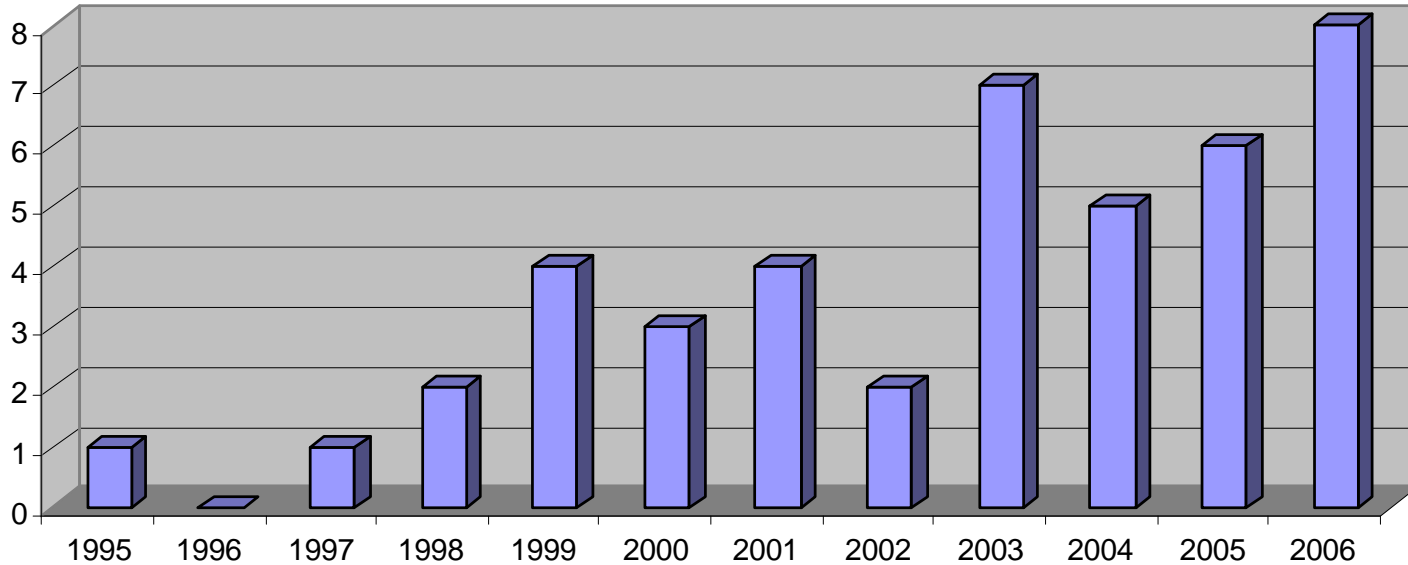
Output: 3D object model

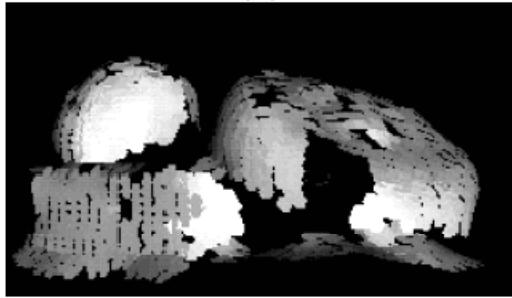


Figures by Carlos Hernandez

# History

number of papers in CVPR, ECCV, and ICCV, by year





Fua  
**1995**



Seitz, Dyer  
**1997**



Narayanan, Rander, Kanade  
**1998**



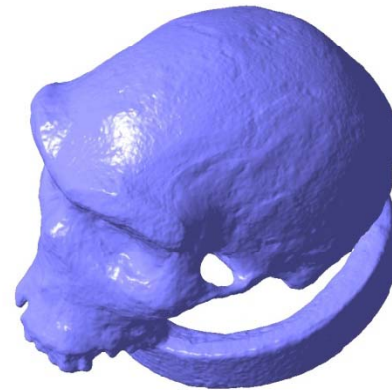
Faugeras, Keriven  
**1998**



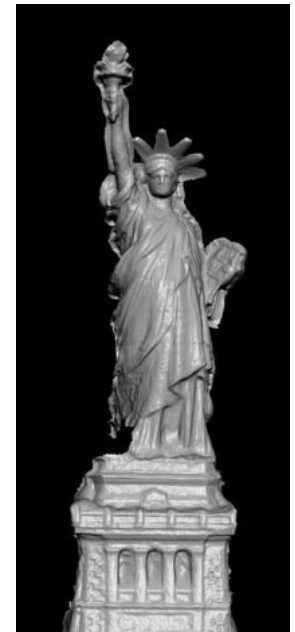
Hernandez, Schmitt  
**2004**



Pons, Keriven, Faugeras  
**2005**



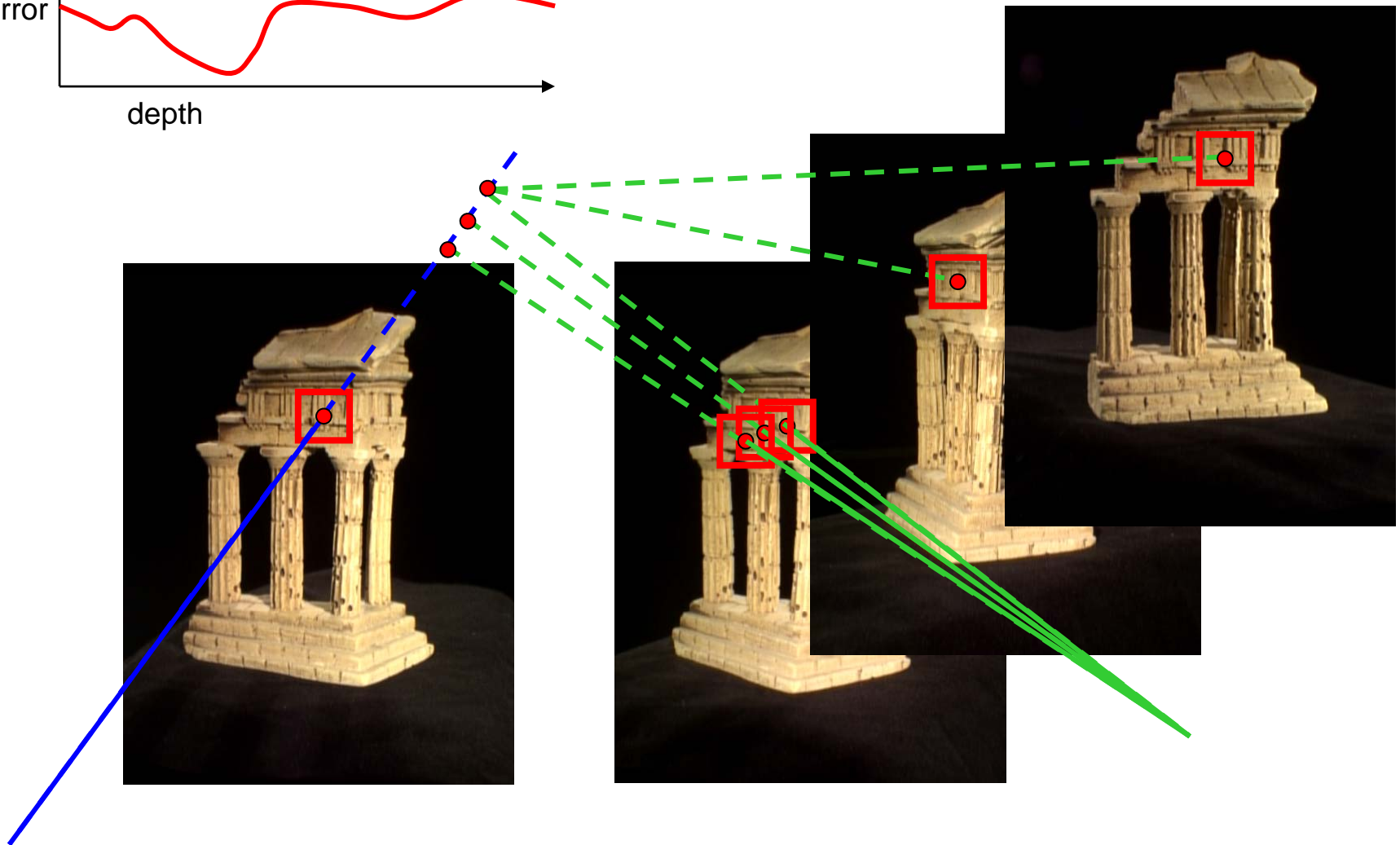
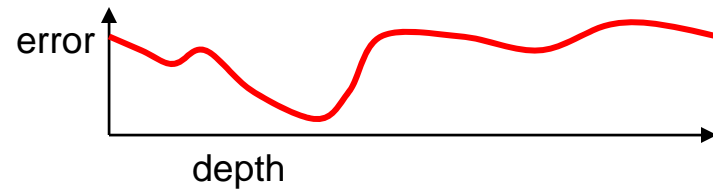
Furukawa, Ponce  
**2006**



Goesele et al.  
**2007** [Seitz]

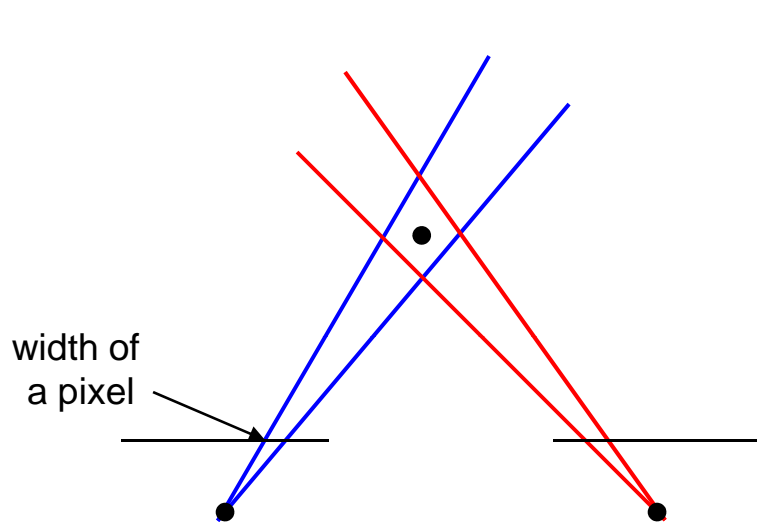
# Stereo: basic idea

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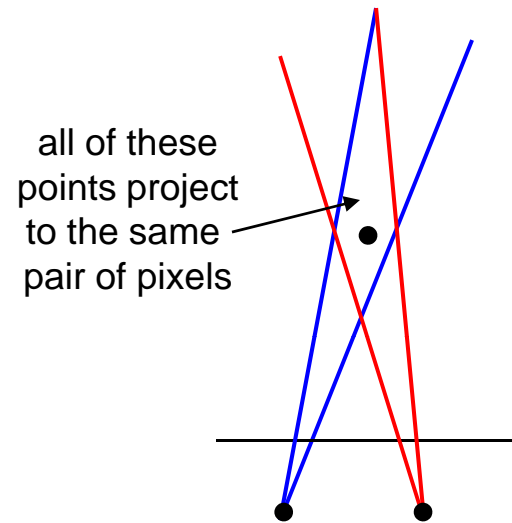


# Choosing the stereo baseline

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**Large Baseline**



**Small Baseline**

What's the optimal baseline?

- Too small: large depth error
- Too large: difficult search problem



# The Effect of Baseline on Depth Estimation

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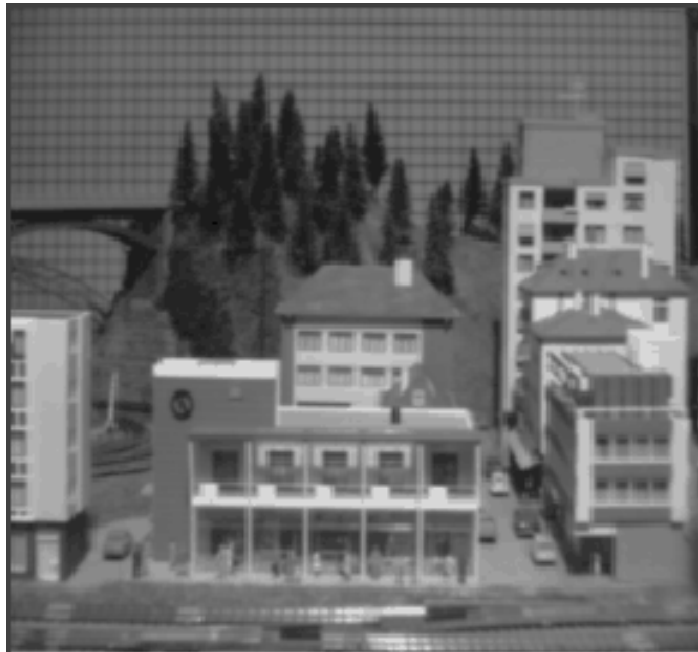
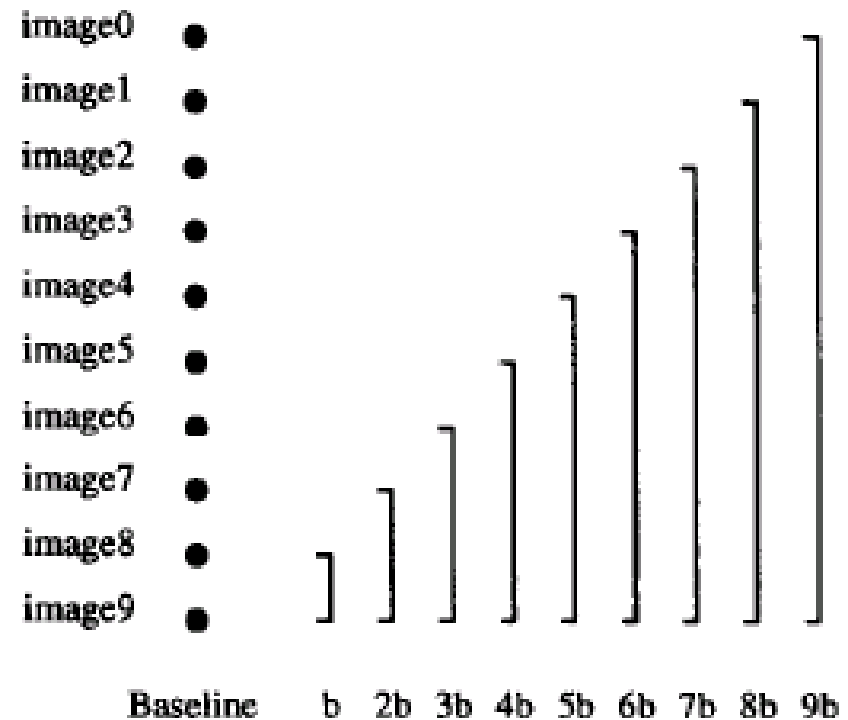
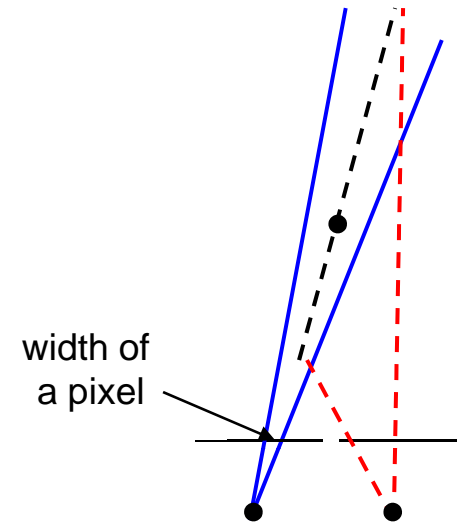
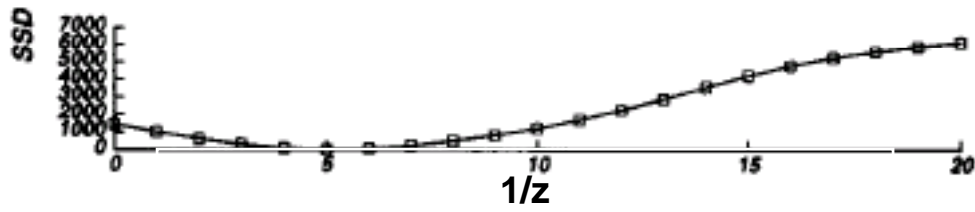
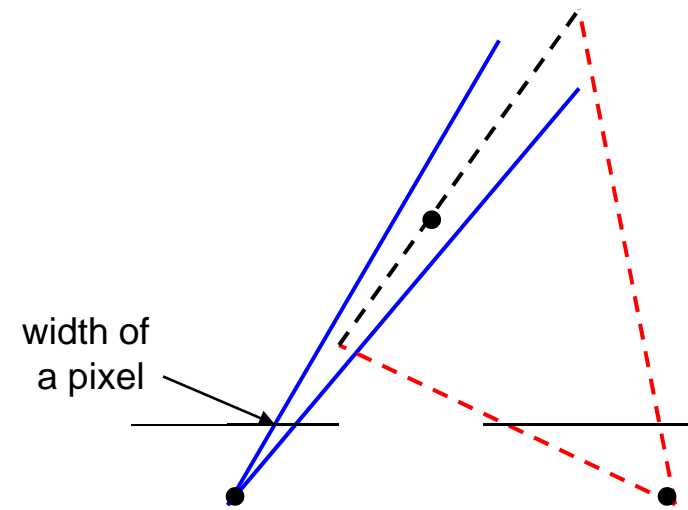
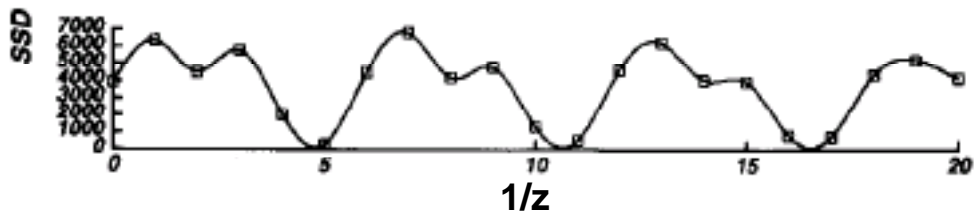


