Today

Experts/Zero-Sum Games Equilibrium.

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Boosting and Experts.

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Routing and Experts.

Two person zero sum games.

$m \times n$ payoff matrix A.

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Row mixed strategy: $x = (x_1, \ldots, x_m)$.

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Payoff for strategy pair (x, y):

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Payoff for strategy pair (x, y):

$$p(x, y) = x^t A y$$

That is,

$$\sum_i x_i \left(\sum_j a_{i,j} y_j \right) = \sum_j \left(\sum_i x_i a_{i,j} \right) y_j.$$

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(No better column strategy, no better row strategy.)

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$\max_j (A^t)^{(j)} \cdot x = (x^*)^t A y^*$.

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Best Response

Column goes first:

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Find y, where best row is not too low..

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Example: Roshambo.

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Row goes first:
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$$C = \min_x \max_y (x^t A y).$$

Best Response

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Row goes first:
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Proof: Better to go second. $\square$

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Doesn't matter who plays first!

Proof of Equilibrium.

Later.

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Later. Still later...

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Aproximate equilibrium ...

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$C(x) = \max_y x^t Ay$

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$R(y) = C(x)$

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$R(y) = C(x) \rightarrow C(x) - R(y) = 0.$

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Approximate Equilibrium: $C(x) - R(y) \leq \varepsilon$.

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$\rightarrow$ "Response y to x is within ε of best response"

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Proof of approximate equilibrium.

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(A) Using geometry.

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Not hard.

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Not hard. Even easy.

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(C) ..and (D).
Not hard. Even easy. Still, head scratching happens.

Games and experts

Again: find $(x*, y*)$, such that

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$L \leq (1+\varepsilon)L^* + \frac{\log n}{\varepsilon}$

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Let x_t be distribution (row strategy) x_t on day t.

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Let $y^* = \frac{1}{T}\sum_t y_t$ and $x^* = \text{argmin}_{x_t} x_t A y_t$.

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Let $y^* = \frac{1}{T}\sum_t y_t$ and $x^* = \text{argmin}_{x_t} x_t A y_t$.

Claim: $(x^*, y)^*$ are 2ε-optimal for matrix A.

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column best response is at least what it is against x_t.

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x_t minimizes the best column response is chosen. Clearly good for row.
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Games and Experts.

Assume: A has payoffs in $[0,1]$.

For $T = \frac{\log n}{\varepsilon^2}$ days:

1) m pure row strategies are experts.
Use multiplicative weights, produce row distribution.
Let x_t be distribution (row strategy) x_t on day t.

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Choose column of A that maximizes row's expected loss.
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Combine bounds.

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Combine bounds. Done!

Approximate Equilibrium!

Experts: x_t is strategy on day t, y_t is best column against x_t.

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Multiplicative Weights: $L \leq (1+\varepsilon)L^* + \frac{\ln n}{\varepsilon}$

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$$TC(x^*) \leq (1+\varepsilon)TR(y^*) + \frac{\ln n}{\varepsilon} \rightarrow C(x^*) \leq (1+\varepsilon)R(y^*) + \frac{\ln n}{\varepsilon T}$$

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$TC(x^*) \leq (1+\varepsilon)TR(y^*) + \frac{\ln n}{\varepsilon} \rightarrow C(x^*) \leq (1+\varepsilon)R(y^*) + \frac{\ln n}{\varepsilon T}$
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$T = \frac{\ln n}{\varepsilon^2}$, $R(y^*) \leq 1$

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$T = \frac{\ln n}{\varepsilon^2}$, $R(y^*) \leq 1$
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Let $x^* = \frac{1}{T}\sum_t x_t$ and $y^* = \frac{1}{T}\sum_t y_t$.

Claim: $(x^*, y)^*$ are 2ε-optimal for matrix A.

Column payoff: $C(x^*) = \max_y x^* Ay$.
Let y_r be best response to $C(x^*)$.
Day t,y_t best response to $x_t \to x_t A y_t \geq x_t A y_r$.
Algorithm loss: $\sum_t x_t A y_t \geq \sum_t x_t A y_r$
$L \geq TC(x^*)$.

Best expert: L^*- best row against all the columns played.

best row against $\sum_t Ay_t$ and $Ty^* = \sum_t y_t$
$\to$ best row against TAy^*.
$\to L^* \leq TR(y^*)$.

Multiplicative Weights: $L \leq (1+\varepsilon)L^* + \frac{\ln n}{\varepsilon}$

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$T = \frac{\ln n}{\varepsilon^2}$, $R(y^*) \leq 1 \rightarrow C(x^*) - R(y^*) \leq 2\varepsilon$.

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“In practice.”

Learning.

Learning just a bit.

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Example: set of labelled points, find hyperplane that separates.

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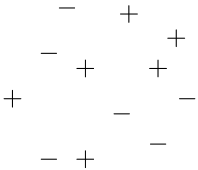

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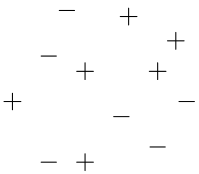

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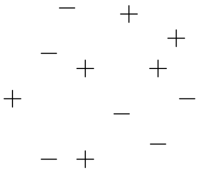

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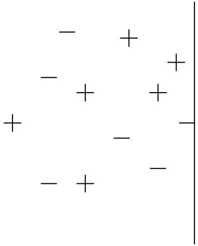

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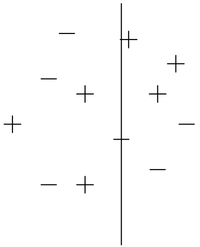

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Arbitrary line. And Scan.

