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Minimize worst expected loss. Best defense.

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"Infinite horizon" is just an assumption of rationality.

Today

Finish Maximum Weight Matching Algorithm.

Today

Finish Maximum Weight Matching Algorithm.
  Exact algorithm with dueling players.

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    Exact algorithm with dueling players.

Multiplicative Weights Framework.

Today

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Multiplicative Weights Framework.
  Very general framework of toll/congestion algorithm.

Matching/Weighted Vertex Cover

Maximum Weight Matching.

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Maximum Weight Matching

Goal: perfect matching on tight edges.

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Algorithm

Start with empty matching, feasible cover function ($p(\cdot)$)

Maximum Weight Matching

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Add tight edges to matching.

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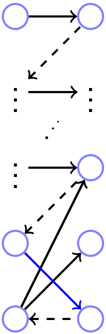

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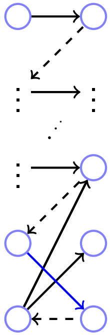

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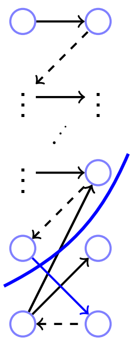

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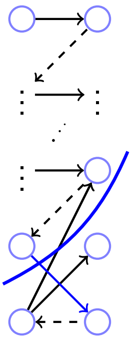

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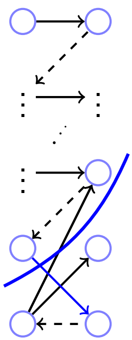

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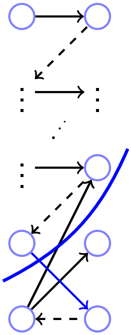

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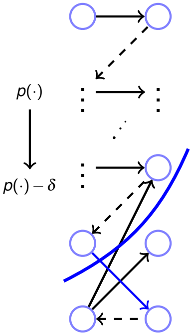

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Lower prices in S_U,

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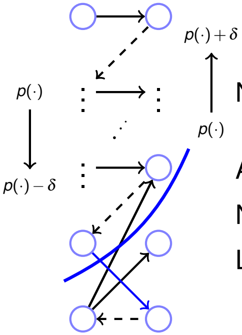

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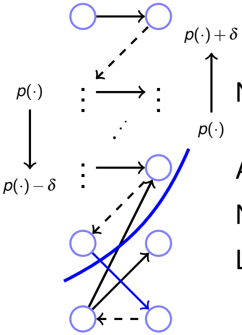

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Lower prices in S_U, raise prices in S_T,
all explored edges still tight,
backward edges still feasible

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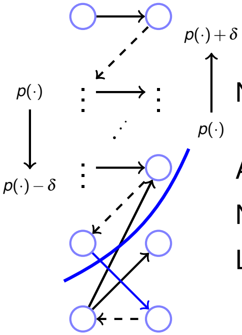

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... and get new tight edge!

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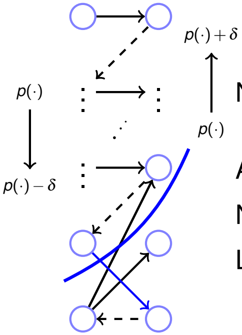

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What's delta?

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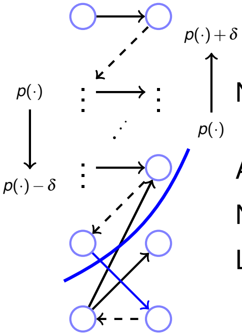

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... and get new tight edge!
What's delta? $w(e) > p(u) + p(v) \rightarrow$
$\delta = min_{e \in (S_U \times T_V)} w(e) - p(u) - p(v)$.

Some details/Runtime

Add 0 value edges, so that optimal solution contains perfect matching.

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Beginning "Matcher" Solution: $M = \{\}$.

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Feasible!

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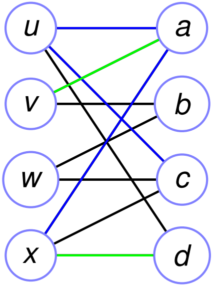
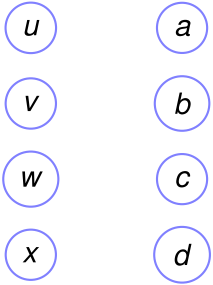
$O(n^2 m)$ time.

Example

Weight legend:
black 1, green 2, blue 3





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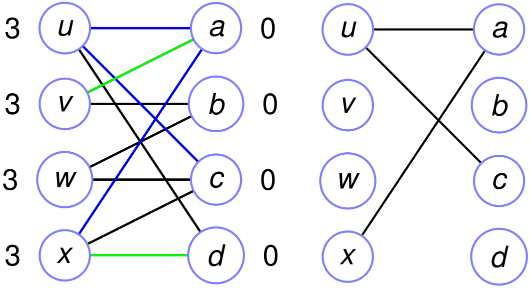
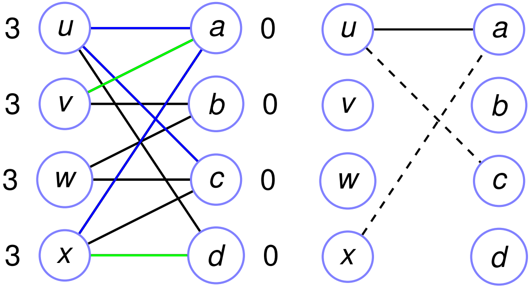


Weight legend:
black 1, green 2, blue 3
Tight edges for inital prices.