Figure 2: An example scene. The grid pattern in the background has ambiguity of matching.





pixel matching score



[Seitz]

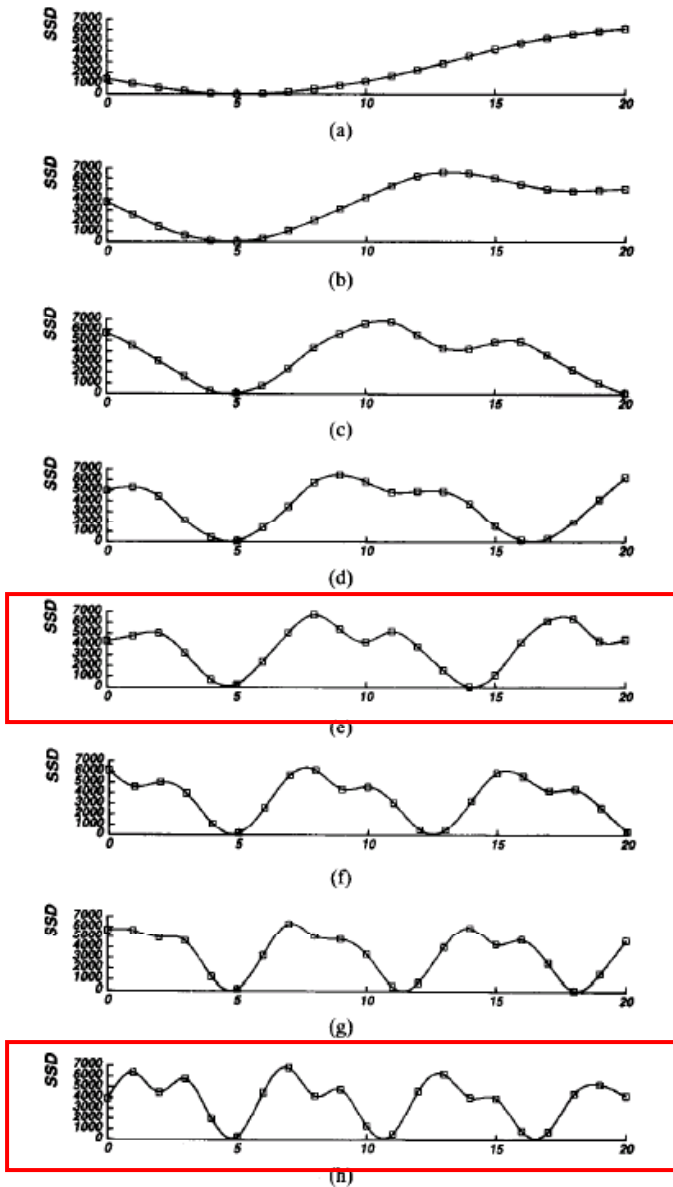


Fig. 5. SSD values versus inverse distance: (a)  $B = b$ ; (b)  $B = 2b$ ; (c)  $B = 3b$ ; (d)  $B = 4b$ ; (e)  $B = 5b$ ; (f)  $B = 6b$ ; (g)  $B = 7b$ ; (h)  $B = 8b$ . The horizontal axis is normalized such that  $8bF = 1$ .

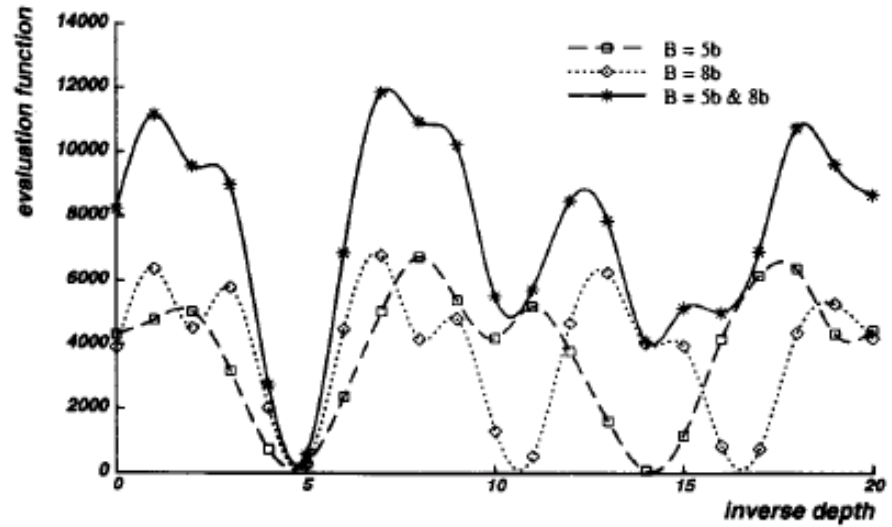


Fig. 6. Combining two stereo pairs with different baselines.

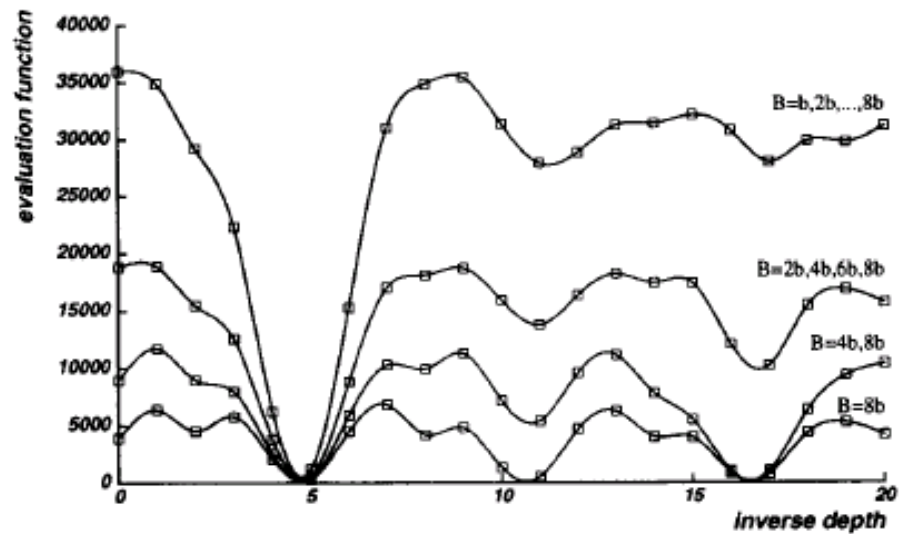


Fig. 7. Combining multiple baseline stereo pairs.

# Multibaseline Stereo

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## Basic Approach

- Choose a reference view
- Use your favorite stereo algorithm BUT
  - > replace two-view SSD with SSSD over all baselines

## Limitations

- Only gives a depth map (not an “object model”)
- Won't work for widely distributed views:



[Seitz]

# Problem: *visibility*

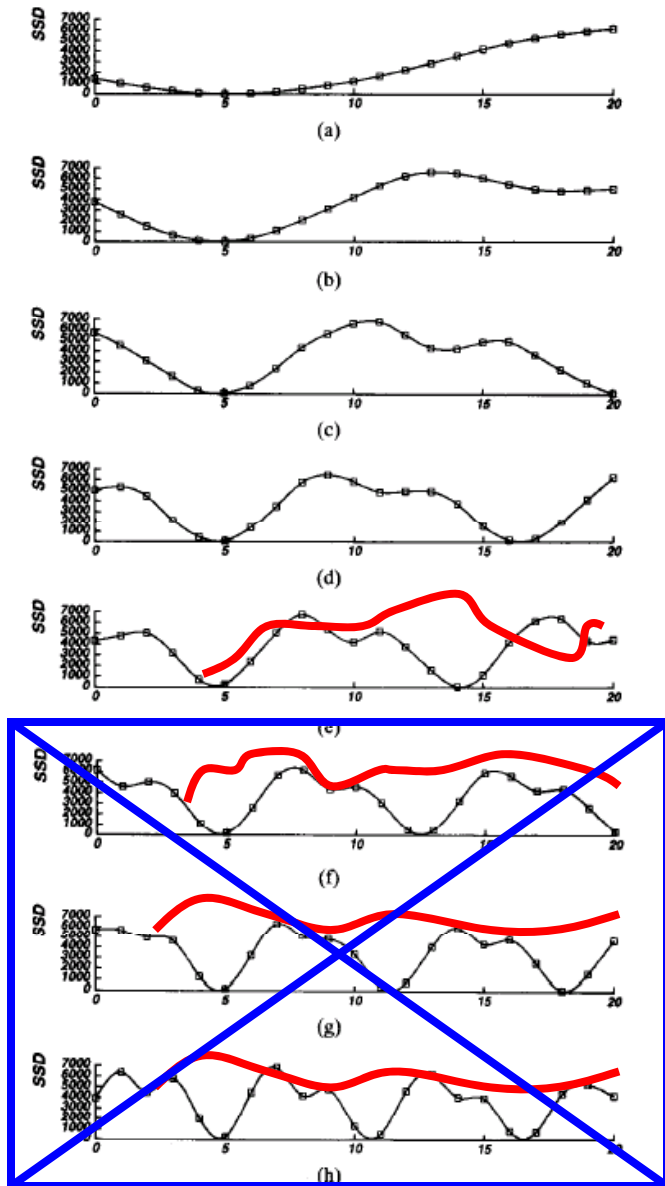


Fig. 5. SSD values versus inverse distance: (a)  $B = b$ ; (b)  $B = 2b$ ; (c)  $B = 3b$ ; (d)  $B = 4b$ ; (e)  $B = 5b$ ; (f)  $B = 6b$ ; (g)  $B = 7b$ ; (h)  $B = 8b$ . The horizontal axis is normalized such that  $8bF = 1$ .

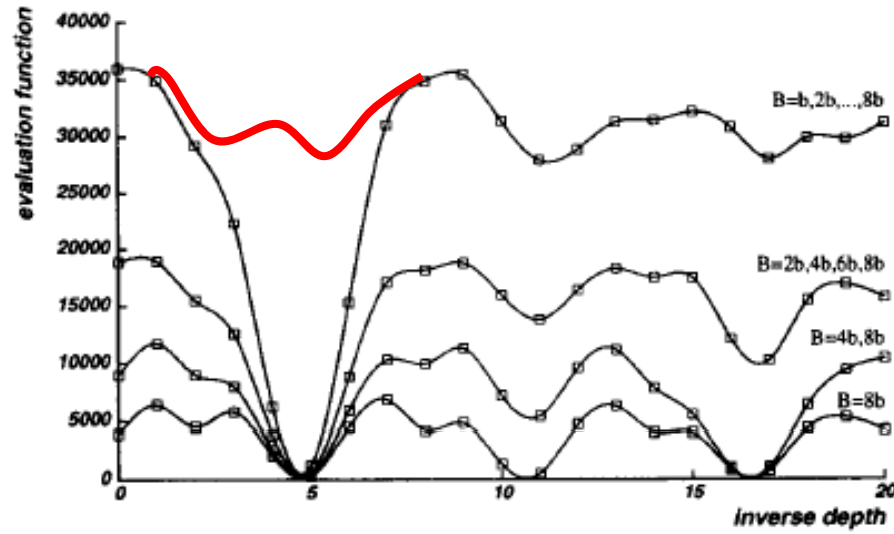


Fig. 7. Combining multiple baseline stereo pairs.

## Some Solutions

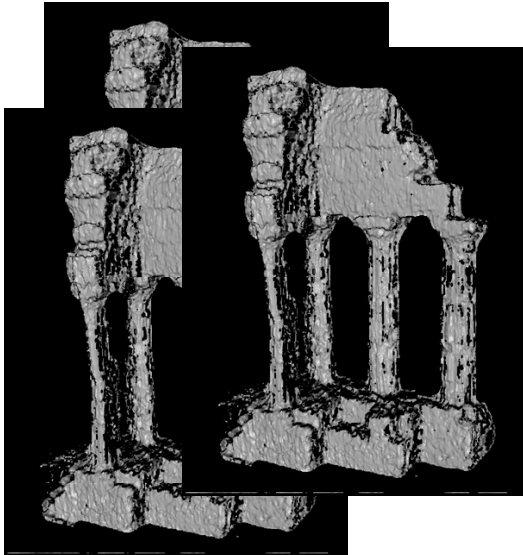
- Match only nearby photos [Narayanan 98]
- Use NCC instead of SSD, Ignore NCC values  $>$  threshold [Hernandez & Schmitt 03]

# Merging Depth Maps

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vrip [Curless and Levoy 1996]

- compute weighted average of depth maps



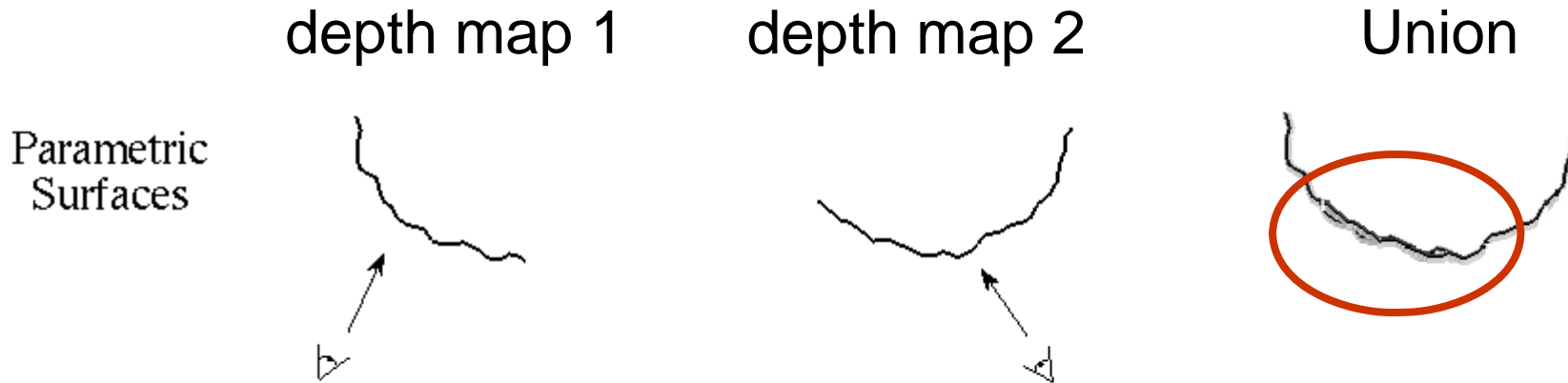
set of depth maps  
(one per view)



merged surface  
mesh

# Merging depth maps

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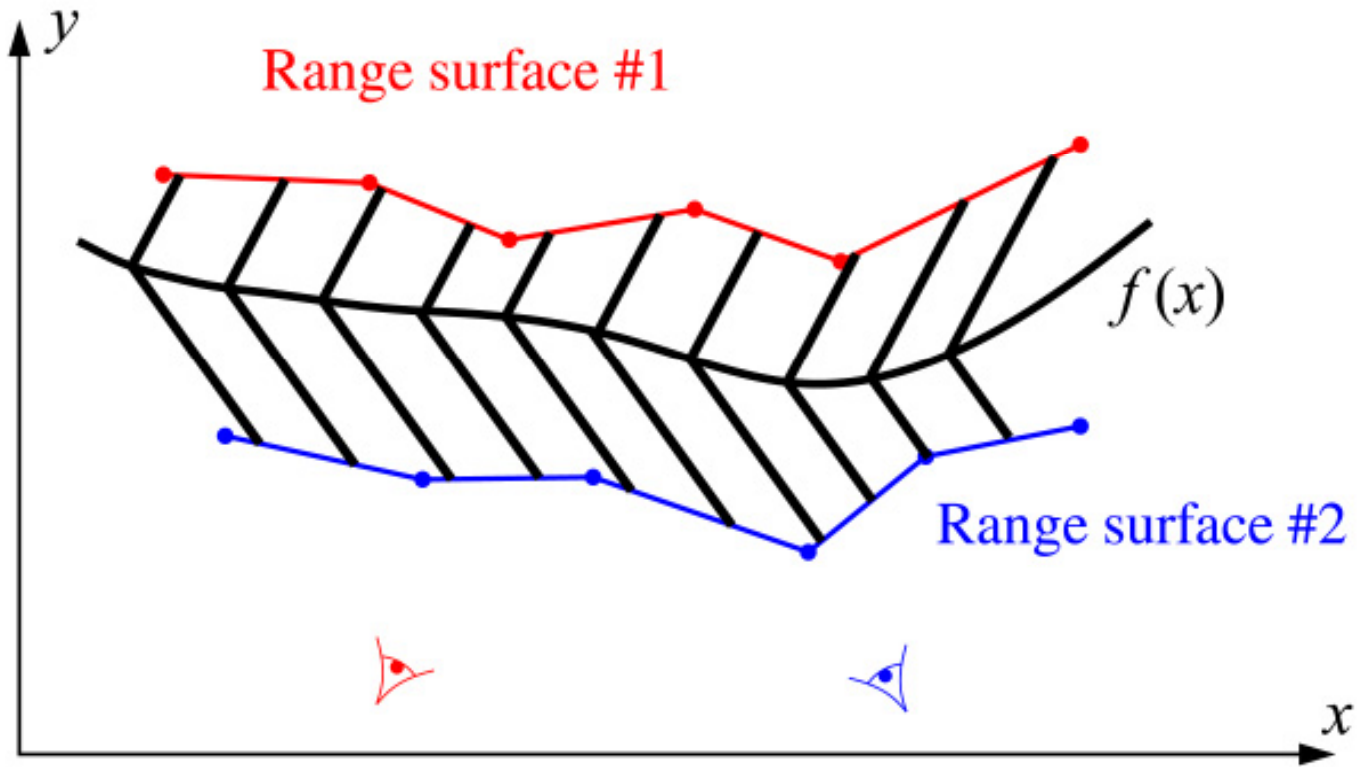
Naïve combination (union) produces artifacts

