Today

Quickly: Matrix View, Taking Dual.

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Matching, algebra, geometry.

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Facility Location.

Rules for School...

or..."Rules for taking duals"

Canonical Form.

<u>Primal LP</u>	<u>Dual LP</u>
max $c \cdot x$	min $y^T b$
$Ax \leq b$	$y^T A \geq c$
$x \geq 0$	$y \geq 0$

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"inequalities" $\leftrightarrow$ "nonnegative variables"

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Another useful trick: Equality constraints.

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"inequalities" $\leftrightarrow$ "nonnegative variables"

"nonnegative variables" $\leftrightarrow$ "inequalities"

Another useful trick: Equality constraints. "equalities" $\leftrightarrow$ "unrestricted variables."

Maximum Weight Matching.

Bipartite Graph $G = (V,E)$, $w : E \rightarrow Z$.

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Find maximum weight perfect matching.

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$$\forall v : \sum_{e=(u,v)} x_e = 1$$

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Variable for each constraint.

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 Variable for each constraint. p_v unrestricted.
 Constraint for each variable.

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Objective function from right hand side.

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Weak duality?

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$\sum_{e \in M} w_e x_e \leq \sum_{e=(u,v) \in M} p_u + p_v \leq \sum_v p_u$.

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Strong Duality?

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Weak duality? Price function upper bounds matching.
$\sum_{e \in M} w_e x_e \leq \sum_{e=(u,v) \in M} p_u + p_v \leq \sum_v p_u$.

Strong Duality? Same value solutions.

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Strong Duality? Same value solutions. Hungarian algorithm

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Strong Duality? Same value solutions. Hungarian algorithm !!!

Matrix View.

x_e variable for $e = (u, v)$.

			x_e		rhs
	$\cdot$	$\cdots$	0	$\cdots$	1
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	1
p_u	$\cdot$	$\cdots$	1	$\cdots$	1
	$\cdot$	$\cdots$	0	$\cdots$	1
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	1
	$\cdot$	$\cdots$	0	$\cdots$	1
p_v	$\cdot$	$\cdots$	1	$\cdots$	1
	$\cdot$	$\cdots$	0	$\cdots$	1
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	1
obj	$\cdot$	$\cdot$	w_e	$\cdot$	

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	$\cdot$	$\cdots$	0	$\cdots$	1
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Row equation: $\sum_{e=(u,v)} x_e = 1$.

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Row equation: $\sum_{e=(u,v)} x_e = 1$. Row (dual) variable:

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Column variable: x_e.

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Row equation: $\sum_{e=(u,v)} x_e = 1$. Row (dual) variable: p_u.

Column variable: x_e. Column (dual) constraint: $p_u + p_v \geq 1$.

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Exercise: objectives?

Complementary Slackness.

$$\max \sum_{e} w_e x_e$$

$$\forall v : \sum_{e=(u,v)} x_e = 1 \qquad\qquad p_v$$

$$x_e \geq 0$$

Dual:

$$\min \textstyle\sum_v p_v$$

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Complementary slackness:
Only match on tight edges.
Nonzero p_u on matched u.

Multicommodity Flow.

Given $G = (V, E)$, and capacity function $c : E \rightarrow Z$, and pairs $(s_1, t_1), \ldots, (s_k, t_k)$ with demands $d_1, \ldots, d_k$.

Multicommodity Flow.

Given $G = (V, E)$, and capacity function $c : E \rightarrow Z$, and pairs $(s_1, t_1), \ldots, (s_k, t_k)$ with demands $d_1, \ldots, d_k$.
Route D_i flow for each s_i, t_i pair, so every edge has $\leq \mu c(e)$ flow with minimum μ.

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variables: f_p flow on path p.
P_i -set of paths with endpoints s_i, t_i.