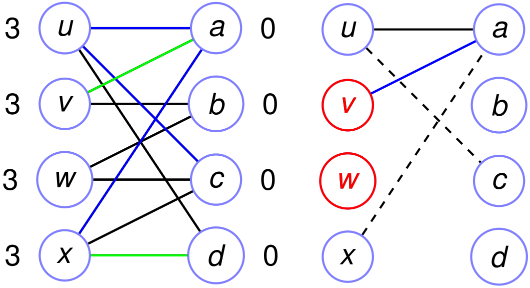
Example



Weight legend:
black 1, green 2, blue 3

Max matching in tight edges.
dashed means matched.

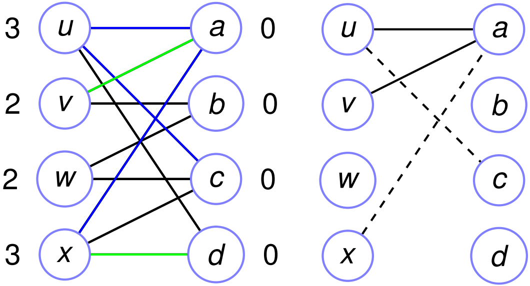
Example



Weight legend:
black 1, green 2, blue 3

No augmenting path →
reachable: $S = \{u, v\}$
Blue edges soon
to be tight!

Example



Weight legend:
black 1, green 2, blue 3

Adjust prices...
new tight edges.

Example

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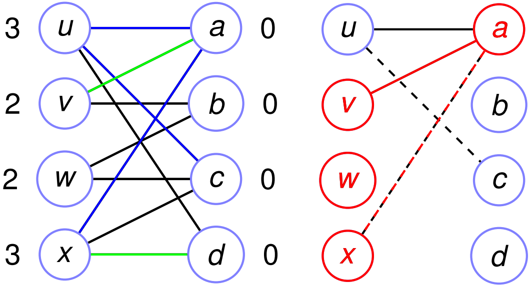
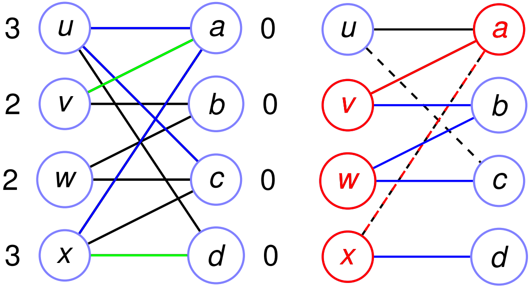


Still no augmenting path.
Reachable $S = \{v, w, x, a\}$

Example



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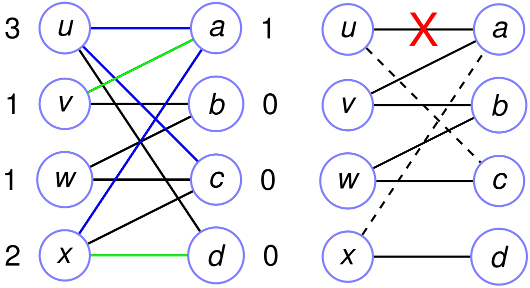
Still no augmenting path.
Reachable $S = \{v, w, x, a\}$
Blue edges minimally non-tight.

Example



Weight legend:
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Adjust prices.
Some more tight edges.
And X shows
a “new” nontight edge.

Example

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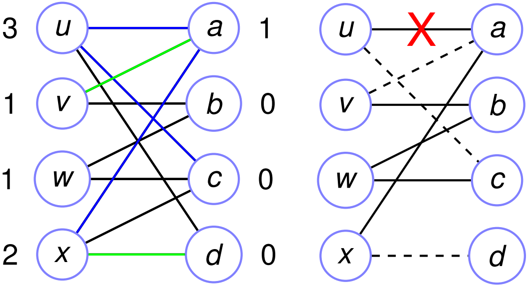
black 1, green 2, blue 3



..and another augmentation...

Example

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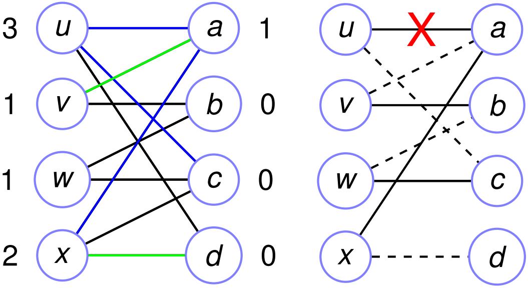

..and finally: a perfect matching.