Better solution: find “average” surface

- Surface that minimizes sum (of squared) distances to the depth maps

# Least squares solution

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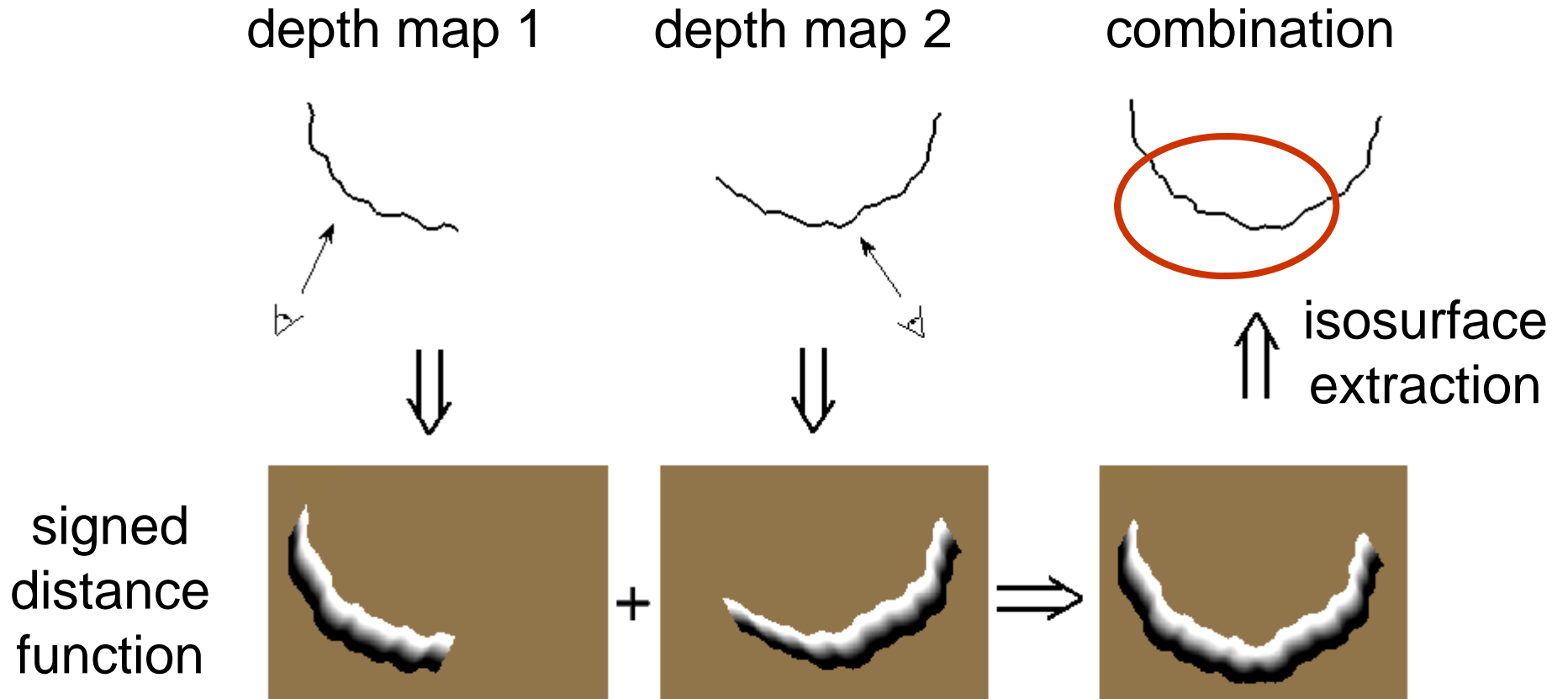


$$E(f) = \sum_{i=1}^N \int d_i^2(x, f) dx$$



# VRIP [Curless & Levoy 1996]

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# Merging Depth Maps: Temple Model

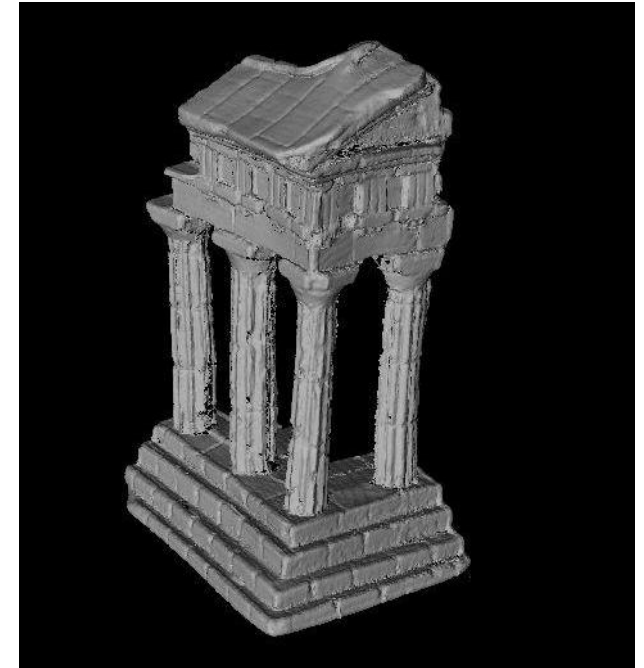
---



input image



317 images  
(hemisphere)



ground truth model

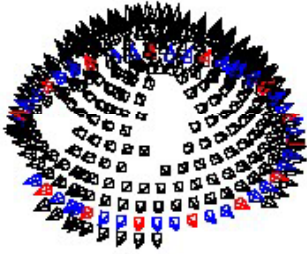

[Goesele, Curless, Seitz, 2006](#)

Multi-View Stereo Evaluation - Microsoft Internet Explorer

File Edit View Favorites Tools Help

Back Forward Stop Refresh Home Search Favorites

Address <http://vision.middlebury.edu/mview/> Go Links



## Multi-View Stereo Evaluation

[Steve Seitz](#), University of Washington  
[Brian Curless](#), University of Washington  
[James Diebel](#), Stanford University  
[Daniel Scharstein](#), Middlebury College  
[Rick Szeliski](#), Microsoft Research

This website accompanies our paper "[A Comparison and Evaluation of Multi-View Stereo Reconstruction Algorithms](#)", to appear in CVPR 2006.

The goal of this project is to provide high quality datasets with which to benchmark and evaluate the performance of multi-view stereo reconstruction algorithms. Each dataset is registered with a ground-truth 3D model acquired via a laser scanning process, to be used as a baseline for measuring accuracy and completeness (the ground truth is not distributed).

- [Data sets](#)
- [How to submit your own results](#)
- [Evaluation results](#)

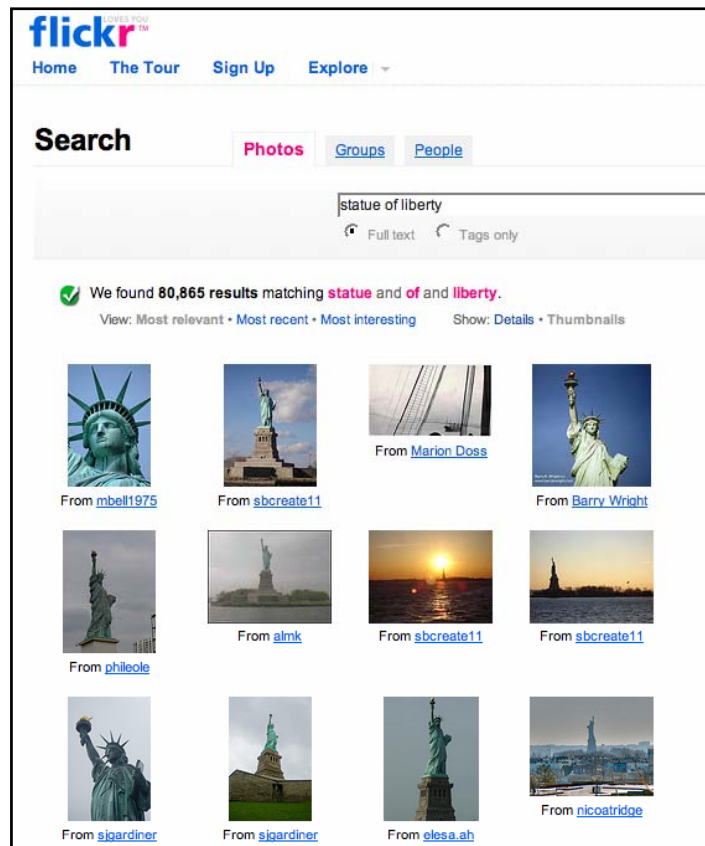
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Internet

# Multi-view stereo from Internet Collections

[Goesele, Snavely, Curless, Hoppe, Seitz, ICCV 2007]



[Seitz]

# Challenges

- appearance variation



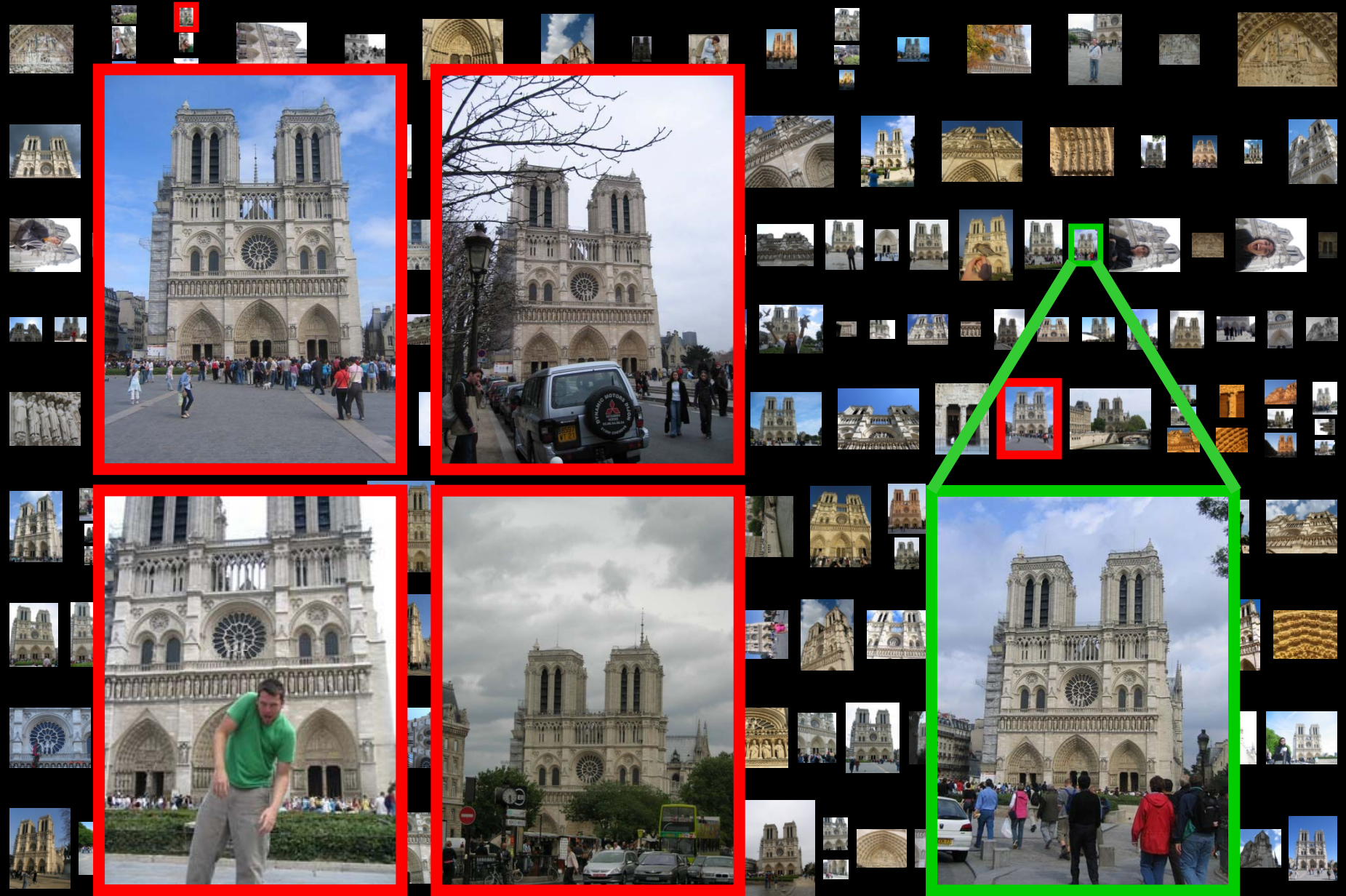
- resolution



- massive collections

82,754 results for photos matching **notre** and **dame** and **paris**.