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$$\min \mu$$
$$\forall e : \sum_{p \ni e} f_p \leq \mu c_e$$
$$\forall i : \sum_{p \in P_i} f_p = D_i$$
$$f_p \geq 0$$

Take the dual.

$$\min \mu$$
$$\forall e : \sum_{p \ni e} f_p \leq \mu c_e$$
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Modify to make it $\geq$, which “go with min.

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And only constants on right hand side.

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$$\forall e : \mu c_e - \sum_{p \ni e} f_p \geq 0$$
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Dual.

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Introduce variable for each constraint.

Dual.

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Introduce variable for each constraint.
Introduce constraint for each var:

Dual.

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μ

Dual.

$$\min \mu$$

$$\forall e : \mu c_e - \sum_{p \ni e} f_p \geq 0 \qquad d_e$$

$$\forall i : \sum_{p \in P_i} f_p = D_i \qquad d_i$$

$$f_p \geq 0$$

Introduce variable for each constraint.
Introduce constraint for each var:
$\mu \rightarrow \sum_e c_e d_e = 1$.

Dual.

$$\min \mu$$

$$\forall e: \mu c_e - \sum_{p \ni e} f_p \geq 0 \qquad d_e$$

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Introduce variable for each constraint.
Introduce constraint for each var:
$\mu \; \rightarrow \sum_e c_e d_e = 1$. $\; f_p \; \rightarrow \forall p \in P_i \; d_i - \sum_{e \in p} d_e \leq 0$.

Dual.

$$\min \mu$$

$$\forall e : \mu c_e - \sum_{p \ni e} f_p \geq 0 \qquad d_e$$

$$\forall i : \sum_{p \in P_i} f_p = D_i \qquad d_i$$

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Dual.

$$\min \mu$$

$$\forall e : \mu c_e - \sum_{p \ni e} f_p \geq 0 \qquad d_e$$

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$\mu \rightarrow \sum_e c_e d_e = 1$. $f_p \rightarrow \forall p \in P_i \; d_i - \sum_{e \in p} d_e \leq 0$.
Objective: right hand sides.

Dual.

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Objective: right hand sides. $\max \sum_i D_i d_i$

$$\max \sum_i D_i d_i$$

$$\forall p \in P_i : d_i \leq \sum_{e \in p} d(e) \qquad \sum_e c_e d_e = 1$$

Dual.

$$\min \mu$$

$$\forall e : \mu c_e - \sum_{p \ni e} f_p \geq 0 \qquad d_e$$

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d_i - shortest s_i, t_i path length.

Dual.

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$$\forall e: \mu c_e - \sum_{p \ni e} f_p \geq 0 \qquad d_e$$

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$$\max \sum_i D_i d_i$$

$$\forall p \in P_i: d_i \leq \sum_{e \in p} d(e) \qquad \sum_e c_e d_e = 1$$

d_i - shortest s_i, t_i path length. Toll problem!

Dual.

$$\min \mu$$

$$\forall e : \mu c_e - \sum_{p \ni e} f_p \geq 0 \qquad d_e$$

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d_i - shortest s_i, t_i path length. Toll problem!
Weak duality: toll lower bounds routing.

Dual.

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$$\forall e : \mu c_e - \sum_{p \ni e} f_p \geq 0 \qquad d_e$$

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d_i - shortest s_i, t_i path length. Toll problem!
Weak duality: toll lower bounds routing.
Strong Duality.

Dual.

$$\min \mu$$

$$\forall e : \mu c_e - \sum_{p \ni e} f_p \geq 0 \qquad d_e$$

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d_i - shortest s_i, t_i path length. Toll problem!
  Weak duality: toll lower bounds routing.
  Strong Duality. Tight lower bound.

Dual.

$$\min \mu$$

$$\forall e : \mu c_e - \sum_{p \ni e} f_p \geq 0 \qquad d_e$$

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d_i - shortest s_i, t_i path length. Toll problem!
Weak duality: toll lower bounds routing.
Strong Duality. Tight lower bound. First lecture.