Example

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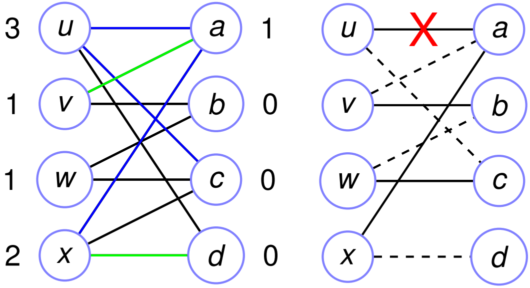

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All matched edges tight.

Example

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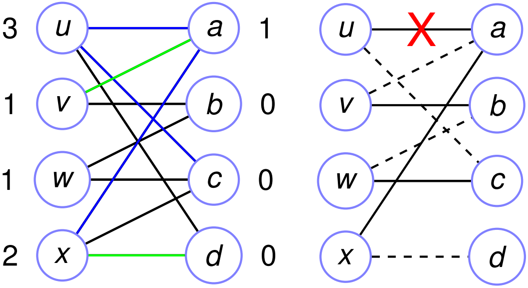

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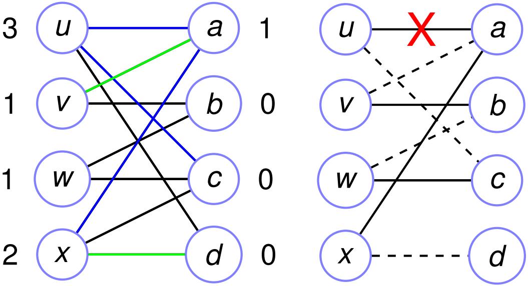
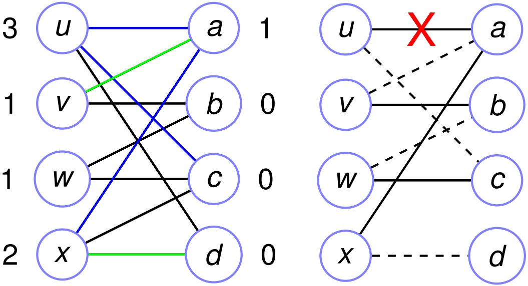


..and finally: a perfect matching.

All matched edges tight.
Perfect matching. Feasible price function.

Example

Weight legend:
black 1, green 2, blue 3



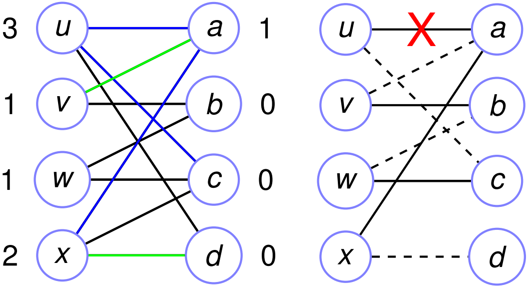
..and finally: a perfect matching.

All matched edges tight.

Perfect matching. Feasible price function. Values the same.

Example

Weight legend:
black 1, green 2, blue 3



..and finally: a perfect matching.

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Optimal!

Example

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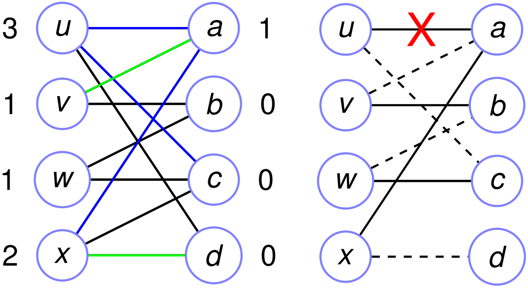


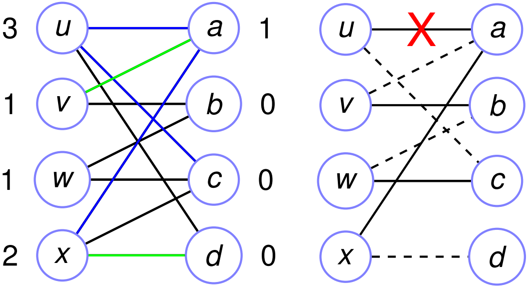
..and finally: a perfect matching.

All matched edges tight.
Perfect matching. Feasible price function. Values the same.
Optimal!

Notice:

Example

Weight legend:
black 1, green 2, blue 3



..and finally: a perfect matching.

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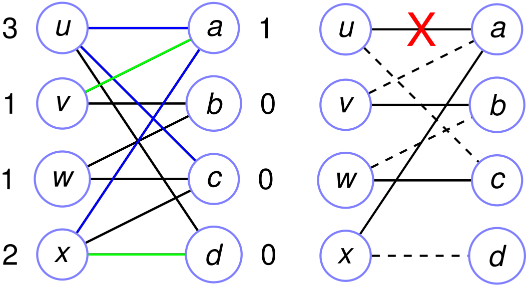
Notice:
no weights on the right problem.

Example

Weight legend:
black 1, green 2, blue 3



..and finally: a perfect matching.

All matched edges tight.

Perfect matching. Feasible price function. Values the same.

Optimal!