# Large Image Collections

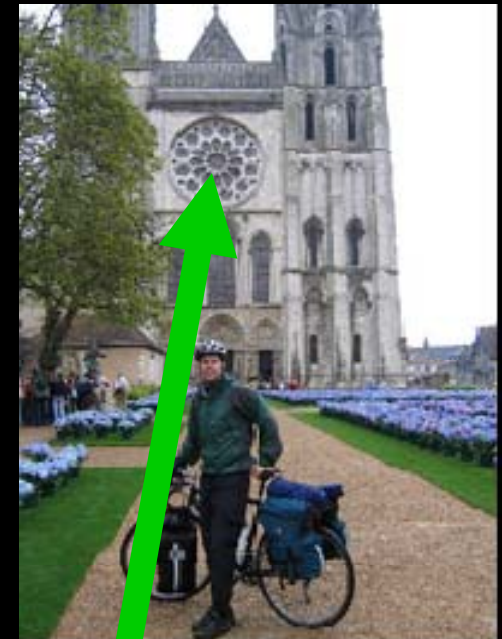


206 Flickr images taken by 92 photographers

[Seitz]



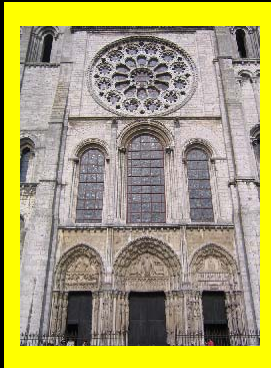
4 best neighboring views



reference view

## Local view selection

- Automatically select neighboring views for each **point** in the image
- Desiderata: good matches AND good baselines



4 best neighboring views

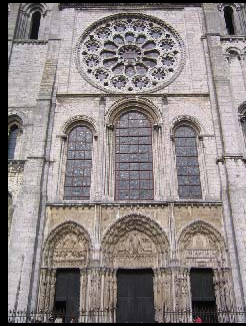


reference view

## Local view selection

- Automatically select neighboring views for each **point** in the image
- Desiderata: good matches AND good baselines





4 best neighboring views

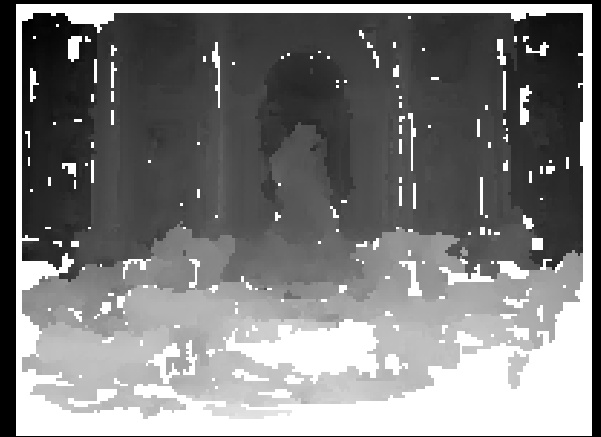
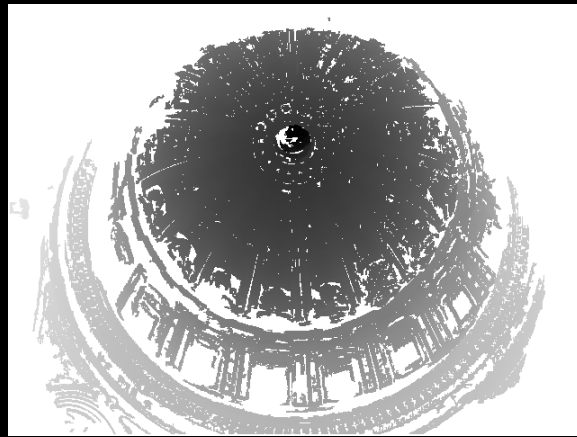


reference view

## Local view selection

- Automatically select neighboring views for each **point** in the image
- Desiderata: good matches AND good baselines

# Results



Mt. Rushmore  
160 images  
60 photographers

St. Peter  
151 images  
50 photographers

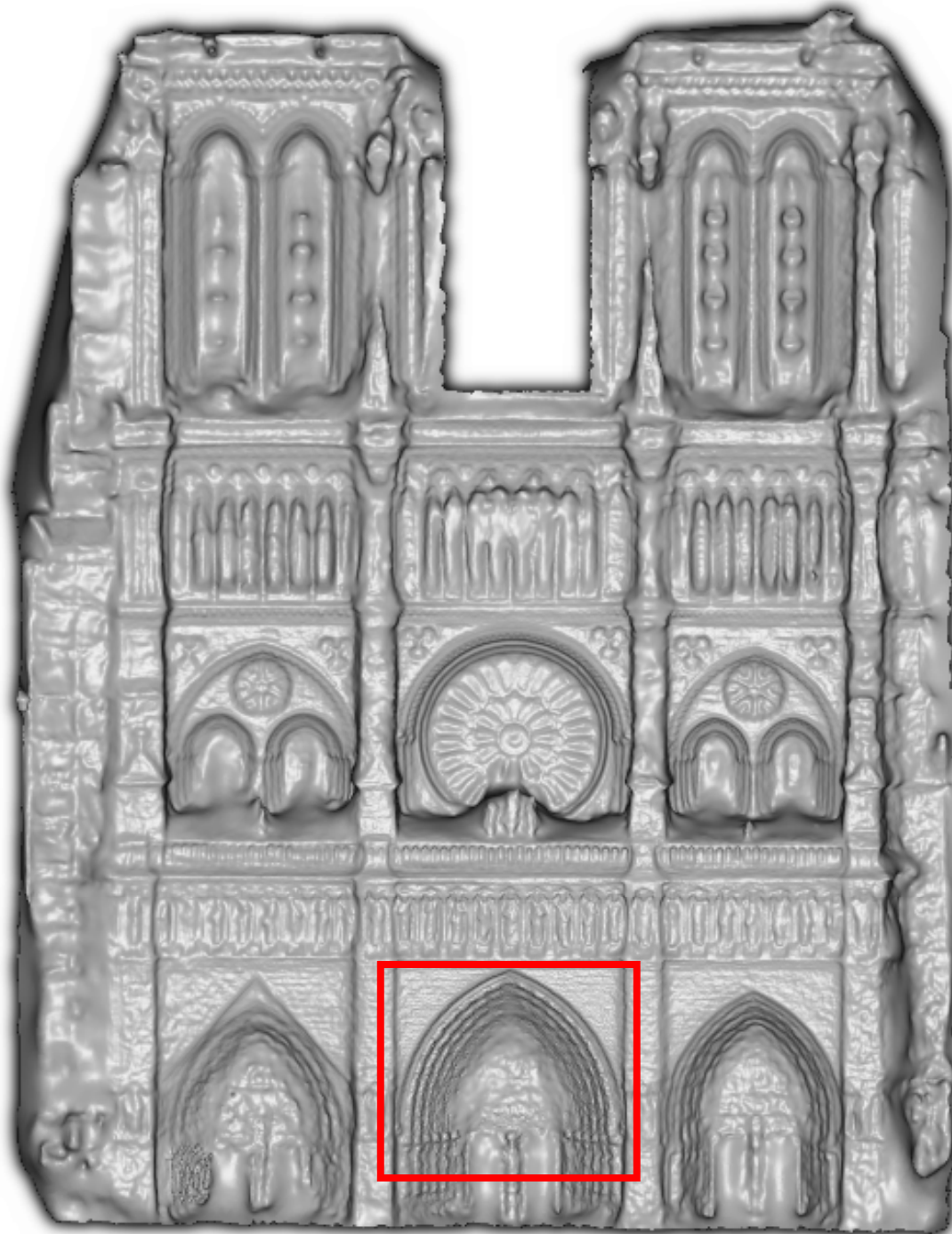
Trevi Fountain  
106 images  
51 photographers

[Seitz]

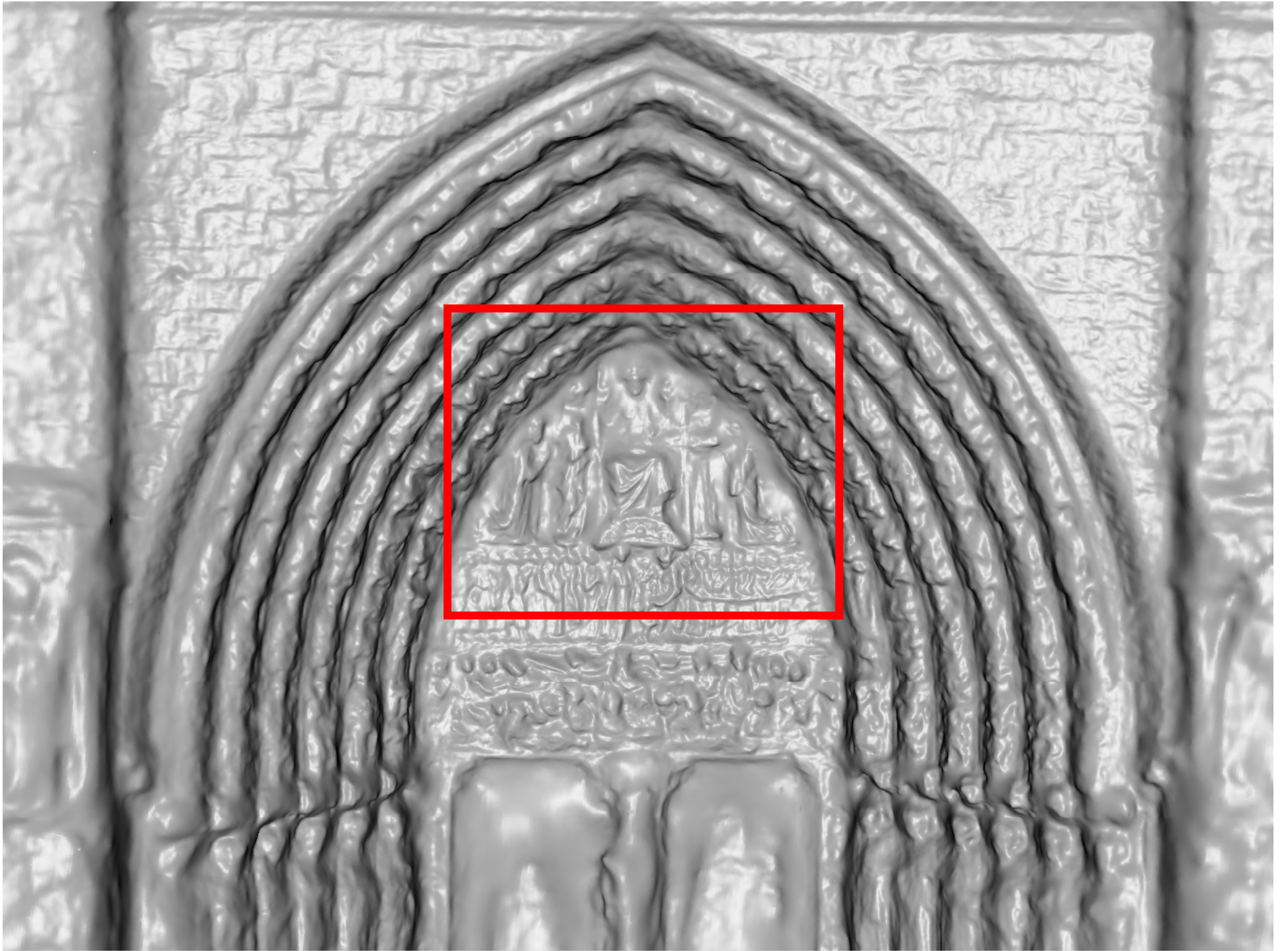
Notre Dame de Paris

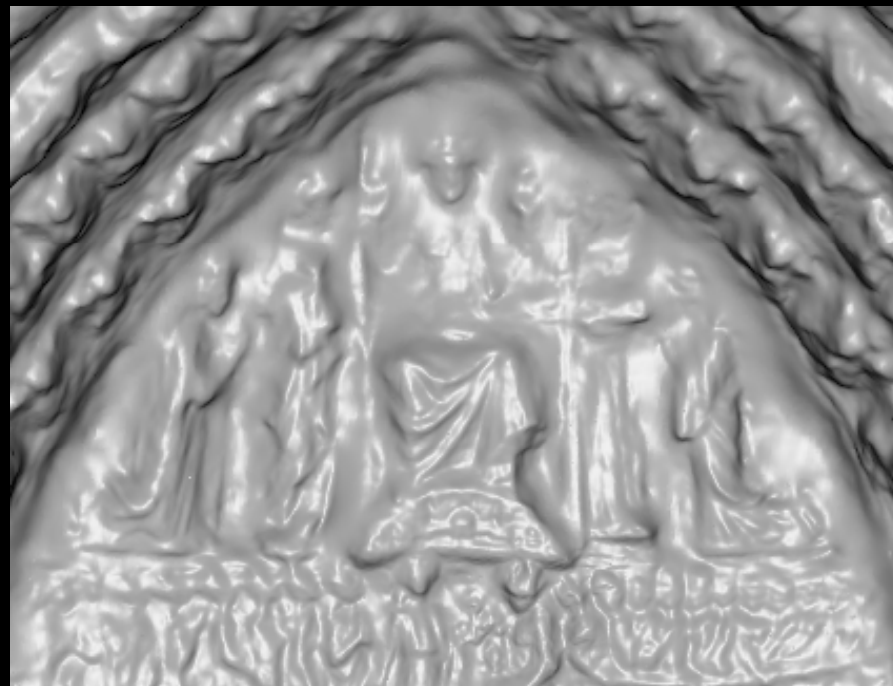
653 images

313 photographers



[Seitz]





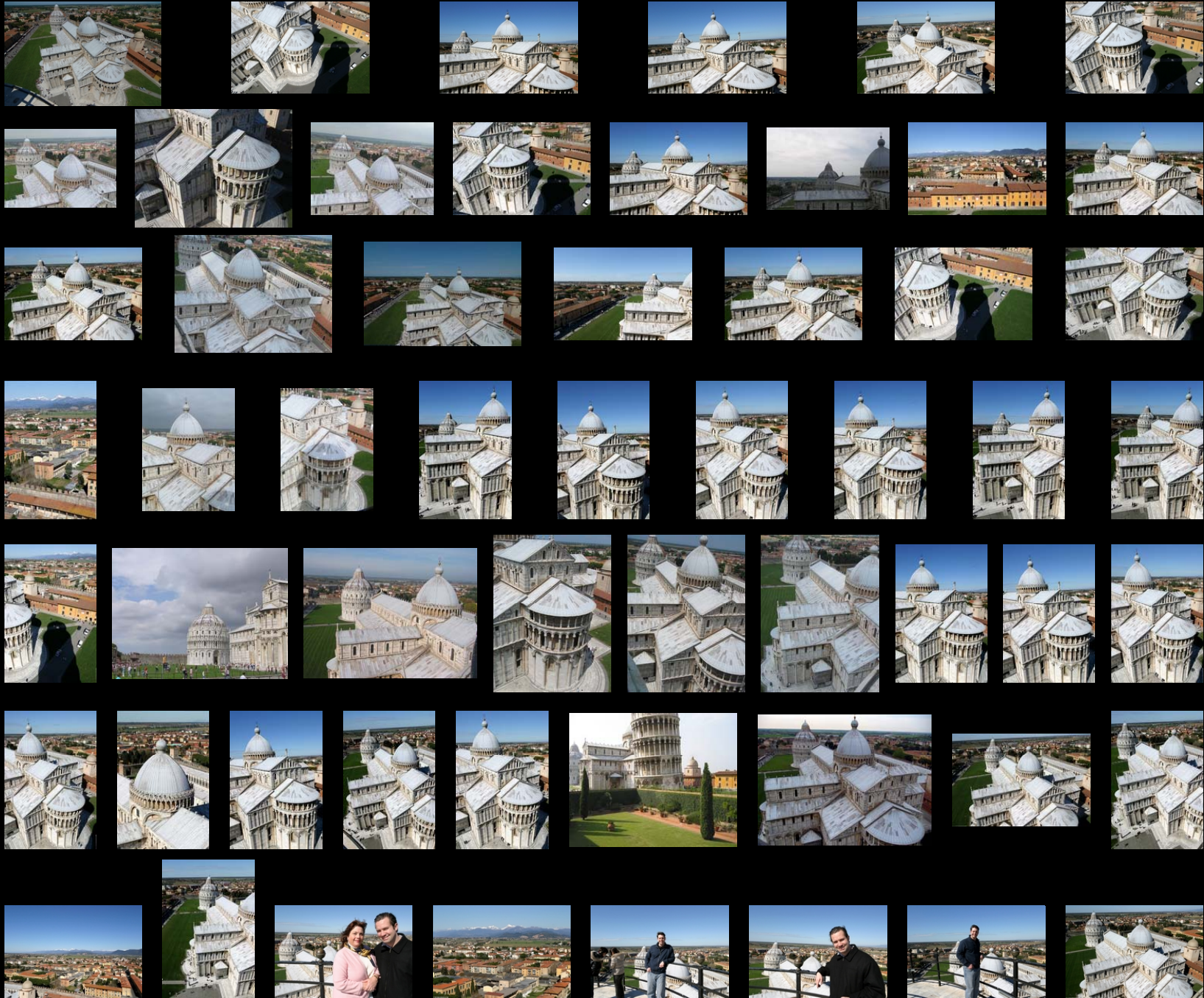


129 *Flickr* images taken by 98 photographers

[Seitz]