Dual.

$$\min \mu$$

$$\forall e : \mu c_e - \sum_{p \ni e} f_p \geq 0 \qquad d_e$$

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d_i - shortest s_i, t_i path length. Toll problem!
Weak duality: toll lower bounds routing.
Strong Duality. Tight lower bound. First lecture. Or Experts.

Dual.

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$$\forall p \in P_i : d_i \leq \sum_{e \in p} d(e) \qquad \sum_e c_e d_e = 1$$

d_i - shortest s_i, t_i path length. Toll problem!

  Weak duality: toll lower bounds routing.
  Strong Duality. Tight lower bound. First lecture. Or Experts.

Complementary Slackness:

Dual.

$$\min \mu$$

$$\forall e : \mu c_e - \sum_{p \ni e} f_p \geq 0 \qquad d_e$$

$$\forall i : \sum_{p \in P_i} f_p = D_i \qquad d_i$$

$$f_p \geq 0$$

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Objective: right hand sides. $\max \sum_i D_i d_i$

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$$\forall p \in P_i : d_i \leq \sum_{e \in p} d(e) \qquad \sum_e c_e d_e = 1$$

d_i - shortest s_i, t_i path length. Toll problem!
Weak duality: toll lower bounds routing.
Strong Duality. Tight lower bound. First lecture. Or Experts.
Complementary Slackness: only route on shortest paths

Dual.

$$\min \mu$$

$$\forall e : \mu c_e - \sum_{p \ni e} f_p \geq 0 \qquad d_e$$

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$$\max \sum_i D_i d_i$$

$$\forall p \in P_i : d_i \leq \sum_{e \in p} d(e) \qquad \sum_e c_e d_e = 1$$

d_i - shortest s_i, t_i path length. Toll problem!
Weak duality: toll lower bounds routing.
Strong Duality. Tight lower bound. First lecture. Or Experts.
Complementary Slackness: only route on shortest paths
only have toll on congested edges.

Matrix View

f_p variable for path $e_1, e_2, \ldots, e_k$. p connects s_i, t_i.

			f_p		μ	rhs
	$\cdot$	$\cdots$	0	$\cdots$	$\cdot$	0
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	0
d_{e_1}	$\cdot$	$\cdots$	-1	$\cdots$	c_{e_1}	0
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	0
d_{e_2}	$\cdot$	$\cdots$	-1	$\cdots$	c_{e_2}	0
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	0
d_{e_k}	$\cdot$	$\cdots$	-1	$\cdots$	c_{e_k}	0
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	0
d_i	$\cdot$	$\cdot$	1	$\cdot$	$\cdots$	D_i
obj	1	1	1	1		

Row constraint: $c_e\mu - \sum_{p \ni e} f_p \geq 0$.

Matrix View

f_p variable for path $e_1, e_2, \ldots, e_k$. p connects s_i, t_i.

			f_p		μ	rhs
	$\cdot$	$\cdots$	0	$\cdots$	$\cdot$	0
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	0
d_{e_1}	$\cdot$	$\cdots$	-1	$\cdots$	c_{e_1}	0
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	0
d_{e_2}	$\cdot$	$\cdots$	-1	$\cdots$	c_{e_2}	0
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	0
d_{e_k}	$\cdot$	$\cdots$	-1	$\cdots$	c_{e_k}	0
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	0
d_i	$\cdot$	$\cdot$	1	$\cdot$	$\cdots$	D_i
obj	1	1	1	1		

Row constraint: $c_e\mu - \sum_{p \ni e} f_p \geq 0$. Row (dual) variable: d_e.

Matrix View

f_p variable for path $e_1, e_2, \ldots, e_k$. p connects s_i, t_i.

			f_p		μ	rhs
	$\cdot$	$\cdots$	0	$\cdots$	$\cdot$	0
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	0
d_{e_1}	$\cdot$	$\cdots$	-1	$\cdots$	c_{e_1}	0
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	0
d_{e_2}	$\cdot$	$\cdots$	-1	$\cdots$	c_{e_2}	0
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	0
d_{e_k}	$\cdot$	$\cdots$	-1	$\cdots$	c_{e_k}	0
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	0
d_i	$\cdot$	$\cdot$	1	$\cdot$	$\cdots$	D_i
obj	1	1	1	1		

Row constraint: $c_e\mu - \sum_{p \ni e} f_p \geq 0$. Row (dual) variable: d_e.