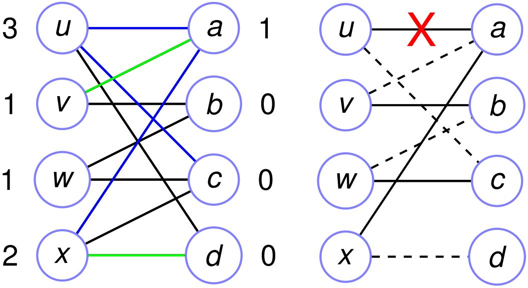
Notice:

no weights on the right problem.

retain previous matching through price changes.

Example

Weight legend:
black 1, green 2, blue 3



..and finally: a perfect matching.

All matched edges tight.
Perfect matching. Feasible price function. Values the same.
Optimal!

Notice:

no weights on the right problem.
retain previous matching through price changes.
retains edges in failed search through price changes.

The multiplicative weights framework.

Expert's framework.

n experts.

Expert's framework.

n experts.

Every day, each offers a prediction.

Expert's framework.

n experts.

Every day, each offers a prediction.

"Rain" or "Shine."

Expert's framework.

n experts.

Every day, each offers a prediction.

"Rain" or "Shine."

Whose advise do you follow?

Expert's framework.

n experts.

Every day, each offers a prediction.

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Whose advise do you follow?

"The one who is correct most often."

Expert's framework.

n experts.

Every day, each offers a prediction.

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Whose advise do you follow?

"The one who is correct most often."

Sort of.

Expert's framework.

n experts.

Every day, each offers a prediction.

"Rain" or "Shine."

Whose advise do you follow?

"The one who is correct most often."

Sort of.

How well do you do?

Infallible expert.

One of the expert’s is infallible!

Infallible expert.

One of the expert's is infallible!

Your strategy?

Infallible expert.

One of the expert's is infallible!

Your strategy?

Choose any expert that has not made a mistake!

Infallible expert.

One of the expert's is infallible!

Your strategy?

Choose any expert that has not made a mistake!

How long to find perfect expert?

Infallible expert.

One of the expert's is infallible!

Your strategy?

Choose any expert that has not made a mistake!

How long to find perfect expert?

Maybe..

Infallible expert.

One of the expert's is infallible!

Your strategy?

Choose any expert that has not made a mistake!

How long to find perfect expert?

Maybe..never!

Infallible expert.

One of the expert's is infallible!

Your strategy?

Choose any expert that has not made a mistake!

How long to find perfect expert?

Maybe..never! Never see a mistake.

Infallible expert.

One of the expert's is infallible!

Your strategy?

Choose any expert that has not made a mistake!

How long to find perfect expert?

Maybe..never! Never see a mistake.

Better model?

Infallible expert.

One of the expert's is infallible!

Your strategy?

Choose any expert that has not made a mistake!

How long to find perfect expert?

Maybe..never! Never see a mistake.

Better model?

How many mistakes could you make?

Infallible expert.

One of the expert's is infallible!

Your strategy?

Choose any expert that has not made a mistake!

How long to find perfect expert?

Maybe..never! Never see a mistake.

Better model?

How many mistakes could you make? Mistake Bound.

Infallible expert.

One of the expert's is infallible!

Your strategy?

Choose any expert that has not made a mistake!

How long to find perfect expert?

Maybe..never! Never see a mistake.

Better model?

How many mistakes could you make? Mistake Bound.

(A) 1
(B) 2
(C) $\log n$
(D) $n-1$

Adversary designs setup to watch who you choose, and make that expert make a mistake.

Infallible expert.

One of the expert's is infallible!

Your strategy?

Choose any expert that has not made a mistake!

How long to find perfect expert?

Maybe..never! Never see a mistake.

Better model?

How many mistakes could you make? Mistake Bound.

(A) 1
(B) 2
(C) $\log n$
(D) $n-1$

Adversary designs setup to watch who you choose, and make that expert make a mistake.

$n-1$!

Concept Alert.

Note.

Concept Alert.

Note.

Adversary:

Concept Alert.

Note.

Adversary:
makes you want to look bad.

Concept Alert.

Note.

Adversary:
makes you want to look bad.
”You could have done so well”...

Concept Alert.

Note.

Adversary:
makes you want to look bad.
”You could have done so well”...
but you didn’t!

Concept Alert.

Note.

Adversary:
makes you want to look bad.
”You could have done so well”...
but you didn’t! ha..

Concept Alert.

Note.

Adversary:

makes you want to look bad.

”You could have done so well”...

but you didn’t! ha..ha!

Concept Alert.

Note.

Adversary:
makes you want to look bad.
"You could have done so well"...
but you didn't! ha..ha!

Analysis of Algorithms: do as well as possible!

Back to mistake bound.

Infallible Experts.

Back to mistake bound.

Infallible Experts.

Alg: Choose one of the perfect experts.

Back to mistake bound.

Infallible Experts.

Alg: Choose one of the perfect experts.

Mistake Bound: $n-1$

Back to mistake bound.

Infallible Experts.

Alg: Choose one of the perfect experts.

Mistake Bound: $n-1$

Lower bound: adversary argument.