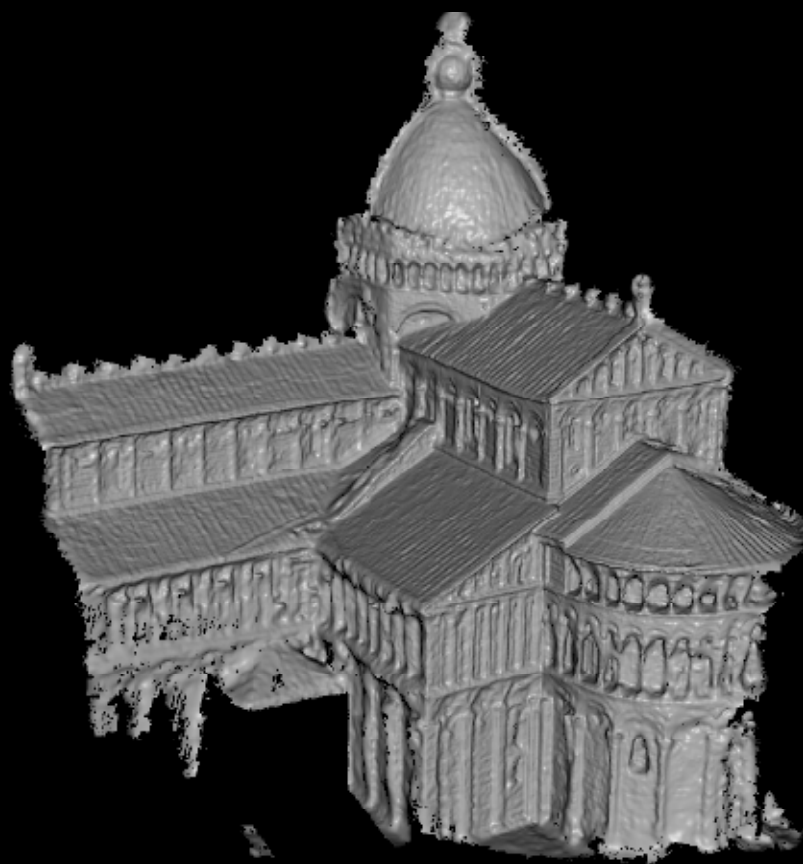
merged model of Venus de Milo



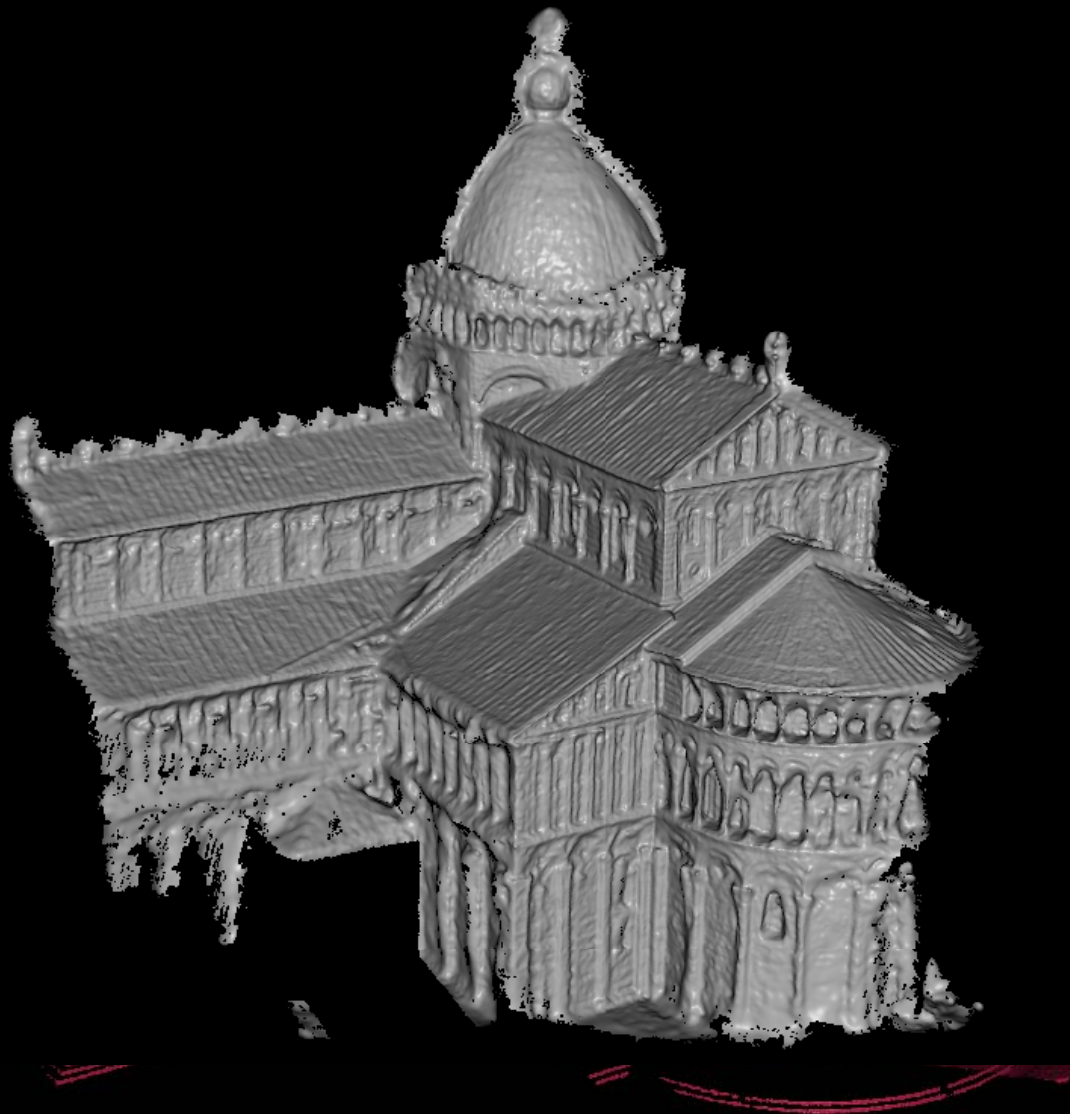
56 Flickr images taken by 8 photographers

[Seitz]





merged model of Pisa Cathedral



Accuracy compared to laser scanned model:  
90% of points within 0.25% of ground truth

# Problem: *visibility*

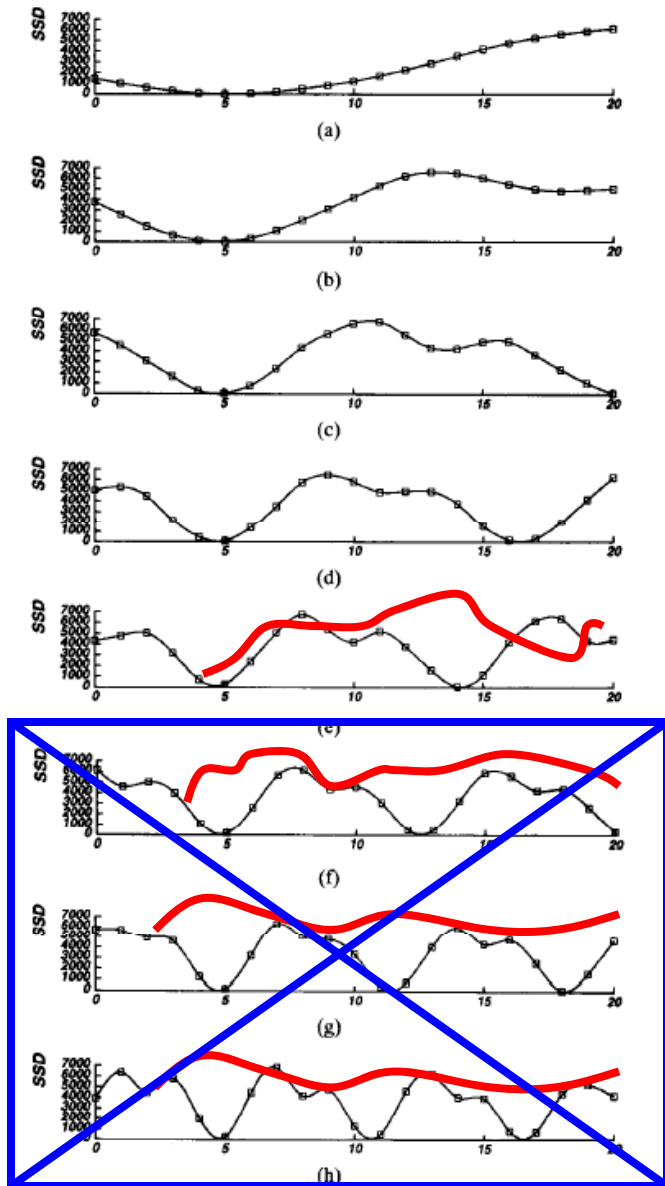


Fig. 5. SSD values versus inverse distance: (a)  $B = b$ ; (b)  $B = 2b$ ; (c)  $B = 3b$ ; (d)  $B = 4b$ ; (e)  $B = 5b$ ; (f)  $B = 6b$ ; (g)  $B = 7b$ ; (h)  $B = 8b$ . The horizontal axis is normalized such that  $8bF = 1$ .

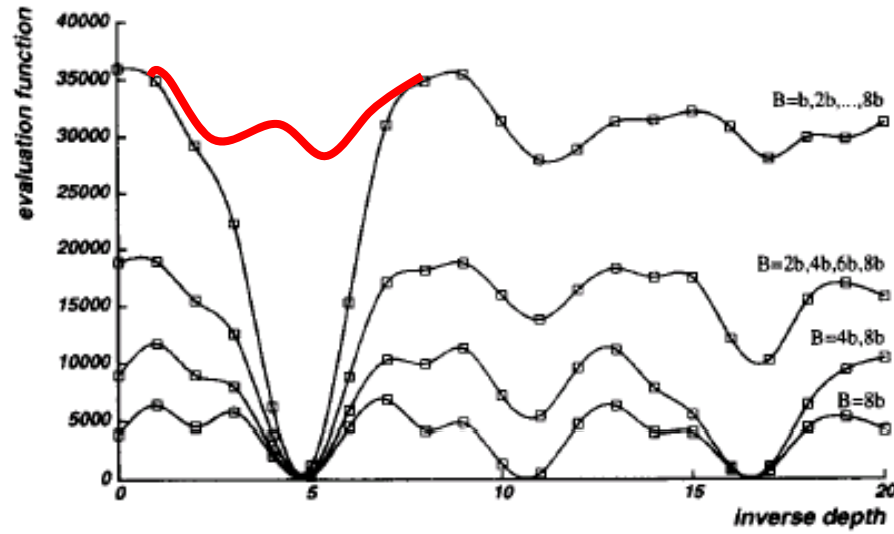


Fig. 7. Combining multiple baseline stereo pairs.

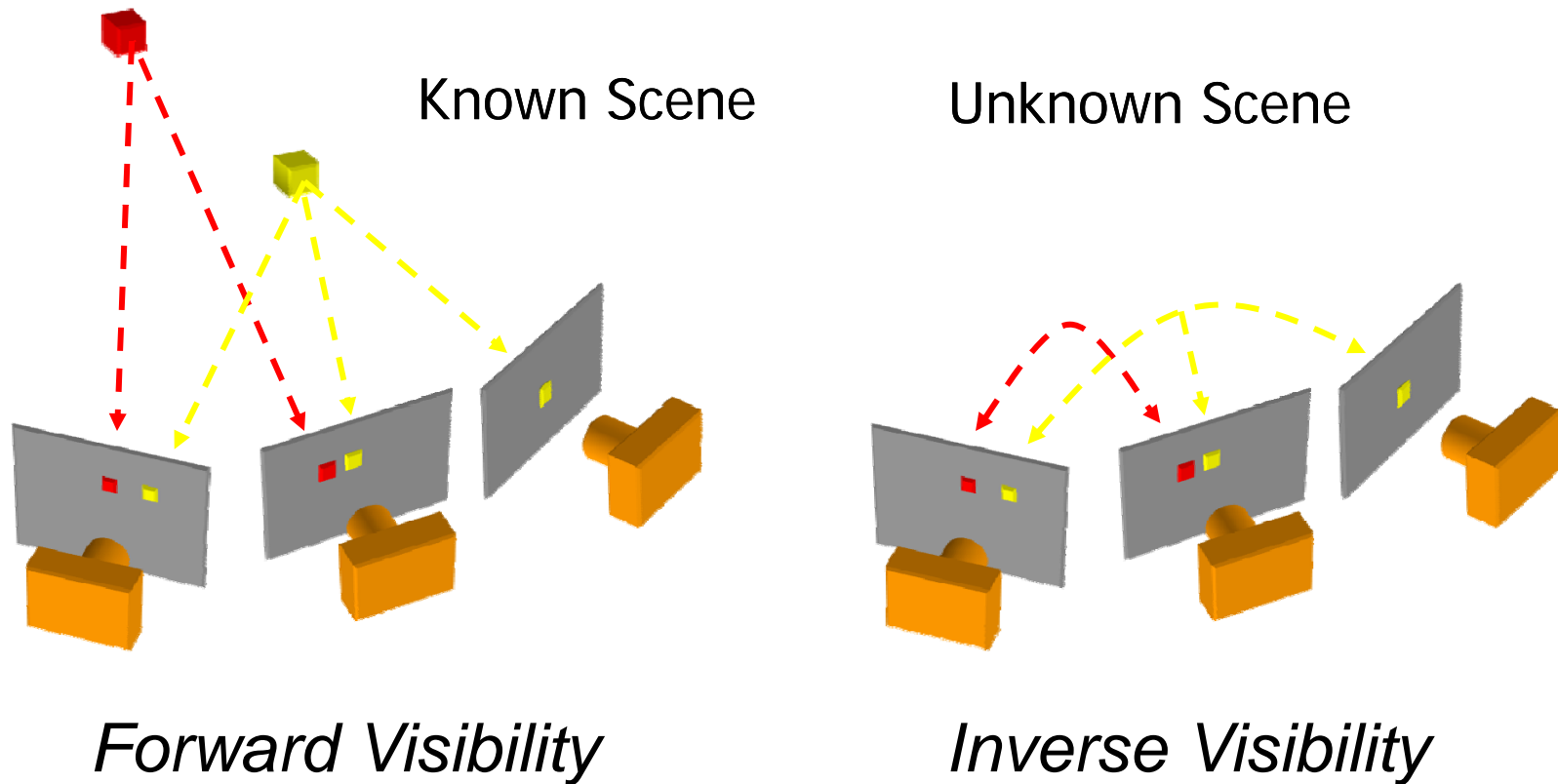
## Some Solutions

- Match only nearby photos [Narayanan 98]
- Use NCC instead of SSD, Ignore NCC values  $>$  threshold [Hernandez & Schmitt 03]

# The visibility problem

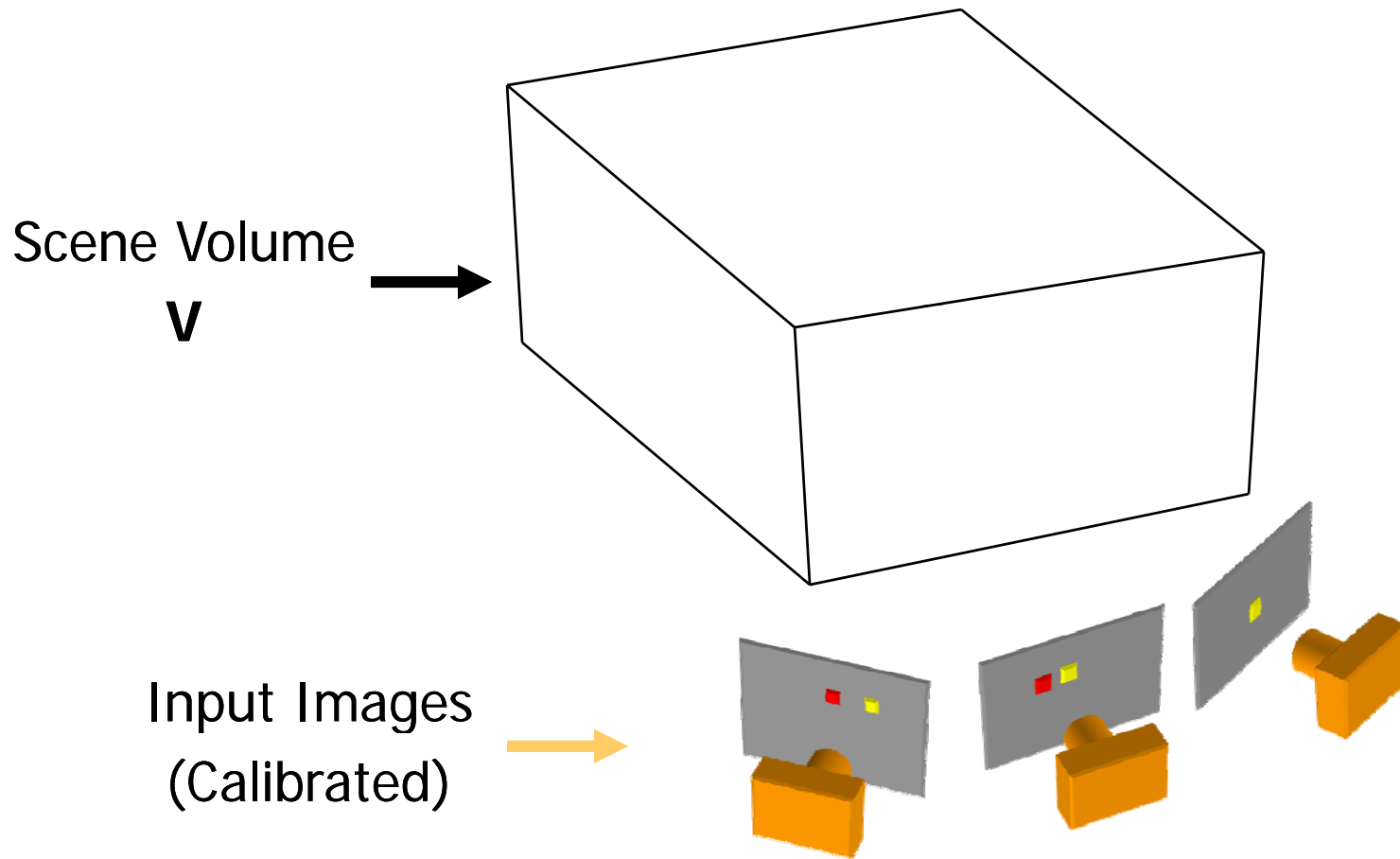
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Which points are visible in which images?