Row constraint: $\sum_{p \in P_i} f_p = D_i$.

Matrix View

f_p variable for path $e_1, e_2, \ldots, e_k$. p connects s_i, t_i.

			f_p		μ	rhs
	$\cdot$	$\cdots$	0	$\cdots$	$\cdot$	0
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	0
d_{e_1}	$\cdot$	$\cdots$	-1	$\cdots$	c_{e_1}	0
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	0
d_{e_2}	$\cdot$	$\cdots$	-1	$\cdots$	c_{e_2}	0
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	0
d_{e_k}	$\cdot$	$\cdots$	-1	$\cdots$	c_{e_k}	0
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	0
d_i	$\cdot$	$\cdot$	1	$\cdot$	$\cdots$	D_i
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			f_p		μ	rhs
	$\cdot$	$\cdots$	0	$\cdots$	$\cdot$	0
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	0
d_{e_1}	$\cdot$	$\cdots$	-1	$\cdots$	c_{e_1}	0
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	0
d_{e_2}	$\cdot$	$\cdots$	-1	$\cdots$	c_{e_2}	0
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	0
d_{e_k}	$\cdot$	$\cdots$	-1	$\cdots$	c_{e_k}	0
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	0
d_i	$\cdot$	$\cdot$	1	$\cdot$	$\cdots$	D_i
obj	1	1	1	1		

Row constraint: $c_e \mu - \sum_{p \ni e} f_p \geq 0$. Row (dual) variable: d_e.

Row constraint: $\sum_{p \in P_i} f_p = D_i$. Row (dual) variable: d_i.

Column variable: f_p.

Matrix View

f_p variable for path $e_1, e_2, \ldots, e_k$. p connects s_i, t_i.

			f_p		μ	rhs
	$\cdot$	$\cdots$	0	$\cdots$	$\cdot$	0
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	0
d_{e_1}	$\cdot$	$\cdots$	-1	$\cdots$	c_{e_1}	0
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	0
d_{e_2}	$\cdot$	$\cdots$	-1	$\cdots$	c_{e_2}	0
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	0
d_{e_k}	$\cdot$	$\cdots$	-1	$\cdots$	c_{e_k}	0
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	0
d_i	$\cdot$	$\cdot$	1	$\cdot$	$\cdots$	D_i
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Row constraint: $c_e\mu - \sum_{p \ni e} f_p \geq 0$. Row (dual) variable: d_e.

Row constraint: $\sum_{p \in P_i} f_p = D_i$. Row (dual) variable: d_i.

Column variable: f_p. Column (dual) constraint:

Matrix View

f_p variable for path $e_1, e_2, \ldots, e_k$. p connects s_i, t_i.

			f_p		μ	rhs
	$\cdot$	$\cdots$	0	$\cdots$	$\cdot$	0
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	0
d_{e_1}	$\cdot$	$\cdots$	-1	$\cdots$	c_{e_1}	0
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	0
d_{e_2}	$\cdot$	$\cdots$	-1	$\cdots$	c_{e_2}	0
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	0
d_{e_k}	$\cdot$	$\cdots$	-1	$\cdots$	c_{e_k}	0
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	0
d_i	$\cdot$	$\cdot$	1	$\cdot$	$\cdots$	D_i
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Row constraint: $\sum_{p \in P_i} f_p = D_i$. Row (dual) variable: d_i.