Back to mistake bound.

Infallible Experts.

Alg: Choose one of the perfect experts.

Mistake Bound: $n-1$

Lower bound: adversary argument.

Upper bound:

Back to mistake bound.

Infallible Experts.

Alg: Choose one of the perfect experts.

Mistake Bound: $n-1$
  Lower bound: adversary argument.
  Upper bound: every mistake finds fallible expert.

Back to mistake bound.

Infallible Experts.

Alg: Choose one of the perfect experts.

Mistake Bound: $n-1$

Lower bound: adversary argument.
Upper bound: every mistake finds fallible expert.

Better Algorithm?

Back to mistake bound.

Infallible Experts.

Alg: Choose one of the perfect experts.

Mistake Bound: $n-1$

Lower bound: adversary argument.

Upper bound: every mistake finds fallible expert.

Better Algorithm?

Making decision, not trying to find expert!

Back to mistake bound.

Infallible Experts.

Alg: Choose one of the perfect experts.

Mistake Bound: $n-1$
  Lower bound: adversary argument.
  Upper bound: every mistake finds fallible expert.

Better Algorithm?

Making decision, not trying to find expert!

Algorithm: Go with the majority of previously correct experts.

Back to mistake bound.

Infallible Experts.

Alg: Choose one of the perfect experts.

Mistake Bound: $n-1$

Lower bound: adversary argument.

Upper bound: every mistake finds fallible expert.

Better Algorithm?

Making decision, not trying to find expert!

Algorithm: Go with the majority of previously correct experts.

What you would do anyway!

Alg 2: find majority of the perfect

How many mistakes could you make?

Alg 2: find majority of the perfect

How many mistakes could you make?

(A) 1

(B) 2

(C) $\log n$

(D) $n-1$

Alg 2: find majority of the perfect

How many mistakes could you make?

(A) 1

(B) 2

(C) $\log n$

(D) $n-1$

At most $\log n$!

Alg 2: find majority of the perfect

How many mistakes could you make?

(A) 1

(B) 2

(C) $\log n$

(D) $n-1$

At most $\log n$!

When alg makes a <u>mistake</u>,
$|$"perfect" experts$|$ drops by a factor of two.

Alg 2: find majority of the perfect

How many mistakes could you make?

(A) 1

(B) 2

(C) $\log n$

(D) $n-1$

At most $\log n$!

When alg makes a mistake,
$|$"perfect" experts$|$ drops by a factor of two.

Initially n perfect experts

Alg 2: find majority of the perfect

How many mistakes could you make?

(A) 1

(B) 2

(C) $\log n$

(D) $n-1$

At most $\log n$!

When alg makes a mistake,
$|$"perfect" experts$|$ drops by a factor of two.

Initially n perfect experts mistake $\rightarrow$ $\leq n/2$ perfect experts

Alg 2: find majority of the perfect

How many mistakes could you make?

(A) 1

(B) 2

(C) $\log n$

(D) $n - 1$

At most $\log n$!

When alg makes a mistake,
$|$"perfect" experts$|$ drops by a factor of two.

Initially n perfect experts mistake $\rightarrow$ $\leq n/2$ perfect experts
mistake $\rightarrow$ $\leq n/4$ perfect experts

Alg 2: find majority of the perfect

How many mistakes could you make?

(A) 1

(B) 2

(C) $\log n$

(D) $n-1$

At most $\log n$!

When alg makes a <u>mistake</u>,
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Alg 2: find majority of the perfect

How many mistakes could you make?

(A) 1

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At most $\log n$!

When alg makes a mistake,
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Initially n perfect experts mistake $\rightarrow$ $\leq n/2$ perfect experts
mistake $\rightarrow$ $\leq n/4$ perfect experts

$\vdots$

mistake $\rightarrow$ ≤ 1 perfect expert

Alg 2: find majority of the perfect

How many mistakes could you make?

(A) 1

(B) 2

(C) $\log n$

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At most $\log n$!

When alg makes a <u>mistake</u>,
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How many mistakes could you make?

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(D) $n-1$

At most $\log n$!

When alg makes a mistake,
$|$"perfect" experts$|$ drops by a factor of two.

Initially n perfect experts mistake $\rightarrow$ $\leq n/2$ perfect experts
mistake $\rightarrow$ $\leq n/4$ perfect experts

$\vdots$

mistake $\rightarrow$ ≤ 1 perfect expert

≥ 1 perfect expert

Alg 2: find majority of the perfect

How many mistakes could you make?

(A) 1

(B) 2

(C) $\log n$

(D) $n-1$

At most $\log n$!

When alg makes a <u>mistake</u>,
$|$"perfect" experts$|$ drops by a factor of two.

Initially n perfect experts mistake $\rightarrow$ $\leq n/2$ perfect experts
mistake $\rightarrow$ $\leq n/4$ perfect experts

$\vdots$

mistake $\rightarrow$ ≤ 1 perfect expert

≥ 1 perfect expert $\rightarrow$ at most $\log n$ mistakes!

Imperfect Experts

Goal?