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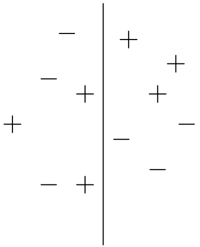

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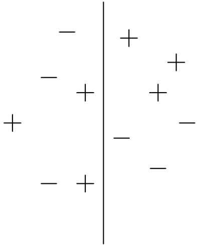

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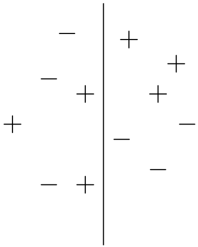

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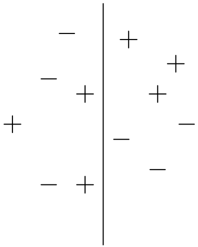

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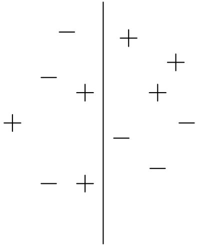

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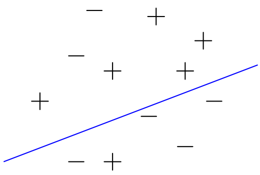

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Input: n labelled points.

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Weak Learner:

produce hypothesis correctly classifies $\frac{1}{2} + \varepsilon$ fraction

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That's a really strong learner!

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Same thing?

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Same thing?

Can one use weak learning to produce strong learner?

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Can one use weak learning to produce strong learner?

Boosting: use a weak learner to produce strong learner.

Poll.

Given a weak learning method (produce ok hypotheses.)

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Can we do this?

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Can we do this?

(A) Yes

(B) No

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If yes.

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Multiplicative Weights!

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Multiplicative Weights!

The endpoint to a line of research.

Experts Picture

Boosting/MW Framework

Experts are points.

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Experts are points. “Adversary” weak learner.

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of classifying random point correctly.

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Strong learner algorithm will come from adversary.

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1. Row player: multiplicative weights($1-\gamma$) on points.

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Claim: $h(x)$ is correct on $1-\mu$ of the points ! ! !

Cool!

Boosting/MW Framework

Experts are points. “Adversary” weak learner.

Points want to be misclassified.

Learner wants to maximize probability
of classifying random point correctly.
Strong learner algorithm will come from adversary.

Do $T = \frac{2}{\gamma^2} \log \frac{1}{\mu}$ rounds

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This subset will be classified correctly with probability $1/2+\varepsilon$.

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Used often in machine learning.

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Set of points on unit ball in *d*-space.

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and find $\frac{1}{2}+\varepsilon$ weighted separating plane.

Experts output is average of hyperplanes ...a hyperplane!

$\frac{1}{2}+\varepsilon$ separating hyperplane?
Assumption: margin γ.

Random hyperplane?
Not likely to be exactly normal to $\mathscr{H}$.
Should get $\frac{1}{2}+\gamma/\sqrt{d}$

$O(\frac{d \log n}{\gamma^2})$ to find separating hyperplane.

Example.

Set of points on unit ball in d-space.

Learner: learns hyperplanes through origin.

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Weak learner: random Wow. That's weak.

Better weak learner?

Hyperplane that separates weighted average of +/- points?

Better weak learner?

Hyperplane that separates weighted average of +/- points?

Change loss a bit, and get better results.

Toll/Congestion

Given: $G = (V, E)$.
Given $(s_1, t_1) \ldots (s_k, t_k)$.
Row: choose routing of all paths.
Column: choose edge.
Row pays if column chooses edge on any path.

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Two person game.

Row for every roting.

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Runtime only dependent on m and T (number of days.)

Congestion minimization and Experts.

Will use gain and $[0, \rho]$ version of experts:

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$G \leq C^* T$ — each day, gain is average congestion $\leq C^*$
since each day cost is toll solution which is at most C^*

$C^* T \geq c_{max} T(1-\varepsilon) - \frac{k \log n}{\varepsilon}$

For $T = \frac{k \log n}{\varepsilon^2}$

$\rightarrow C^* \frac{1}{1-\varepsilon} + \varepsilon \geq c_{\max}$ plus $\frac{1}{1-\varepsilon} \leq 1+\varepsilon$

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$c_{max} - C^* \leq \varepsilon C + \varepsilon/(1-\varepsilon)$ □

Better setup.

Runtime: $O(km)$ to route in each step.

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$O(k \log n (\frac{1}{\varepsilon^2}))$ steps

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Homework: $O(km \log n)$ algorithm.

Fractional versus Integer.

Did we solve path routing?

Fractional versus Integer.

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Yes?

Fractional versus Integer.

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Yes? No?

Fractional versus Integer.

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Yes? No?

No!

Fractional versus Integer.

Did we solve path routing?
Yes? No?

No! Average of T routings.

Fractional versus Integer.

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No! Average of T routings.
  We approximately solved fractional routing problem.

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No solution to the path routing problem that is $(1+\varepsilon)$ optimal!

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Homework 2. Problem 1.

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Yes? No?

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Homework 2. Problem 1.

Decent solution to path routing problem?

Randomized Rounding

For each s_i, t_i, choose path p_i with probability $f(p_i)$.

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Concentration (law of large numbers)

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Concentration results?

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Concentration results? later.

See you on Tuesday.