# Volumetric stereo

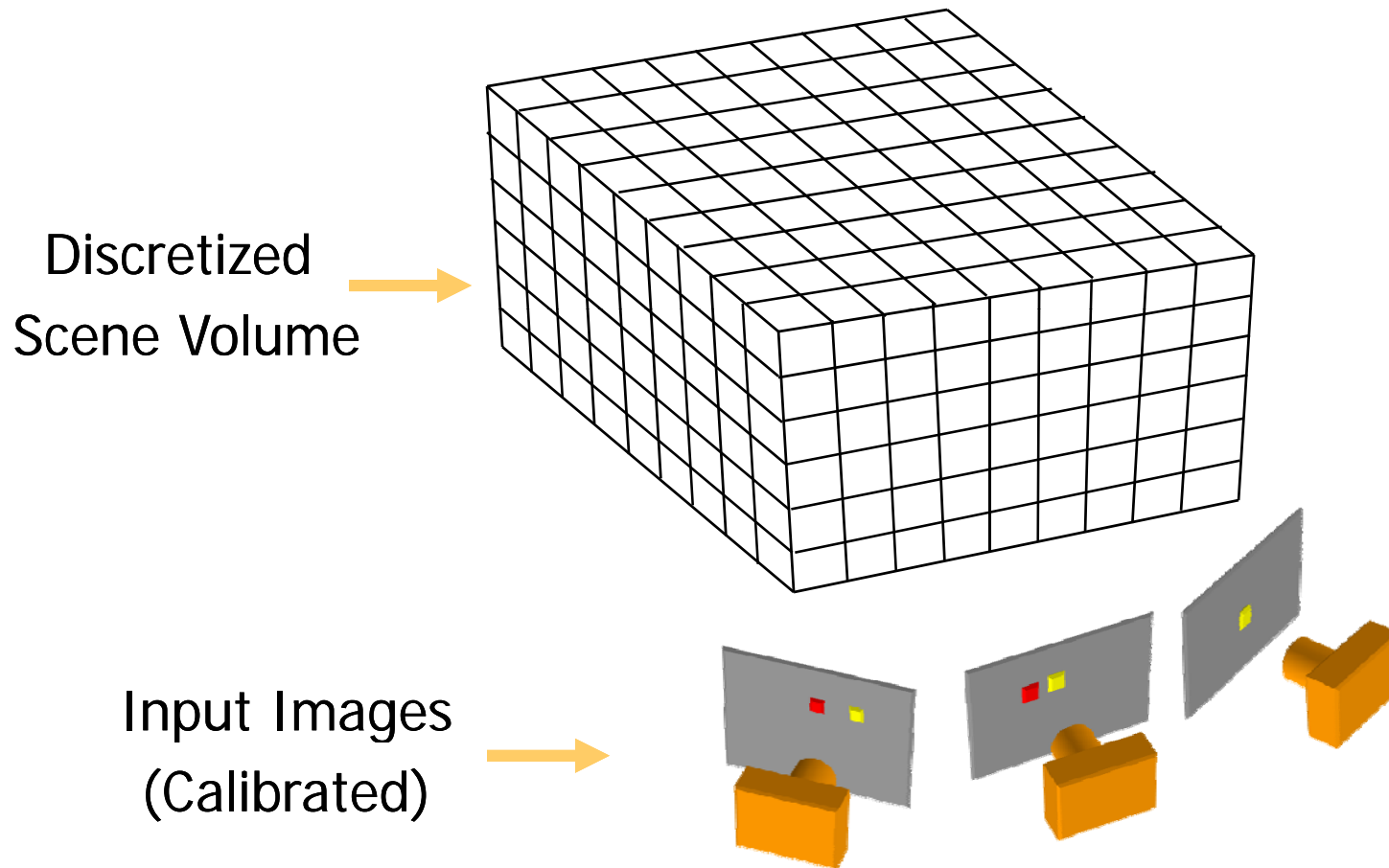
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**Goal: Determine occupancy, “color” of points in  $V$**

# Discrete formulation: Voxel Coloring

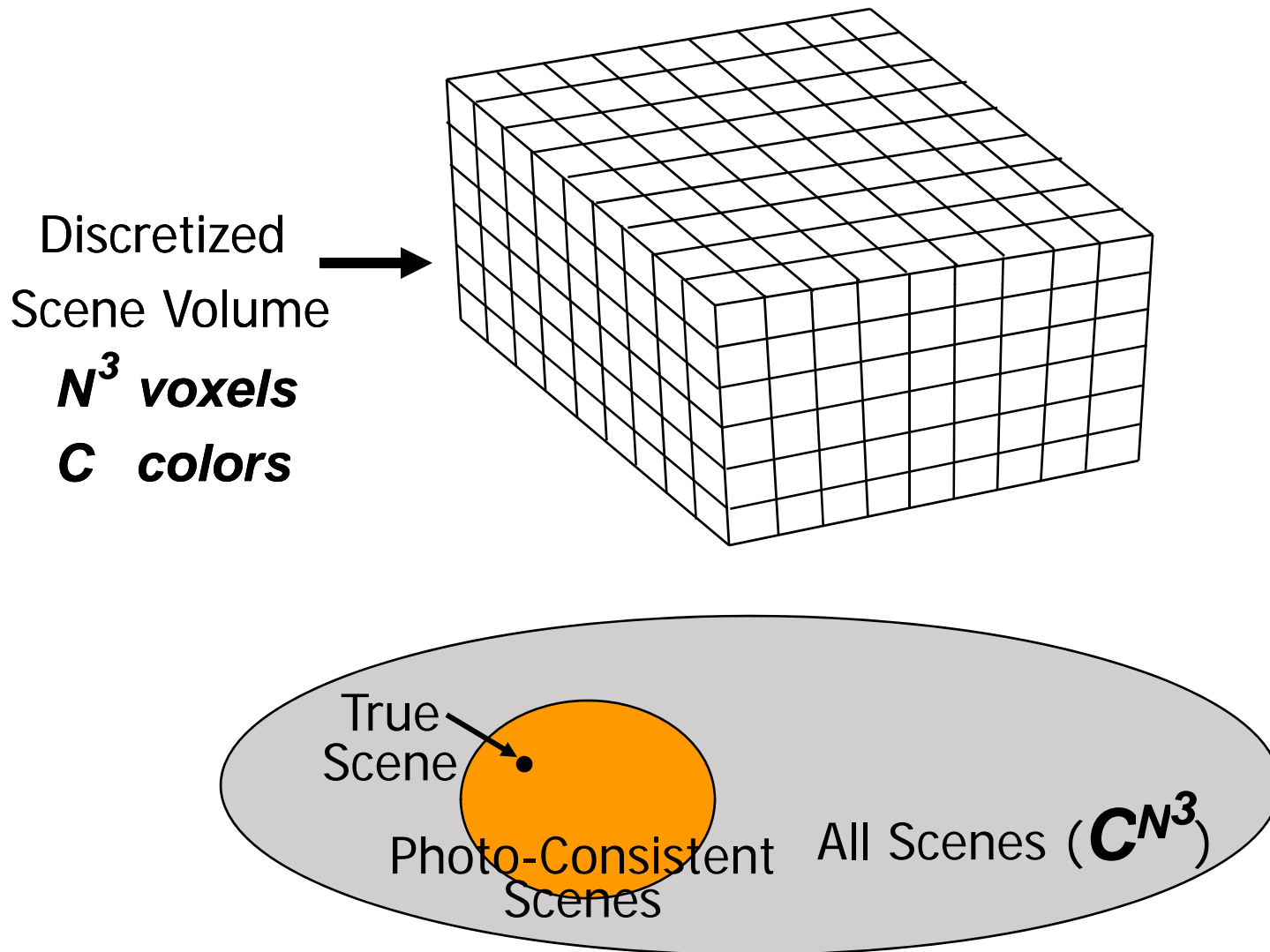
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**Goal:** Assign RGBA values to voxels in  $V$   
*photo-consistent* with images

# Complexity and computability

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# Issues

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## Theoretical Questions

- Identify class of *all* photo-consistent scenes

## Practical Questions

- How do we compute photo-consistent models?



# Voxel coloring solutions

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## 1. $C=2$ (shape from silhouettes)

- Volume intersection [Baumgart 1974]
  - > For more info: *Rapid octree construction from image sequences*. R. Szeliski, CVGIP: Image Understanding, 58(1):23-32, July 1993. (this paper is apparently not available online) or
  - > W. Matusik, C. Buehler, R. Raskar, L. McMillan, and S. J. Gortler, *Image-Based Visual Hulls*, SIGGRAPH 2000 ( [pdf 1.6 MB](#) )

## 2. $C$ unconstrained, viewpoint constraints

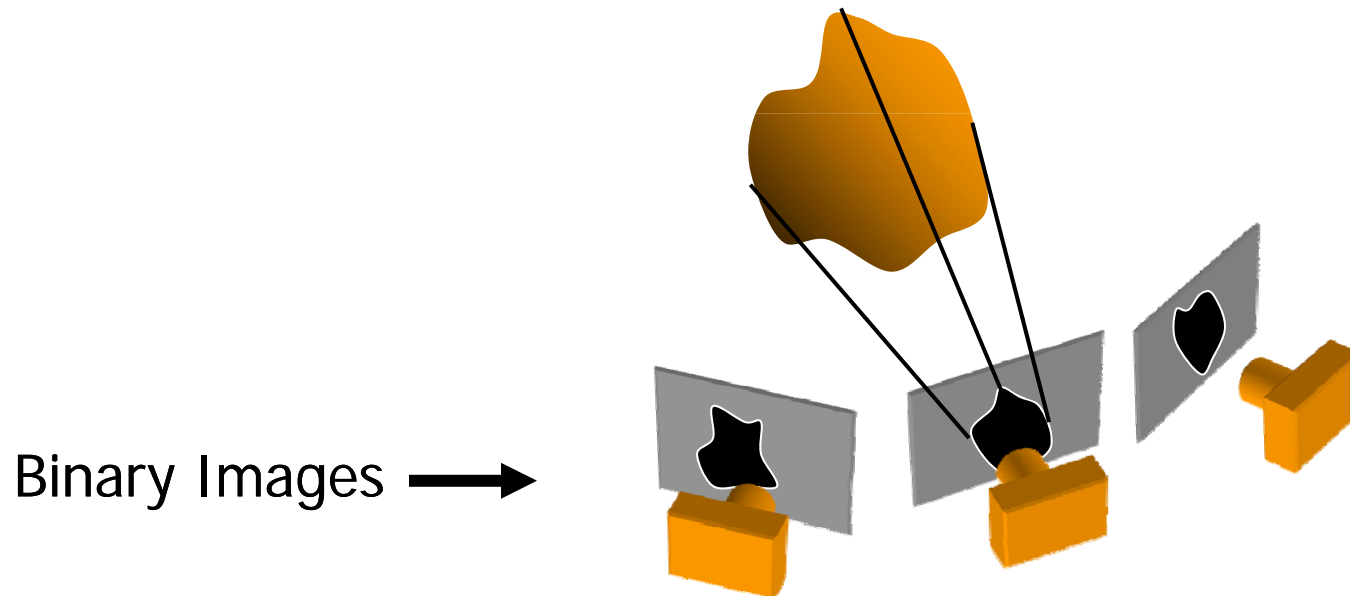
- Voxel coloring algorithm [Seitz & Dyer 97]

## 3. General Case

- Space carving [Kutulakos & Seitz 98]

# Reconstruction from Silhouettes ( $C = 2$ )

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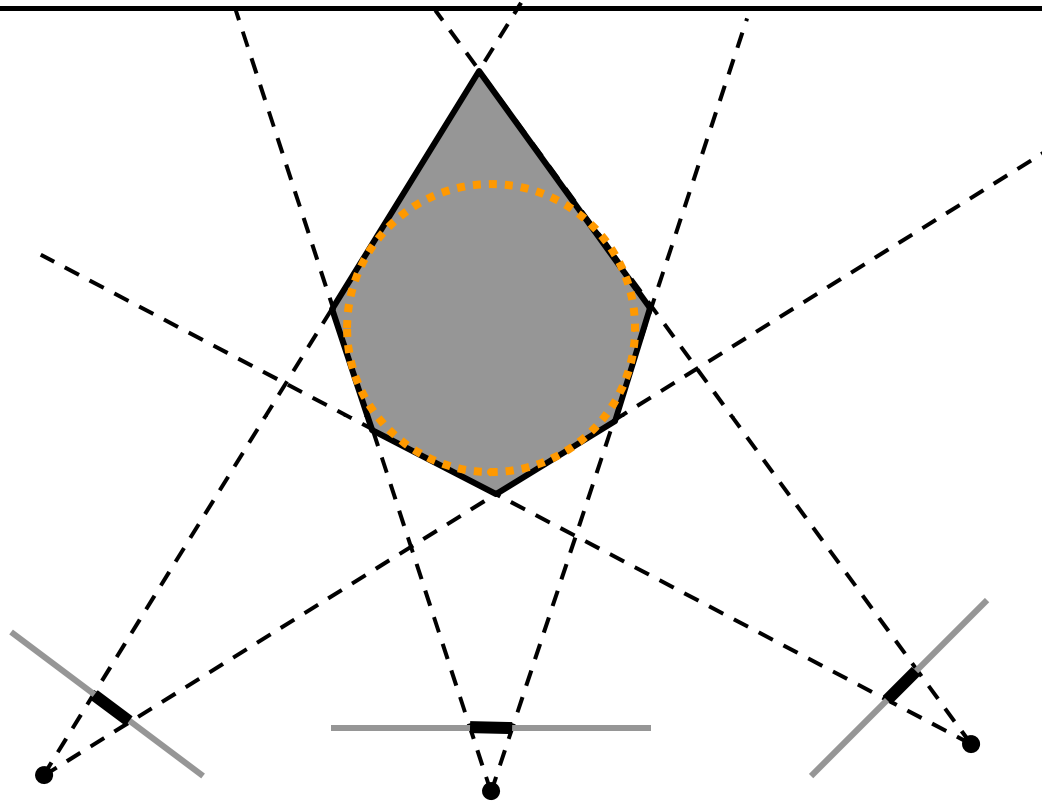


## Approach:

- *Backproject* each silhouette
- Intersect backprojected volumes

# Volume intersection

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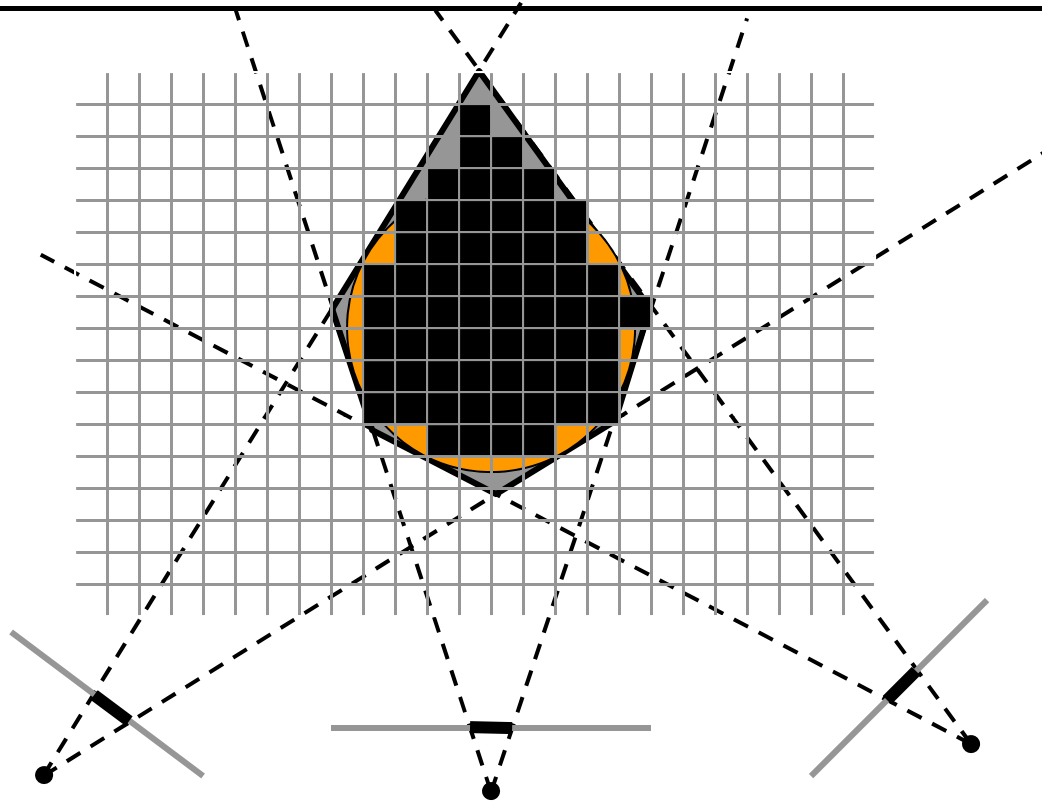


## Reconstruction Contains the True Scene

- But is generally not the same
- In the limit (all views) get *visual hull*
  - > Complement of all lines that don't intersect S

# Voxel algorithm for volume intersection

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Color voxel black if on silhouette in every image

- $O(N^3)$ , for  $M$  images,  $N^3$  voxels  $O(MN^3)$
- Don't have to search  $2^{N^3}$  possible scenes!