Imperfect Experts

Goal?

Do as well as the best expert!

Imperfect Experts

Goal?

Do as well as the best expert!

Algorithm.

Imperfect Experts

Goal?

Do as well as the best expert!

Algorithm. Suggestions?

Imperfect Experts

Goal?

Do as well as the best expert!

Algorithm. Suggestions?

Go with majority?

Imperfect Experts

Goal?

Do as well as the best expert!

Algorithm. Suggestions?

Go with majority?

Penalize inaccurate experts?

Imperfect Experts

Goal?

Do as well as the best expert!

Algorithm. Suggestions?

Go with majority?

Penalize inaccurate experts?

Best expert is penalized the least.

Imperfect Experts

Goal?

Do as well as the best expert!

Algorithm. Suggestions?

Go with majority?

Penalize inaccurate experts?

Best expert is penalized the least.

1. Initially: $w_i = 1$.

Imperfect Experts

Goal?

Do as well as the best expert!

Algorithm. Suggestions?

Go with majority?

Penalize inaccurate experts?

Best expert is penalized the least.

1. Initially: $w_i = 1$.
2. Predict with weighted majority of experts.

Imperfect Experts

Goal?

Do as well as the best expert!

Algorithm. Suggestions?

Go with majority?

Penalize inaccurate experts?

Best expert is penalized the least.

1. Initially: $w_i = 1$.
2. Predict with weighted majority of experts.
3. $w_i \rightarrow w_i/2$ if wrong.

Imperfect Experts

Goal?

Do as well as the best expert!

Algorithm. Suggestions?

Go with majority?

Penalize inaccurate experts?

Best expert is penalized the least.

1. Initially: $w_i = 1$.
2. Predict with weighted majority of experts.
3. $w_i \rightarrow w_i/2$ if wrong.

Analysis: weighted majority

Analysis: weighted majority

1. Initially: $w_i = 1$.
2. Predict with weighted majority of experts.
3. $w_i \to w_i/2$ if wrong.

Analysis: weighted majority

Goal: Best expert makes m mistakes.

1. Initially: $w_i = 1$.
2. Predict with weighted majority of experts.
3. $w_i \to w_i/2$ if wrong.

Analysis: weighted majority

Goal: Best expert makes m mistakes.

Potential function:

1. Initially: $w_i = 1$.
2. Predict with weighted majority of experts.
3. $w_i \rightarrow w_i/2$ if wrong.

Analysis: weighted majority

Goal: Best expert makes m mistakes.

Potential function: $\sum_i w_i$.

1. Initially: $w_i = 1$.
2. Predict with weighted majority of experts.
3. $w_i \to w_i/2$ if wrong.

Analysis: weighted majority

Goal: Best expert makes m mistakes.

Potential function: $\sum_i w_i$. Initially n.

1. Initially: $w_i = 1$.
2. Predict with weighted majority of experts.
3. $w_i \to w_i/2$ if wrong.

Analysis: weighted majority

Goal: Best expert makes m mistakes.

Potential function: $\sum_i w_i$. Initially n.

For best expert, b, $w_b \geq \frac{1}{2^m}$.

1. Initially: $w_i = 1$.
2. Predict with weighted majority of experts.
3. $w_i \rightarrow w_i/2$ if wrong.

Analysis: weighted majority

Goal: Best expert makes m mistakes.

Potential function: $\sum_i w_i$. Initially n.

For best expert, b, $w_b \geq \frac{1}{2^m}$.

Each mistake:

1. Initially: $w_i = 1$.
2. Predict with weighted majority of experts.
3. $w_i \rightarrow w_i/2$ if wrong.

Analysis: weighted majority

Goal: Best expert makes m mistakes.

Potential function: $\sum_i w_i$. Initially n.

For best expert, b, $w_b \geq \frac{1}{2^m}$.

Each mistake:
total weight of incorrect experts reduced by

1. Initially: $w_i = 1$.
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3. $w_i \rightarrow w_i/2$ if wrong.

Analysis: weighted majority

Goal: Best expert makes m mistakes.

Potential function: $\sum_i w_i$. Initially n.

For best expert, b, $w_b \geq \frac{1}{2^m}$.

Each mistake:
total weight of incorrect experts reduced by
-1?

1. Initially: $w_i = 1$.
2. Predict with weighted majority of experts.
3. $w_i \rightarrow w_i/2$ if wrong.

Analysis: weighted majority

Goal: Best expert makes m mistakes.

Potential function: $\sum_i w_i$. Initially n.

For best expert, b, $w_b \geq \frac{1}{2^m}$.

Each mistake:
total weight of incorrect experts reduced by
-1? -2?

1. Initially: $w_i = 1$.
2. Predict with weighted majority of experts.
3. $w_i \to w_i/2$ if wrong.

Analysis: weighted majority

Goal: Best expert makes m mistakes.

Potential function: $\sum_i w_i$. Initially n.

For best expert, b, $w_b \geq \frac{1}{2^m}$.

Each mistake:
total weight of incorrect experts reduced by
-1? -2? factor of $\frac{1}{2}$?

1. Initially: $w_i = 1$.
2. Predict with weighted majority of experts.
3. $w_i \to w_i/2$ if wrong.

Analysis: weighted majority

Goal: Best expert makes m mistakes.

Potential function: $\sum_i w_i$. Initially n.

For best expert, b, $w_b \geq \frac{1}{2^m}$.

Each mistake:
total weight of incorrect experts reduced by
-1? -2? factor of $\frac{1}{2}$?
each incorrect expert weight multiplied by $\frac{1}{2}$!

1. Initially: $w_i = 1$.
2. Predict with weighted majority of experts.
3. $w_i \rightarrow w_i/2$ if wrong.

Analysis: weighted majority

Goal: Best expert makes m mistakes.

Potential function: $\sum_i w_i$. Initially n.

For best expert, b, $w_b \geq \frac{1}{2^m}$.

Each mistake:
total weight of incorrect experts reduced by
-1? -2? factor of $\frac{1}{2}$?
each incorrect expert weight multiplied by $\frac{1}{2}$!
total weight decreases by

1. Initially: $w_i = 1$.
2. Predict with weighted majority of experts.
3. $w_i \rightarrow w_i/2$ if wrong.

Analysis: weighted majority

Goal: Best expert makes m mistakes.

Potential function: $\sum_i w_i$. Initially n.

For best expert, b, $w_b \geq \frac{1}{2^m}$.

Each mistake:
total weight of incorrect experts reduced by
-1? -2? factor of $\frac{1}{2}$?
each incorrect expert weight multiplied by $\frac{1}{2}$!
total weight decreases by
factor of $\frac{1}{2}$? factor of $\frac{3}{4}$?

1. Initially: $w_i = 1$.
2. Predict with weighted majority of experts.
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Analysis: weighted majority

Goal: Best expert makes m mistakes.

Potential function: $\sum_i w_i$. Initially n.

For best expert, b, $w_b \geq \frac{1}{2^m}$.

Each mistake:
total weight of incorrect experts reduced by
-1? -2? factor of $\frac{1}{2}$?
each incorrect expert weight multiplied by $\frac{1}{2}$!
total weight decreases by
factor of $\frac{1}{2}$? factor of $\frac{3}{4}$?
mistake $\rightarrow \geq$ half weight with incorrect experts.

1. Initially: $w_i = 1$.
2. Predict with weighted majority of experts.
3. $w_i \rightarrow w_i/2$ if wrong.

Analysis: weighted majority

Goal: Best expert makes m mistakes.

Potential function: $\sum_i w_i$. Initially n.

For best expert, b, $w_b \geq \frac{1}{2^m}$.

Each mistake:
total weight of incorrect experts reduced by
-1? -2? factor of $\frac{1}{2}$?
each incorrect expert weight multiplied by $\frac{1}{2}$!
total weight decreases by
factor of $\frac{1}{2}$? factor of $\frac{3}{4}$?
mistake $\rightarrow \geq$ half weight with incorrect experts.

Mistake $\rightarrow$ potential function decreased by $\frac{3}{4}$.

1. Initially: $w_i = 1$.
2. Predict with weighted majority of experts.
3. $w_i \rightarrow w_i/2$ if wrong.

Analysis: weighted majority

Goal: Best expert makes m mistakes.

Potential function: $\sum_i w_i$. Initially n.

For best expert, b, $w_b \geq \frac{1}{2^m}$.

Each mistake:
total weight of incorrect experts reduced by
-1? -2? factor of $\frac{1}{2}$?
each incorrect expert weight multiplied by $\frac{1}{2}$!
total weight decreases by
factor of $\frac{1}{2}$? factor of $\frac{3}{4}$?
mistake $\rightarrow \geq$ half weight with incorrect experts.

Mistake $\rightarrow$ potential function decreased by $\frac{3}{4}$.

We have

$$\frac{1}{2^m} \leq \sum_i w_i \leq \left(\frac{3}{4}\right)^M n.$$

where M is number of algorithm mistakes.

1. Initially: $w_i = 1$.
2. Predict with weighted majority of experts.
3. $w_i \rightarrow w_i/2$ if wrong.

Analysis: continued.

$$\frac{1}{2^m} \leq \sum_i w_i \leq \left(\frac{3}{4}\right)^M n.$$

Analysis: continued.

$$\frac{1}{2^m} \leq \sum_i w_i \leq \left(\frac{3}{4}\right)^M n.$$

m - best expert mistakes

Analysis: continued.

$$\frac{1}{2^m} \leq \sum_i w_i \leq \left(\frac{3}{4}\right)^M n.$$

m - best expert mistakes M algorithm mistakes.

Analysis: continued.

$$\frac{1}{2^m} \leq \sum_i w_i \leq \left(\frac{3}{4}\right)^M n.$$

m - best expert mistakes M algorithm mistakes.

$$\frac{1}{2^m} \leq \left(\frac{3}{4}\right)^M n.$$

Analysis: continued.

$$\frac{1}{2^m} \leq \sum_i w_i \leq \left(\frac{3}{4}\right)^M n.$$

m - best expert mistakes M algorithm mistakes.

$$\frac{1}{2^m} \leq \left(\frac{3}{4}\right)^M n.$$

Take log of both sides.