# Properties of Volume Intersection

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## Pros

- Easy to implement, fast
- Accelerated via octrees [Szeliski 1993] or interval techniques [Matusik 2000]

## Cons

- No concavities
- Reconstruction is not photo-consistent
- Requires identification of silhouettes

# Voxel Coloring Solutions

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## 1. $C=2$ (silhouettes)

- Volume intersection [Baumgart 1974]

## 2. $C$ unconstrained, viewpoint constraints

- Voxel coloring algorithm [Seitz & Dyer 97]
  - > For more info: <http://www.cs.washington.edu/homes/seitz/papers/ijcv99.pdf>

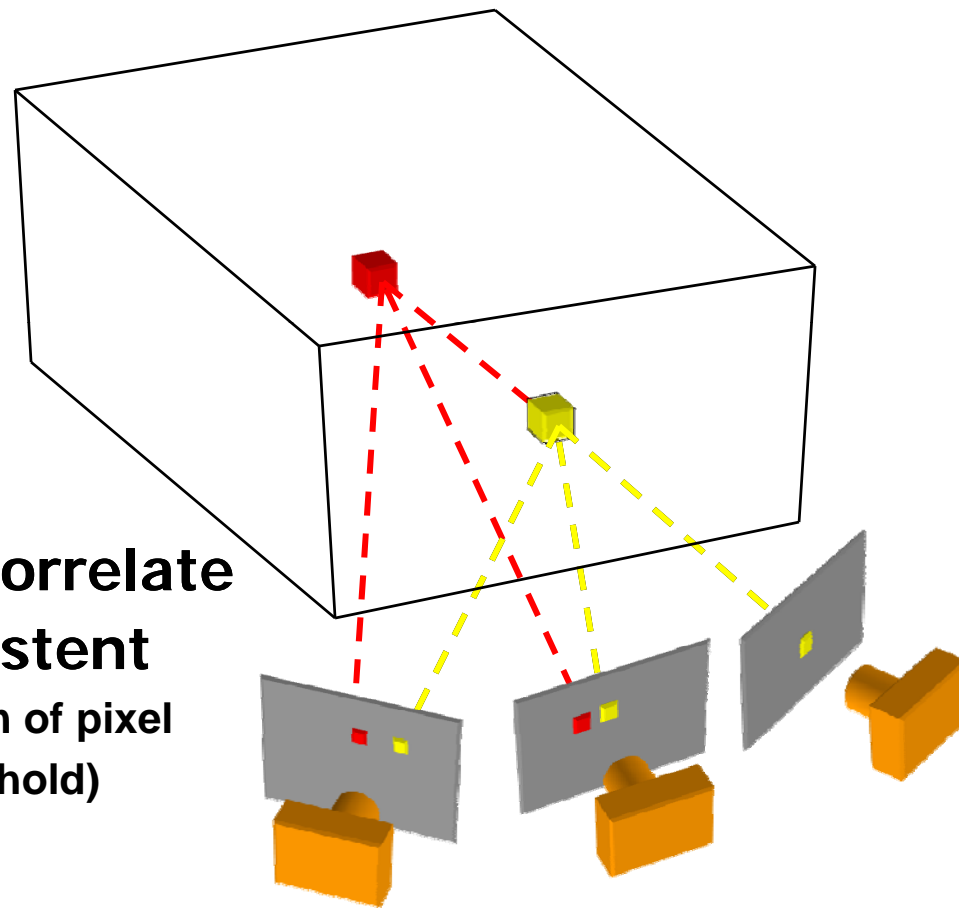
## 3. General Case

- Space carving [Kutulakos & Seitz 98]

# Voxel Coloring Approach

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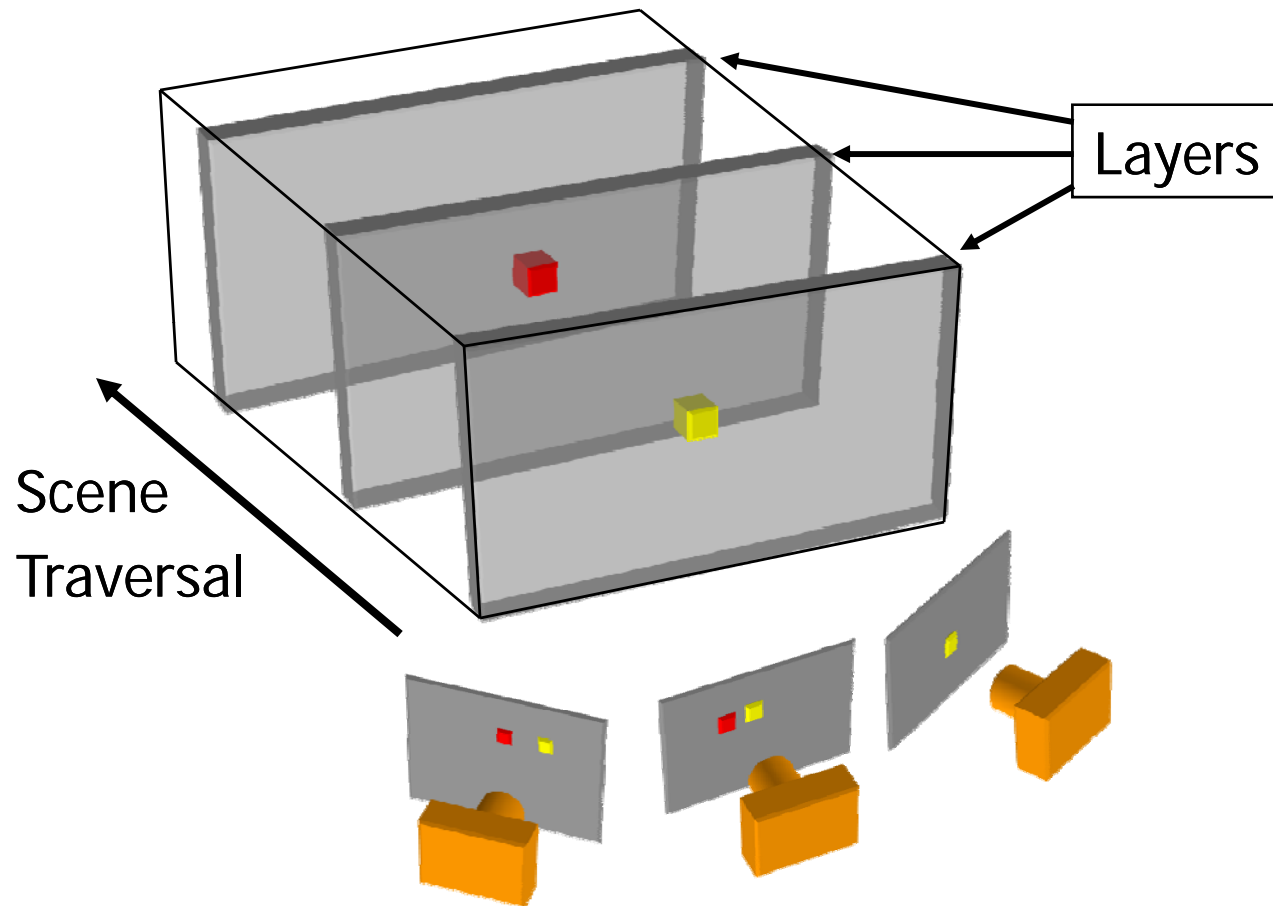
1. Choose voxel
2. Project and correlate
3. Color if consistent  
(standard deviation of pixel colors below threshold)



**Visibility Problem:** in which images is each voxel visible?

# Depth Ordering: visit occluders first!

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**Condition:** depth order is the *same for all input views*



# Calibrated Image Acquisition

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*Calibrated Turntable*



**Selected Dinosaur Images**



**Selected Flower Images**

# Voxel Coloring Results ([Video](#))

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## Dinosaur Reconstruction

72 K voxels colored  
7.6 M voxels tested  
7 min. to compute  
on a 250MHz SGI



## Flower Reconstruction

70 K voxels colored  
7.6 M voxels tested  
7 min. to compute  
on a 250MHz SGI

# Improvements

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## Unconstrained camera viewpoints

- Space carving [[Kutulakos & Seitz 98](#)]

## Evolving a surface

- Level sets [[Faugeras & Keriven 98](#)]
- More recent [work](#) by Pons et al.

## Global optimization

- Graph cut approaches
  - > [[Kolmogoriv & Zabih, ECCV 2002](#)]
  - > [[Vogiatzis et al., PAMI 2007](#)]

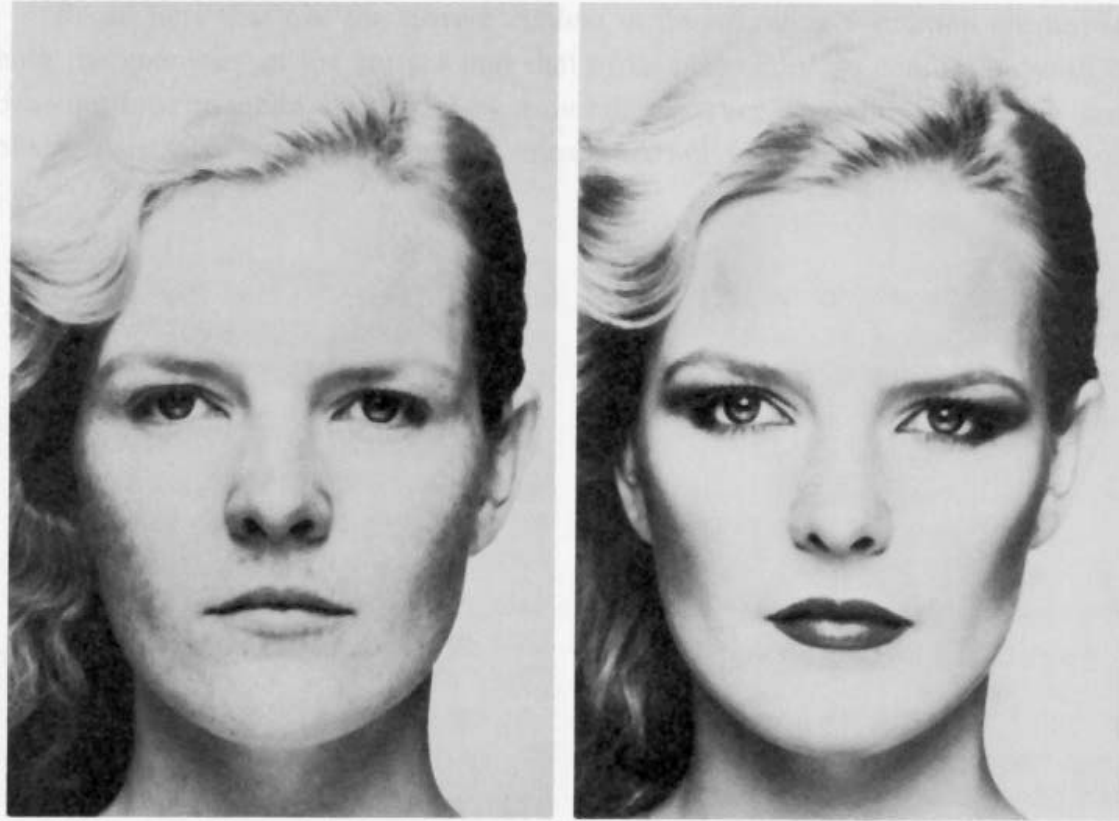
## Modeling shiny (and other reflective) surfaces

- e.g., [Zickler et al., \*Helmholtz Stereopsis\*](#)

See today's reading for an overview of the state of the art

# Photometric Stereo

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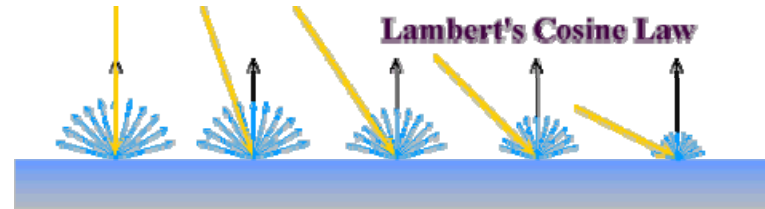
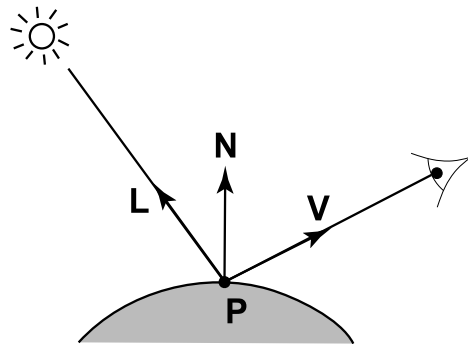
Merle Norman Cosmetics, Los Angeles

## Readings

- R. Woodham, *Photometric Method for Determining Surface Orientation from Multiple Images*. *Optical Engineering* 19(1)139-144 (1980). ([PDF](#))

# Diffuse reflection

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$$R_e = k_d \mathbf{N} \cdot \mathbf{L} R_i$$

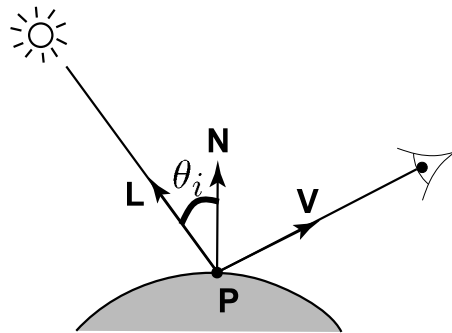
image intensity of  $\mathbf{P}$   $\longrightarrow I = k_d \mathbf{N} \cdot \mathbf{L}$

## Simplifying assumptions

- $I = R_e$ : camera response function  $f$  is the identity function:
  - can always achieve this in practice by solving for  $f$  and applying  $f^{-1}$  to each pixel in the image
- $R_i = 1$ : light source intensity is 1
  - can achieve this by dividing each pixel in the image by  $R_i$

# Shape from shading

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Suppose  $k_d = 1$

$$\begin{aligned} I &= k_d \mathbf{N} \cdot \mathbf{L} \\ &= \mathbf{N} \cdot \mathbf{L} \\ &= \cos \theta_i \end{aligned}$$

You can directly measure angle between normal and light source

- Not quite enough information to compute surface shape
- But can be if you add some additional info, for example
  - assume a few of the normals are known (e.g., along silhouette)
  - constraints on neighboring normals—“integrability”
  - smoothness
- Hard to get it to work well in practice
  - plus, how many real objects have constant albedo?