Analysis: continued.

$\frac{1}{2^m} \leq \sum_i w_i \leq \left(\frac{3}{4}\right)^M n.$

m - best expert mistakes M algorithm mistakes.

$\frac{1}{2^m} \leq \left(\frac{3}{4}\right)^M n.$

Take log of both sides.

$-m \leq -M \log(4/3) + logn.$

Analysis: continued.

$$\frac{1}{2^m} \leq \sum_i w_i \leq \left(\frac{3}{4}\right)^M n.$$

m - best expert mistakes M algorithm mistakes.

$$\frac{1}{2^m} \leq \left(\frac{3}{4}\right)^M n.$$

Take log of both sides.

$$-m \leq -M \log(4/3) + logn.$$

Solve for M.

$$M \leq (m + logn)/\log(4/3)$$

Analysis: continued.

$$\frac{1}{2^m} \leq \sum_i w_i \leq \left(\frac{3}{4}\right)^M n.$$

m - best expert mistakes M algorithm mistakes.

$$\frac{1}{2^m} \leq \left(\frac{3}{4}\right)^M n.$$

Take log of both sides.

$$-m \leq -M \log(4/3) + logn.$$

Solve for M.

$$M \leq (m + logn)/\log(4/3) \leq 2.4(m + \log n)$$

Analysis: continued.

$\frac{1}{2^m} \le \sum_i w_i \le \left(\frac{3}{4}\right)^M n.$

m - best expert mistakes M algorithm mistakes.

$\frac{1}{2^m} \le \left(\frac{3}{4}\right)^M n.$

Take log of both sides.

$-m \le -M \log(4/3) + logn.$

Solve for M.

$M \le (m + logn)/\log(4/3) \le 2.4(m + \log n)$

Multiple by $1 - \varepsilon$ for incorrect experts...

Analysis: continued.

$$\frac{1}{2^m} \leq \sum_i w_i \leq \left(\frac{3}{4}\right)^M n.$$

m - best expert mistakes M algorithm mistakes.

$$\frac{1}{2^m} \leq \left(\frac{3}{4}\right)^M n.$$

Take log of both sides.

$$-m \leq -M \log(4/3) + logn.$$

Solve for M.

$$M \leq (m + logn)/\log(4/3) \leq 2.4(m + \log n)$$

Multiple by $1 - \varepsilon$ for incorrect experts...

$$(1-\varepsilon)^m \leq \left(1 - \frac{\varepsilon}{2}\right)^M n.$$

Analysis: continued.

$$\frac{1}{2^m} \leq \sum_i w_i \leq \left(\frac{3}{4}\right)^M n.$$

m - best expert mistakes M algorithm mistakes.

$$\frac{1}{2^m} \leq \left(\frac{3}{4}\right)^M n.$$

Take log of both sides.

$$-m \leq -M \log(4/3) + logn.$$

Solve for M.

$$M \leq (m + logn)/\log(4/3) \leq 2.4(m + \log n)$$

Multiple by $1 - \varepsilon$ for incorrect experts...

$$(1-\varepsilon)^m \leq \left(1 - \frac{\varepsilon}{2}\right)^M n.$$

Massage...

Analysis: continued.

$$\frac{1}{2^m} \leq \sum_i w_i \leq \left(\frac{3}{4}\right)^M n.$$

m - best expert mistakes M algorithm mistakes.

$$\frac{1}{2^m} \leq \left(\frac{3}{4}\right)^M n.$$

Take log of both sides.

$$-m \leq -M \log(4/3) + logn.$$

Solve for M.

$$M \leq (m + logn)/\log(4/3) \leq 2.4(m + \log n)$$

Multiple by $1-\varepsilon$ for incorrect experts...

$$(1-\varepsilon)^m \leq \left(1 - \frac{\varepsilon}{2}\right)^M n.$$

Massage...

$$M \leq 2(1+\varepsilon)m + \frac{2\ln n}{\varepsilon}$$

Analysis: continued.

$$\frac{1}{2^m} \le \sum_i w_i \le \left(\frac{3}{4}\right)^M n.$$

m - best expert mistakes M algorithm mistakes.

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Approaches a factor of two of best expert performance!

Best Analysis?

Two experts: A,B

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Bad example?

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Which is worse?

(A) A right on even, B right on odd.

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Factor of (almost) two worse!

Randomization

Better approach?

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Proof Idea: $\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \cdots$

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Losses in $[0, 1]$.

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Expert i loses $\ell_i^t \in [0,1]$ in round t.

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$$(1-\varepsilon)^{L^*} \leq W(T) \leq n \;\; \prod_t (1-\varepsilon L_t)$$

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No factor of 2 loss!

Gains.

Why so negative?

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Multiplicative weights with $(1+\varepsilon)^{g_i^t}$.

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Not $[0,1]$, say $[0,\rho]$.

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Multiplicative weights framework!

Applications next!