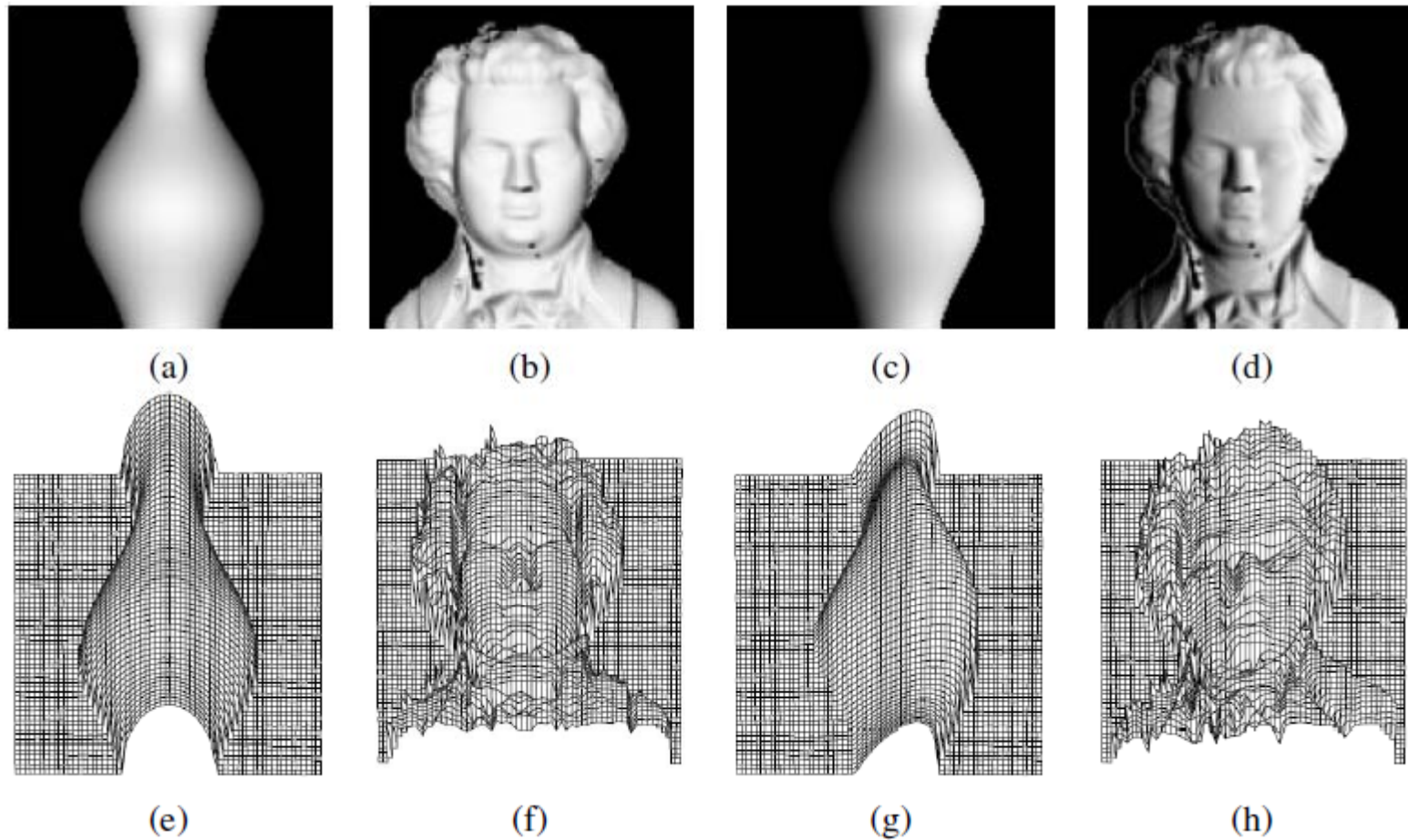


Figure 12.2: *Synthetic shape from shading example (Zhang et al. 1999a): (a–d) shaded images, with light from in front  $(0, 0, 1)$  and from the front right  $(1, 0, 1)$ ; (e–f) corresponding shape from shading reconstructions using the technique of Tsai and Shah (1994).*

# Shape from Shading

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Most shape from shading algorithms assume that the surface under consideration is of a uniform albedo and reflectance, and that the light source directions are either known or can be calibrated by the use of a reference object. Under the assumptions of distant light sources and observer, the variation in intensity (*irradiance equation*) become purely a function of the local surface orientation,

$$I(x, y) = R(p(x, y), q(x, y)), \quad (12.1)$$

where  $(p, q) = (z_x, z_y)$  are the depth map derivatives and  $R(p, q)$  is called the *reflectance map*. For example, a diffuse (Lambertian) surface has a reflectance map that is the (non-negative) dot product (2.87) between the surface normal  $\hat{n} = (p, q, 1)/\sqrt{1 + p^2 + q^2}$  and the light source direction  $v = (v_x, v_y, v_z)$ ,

$$R(p, q) = \max \left( 0, \rho \frac{pv_x + qv_y + v_z}{\sqrt{1 + p^2 + q^2}} \right), \quad (12.2)$$

where  $\rho$  is the surface reflectance factor (albedo).



# Shape from Shading

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In principle, (12.1–12.2) can be used to estimate  $(p, q)$  using non-linear least squares or some other method. Unfortunately, unless additional constraints are imposed, there are more unknowns per pixel  $(p, q)$  than there are measurements  $(I)$ . One commonly used constraint is the smoothness constraint,

$$\mathcal{E}_s = \int p_x^2 + p_y^2 + q_x^2 + q_y^2 dx dy = \int \|\nabla p\|^2 + \|\nabla q\|^2 dx dy, \quad (12.3)$$

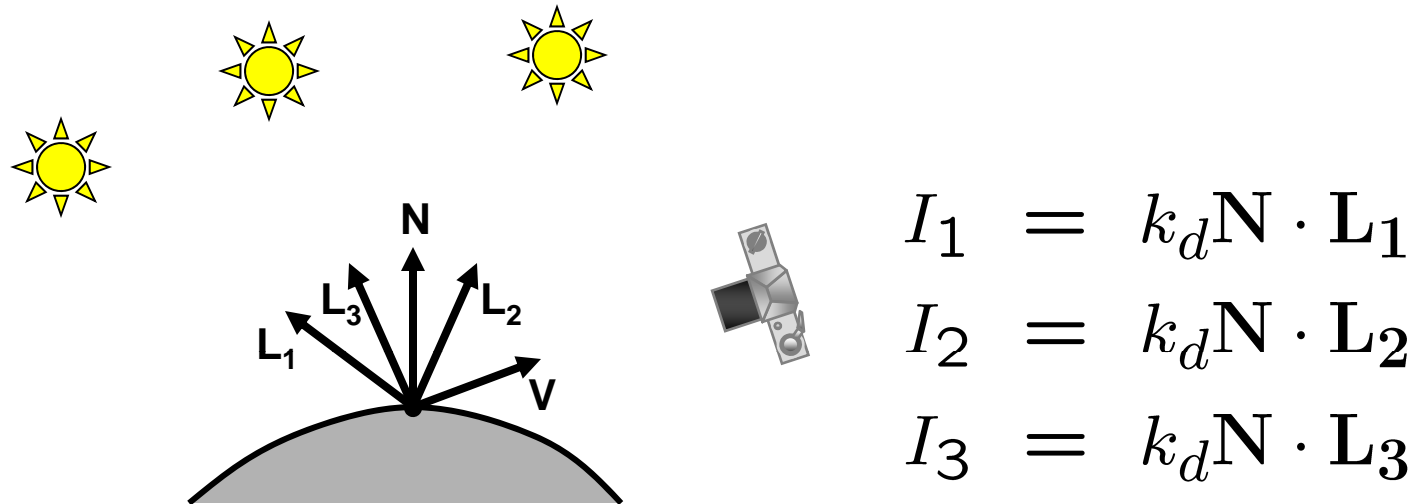
which we already saw in §3.6.1 (3.93). The other is the *integrability constraint*,

$$\mathcal{E}_i = \int (p_y - q_x)^2 dx dy, \quad (12.4)$$

which arises naturally, since for a valid depth map  $z(x, y)$  with  $(p, q) = (z_x, z_y)$ , we have  $p_y = z_{xy} = z_{yx} = q_x$ .

# Photometric stereo

---



Can write this as a matrix equation:

$$\begin{bmatrix} I_1 & I_2 & I_3 \end{bmatrix} = k_d \mathbf{N}^T \begin{bmatrix} \mathbf{L}_1 & \mathbf{L}_2 & \mathbf{L}_3 \end{bmatrix}$$

# Solving the equations

---

$$\underbrace{\begin{bmatrix} I_1 & I_2 & I_3 \end{bmatrix}}_{\substack{\mathbf{I} \\ 1 \times 3}} = k_d \underbrace{\mathbf{N}^T}_{\substack{\mathbf{G} \\ 1 \times 3}} \underbrace{\begin{bmatrix} \mathbf{L}_1 & \mathbf{L}_2 & \mathbf{L}_3 \end{bmatrix}}_{\substack{\mathcal{L} \\ 3 \times 3}}$$

$$\mathbf{G} = \mathbf{I}\mathcal{L}^{-1}$$

$$k_d = \|\mathbf{G}\|$$

$$\mathbf{N} = \frac{1}{k_d} \mathbf{G}$$

# More than three lights

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Get better results by using more lights

$$\begin{bmatrix} I_1 & \dots & I_n \end{bmatrix} = k_d \mathbf{N}^T \begin{bmatrix} \mathbf{L}_1 & \dots & \mathbf{L}_n \end{bmatrix}$$

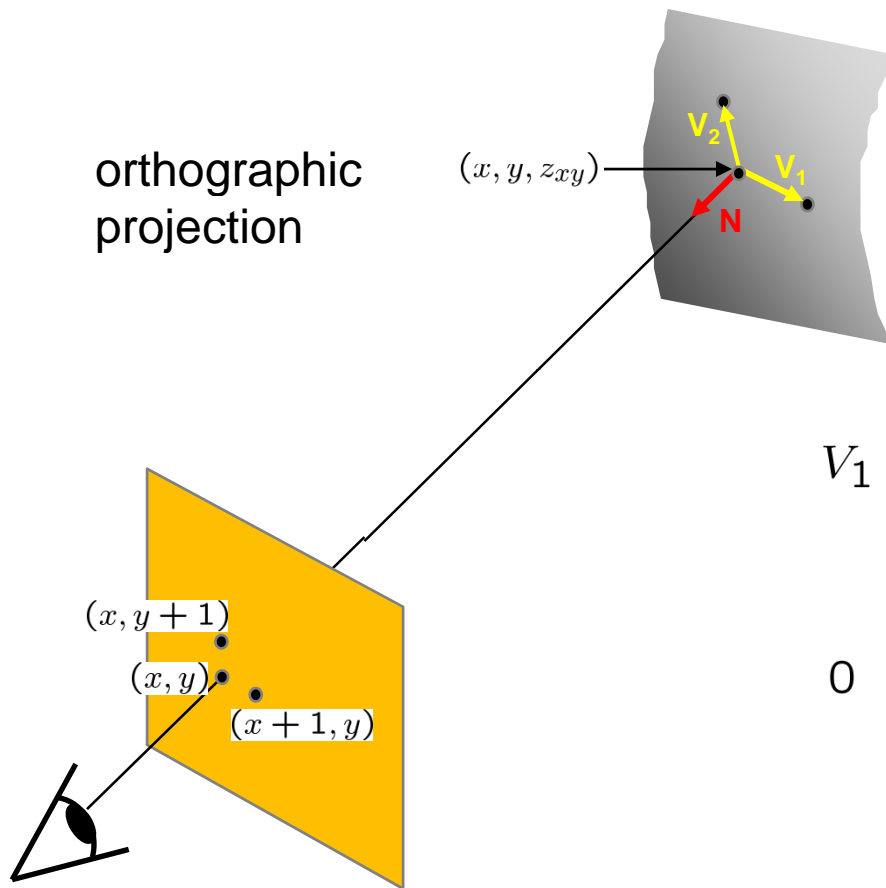
Least squares solution:

$$\begin{aligned} \mathbf{I} &= \mathbf{G}\mathbf{L} \\ \mathbf{I}\mathbf{L}^T &= \mathbf{G}\mathbf{L}\mathbf{L}^T \\ \mathbf{G} &= (\mathbf{I}\mathbf{L}^T)(\mathbf{L}\mathbf{L}^T)^{-1} \end{aligned}$$

Solve for  $\mathbf{N}$ ,  $k_d$  as before

# Depth from normals

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orthographic  
projection

$(x, y, z_{xy})$

$$\begin{aligned} V_1 &= (x + 1, y, z_{x+1,y}) - (x, y, z_{xy}) \\ &= (1, 0, z_{x+1,y} - z_{xy}) \end{aligned}$$

$$\begin{aligned} 0 &= N \cdot V_1 \\ &= (n_x, n_y, n_z) \cdot (1, 0, z_{x+1,y} - z_{xy}) \\ &= n_x + n_z(z_{x+1,y} - z_{xy}) \end{aligned}$$

Get a similar equation for  $V_2$

- Each normal gives us two linear constraints on  $z$
- compute  $z$  values by solving a matrix equation

# Results...

---



Input  
(1 of 12)



Normals



Normals



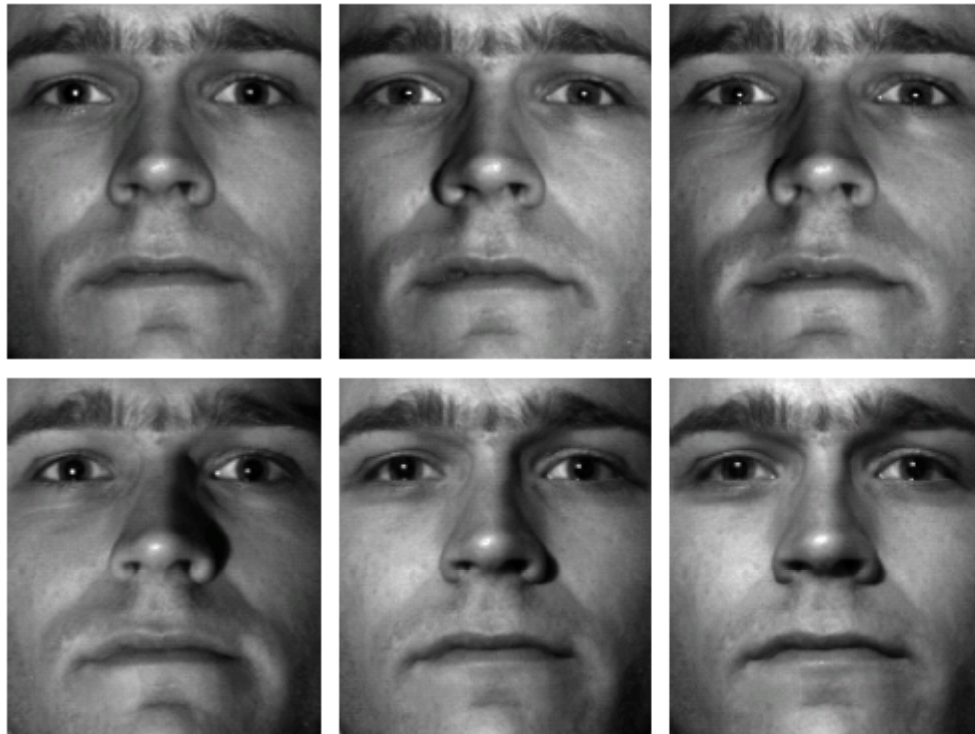
Shaded  
rendering



Textured  
rendering

# Results...

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from Athos Georghiades  
<http://cvc.yale.edu/people/Athos.html>

# Limitations

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## Big problems

- doesn't work for shiny things, semi-translucent things
- shadows, inter-reflections

## Smaller problems

- camera and lights have to be distant
- calibration requirements
  - measure light source directions, intensities
  - camera response function

Newer work addresses some of these issues

Some pointers for further reading:

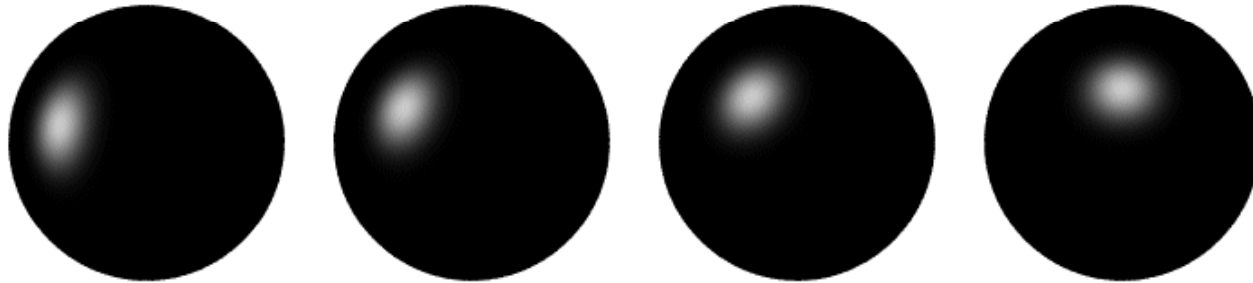
- Zickler, Belhumeur, and Kriegman, "[\*Helmholtz Stereopsis: Exploiting Reciprocity for Surface Reconstruction\*](#)." IJCV, Vol. 49 No. 2/3, pp 215-227.
- Hertzmann & Seitz, "[\*Example-Based Photometric Stereo: Shape Reconstruction with General, Varying BRDFs\*](#)." IEEE Trans. PAMI 2005



# Computing light source directions

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Trick: place a chrome sphere in the scene



- the location of the highlight tells you where the light source is

# An Aside: Digital Forensics (H. Farid)

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[H. Farid, Dartmouth]

This photograph of the host and judges for the popular television show American Idol was scheduled for publication when it caught the attention of a photo-editor who was concerned that the image was doctored.

We found traces of tampering when we examined each eye. First, the inconsistencies in the shape of the specular highlight on the eyes suggest that each person was originally photographed under different lighting conditions. Second, from the location of these highlights, we can mathematically determine the 3-D location of the lights under which each person was originally photographed. We have determined that the positions of the lights in this image are inconsistent and conclude that this image is a composite of at least three images.



[H. Farid, Dartmouth]