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	$\cdot$	$\cdots$	0	$\cdots$	$\cdot$	0
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	0
d_{e_1}	$\cdot$	$\cdots$	-1	$\cdots$	c_{e_1}	0
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	0
d_{e_2}	$\cdot$	$\cdots$	-1	$\cdots$	c_{e_2}	0
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	0
d_{e_k}	$\cdot$	$\cdots$	-1	$\cdots$	c_{e_k}	0
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	0
d_i	$\cdot$	$\cdot$	1	$\cdot$	$\cdots$	D_i
obj	1	1	1	1		

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Column variable: μ.

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			f_p		μ	rhs
	$\cdot$	$\cdots$	0	$\cdots$	$\cdot$	0
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	0
d_{e_1}	$\cdot$	$\cdots$	-1	$\cdots$	c_{e_1}	0
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	0
d_{e_2}	$\cdot$	$\cdots$	-1	$\cdots$	c_{e_2}	0
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	0
d_{e_k}	$\cdot$	$\cdots$	-1	$\cdots$	c_{e_k}	0
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	0
d_i	$\cdot$	$\cdot$	1	$\cdot$	$\cdots$	D_i
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			f_p		μ	rhs
	$\cdot$	$\cdots$	0	$\cdots$	$\cdot$	0
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	0
d_{e_1}	$\cdot$	$\cdots$	-1	$\cdots$	c_{e_1}	0
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	0
d_{e_2}	$\cdot$	$\cdots$	-1	$\cdots$	c_{e_2}	0
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	0
d_{e_k}	$\cdot$	$\cdots$	-1	$\cdots$	c_{e_k}	0
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	0
d_i	$\cdot$	$\cdot$	1	$\cdot$	$\cdots$	D_i
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Row constraint: $c_e\mu - \sum_{p \ni e} f_p \geq 0$. Row (dual) variable: d_e.

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Column variable: f_p. Column (dual) constraint: $d_i - \sum_{e \in p} d_e \leq 0$.

Column variable: μ. Column (dual) constraint: $\sum_e d(e)c(e) = 1$.

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			f_p		μ	rhs
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	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	0
d_{e_1}	$\cdot$	$\cdots$	-1	$\cdots$	c_{e_1}	0
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	0
d_{e_2}	$\cdot$	$\cdots$	-1	$\cdots$	c_{e_2}	0
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	0
d_{e_k}	$\cdot$	$\cdots$	-1	$\cdots$	c_{e_k}	0
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	0
d_i	$\cdot$	$\cdot$	1	$\cdot$	$\cdots$	D_i
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Column variable: μ. Column (dual) constraint: $\sum_e d(e)c(e) = 1$.

Exercise: objectives?

Exponential size.

Multicommodity flow.

$$\min \mu$$

$$\forall e : \mu c_e - \sum_{p \ni e} f_p \geq 0$$

$$\forall i : \sum_{p \in P_i} f_p = d_i$$

$$f_p \geq 0$$

Exponential size.

Multicommodity flow.

$$\min \mu$$

$$\forall e : \mu c_e - \sum_{p \ni e} f_p \geq 0$$

$$\forall i : \sum_{p \in P_i} f_p = d_i$$

$$f_p \geq 0$$

Dual is.

$$\max \sum_i D_i d_i$$

$$\forall p \in P_i : d_i \leq \sum_{e \in p} d(e)$$

Exponential size.

Multicommodity flow.

$$\min \mu$$

$$\forall e : \mu c_e - \sum_{p \ni e} f_p \geq 0$$

$$\forall i : \sum_{p \in P_i} f_p = d_i$$

$$f_p \geq 0$$

Dual is.

$$\max \sum_i D_i d_i$$

$$\forall p \in P_i : d_i \leq \sum_{e \in p} d(e)$$

Exponential sized programs?

Exponential size.

Multicommodity flow.

$$\min \mu$$
$$\forall e : \mu c_e - \sum_{p \ni e} f_p \geq 0$$
$$\forall i : \sum_{p \in P_i} f_p = d_i$$
$$f_p \geq 0$$

Dual is.

$$\max \sum_i D_i d_i$$
$$\forall p \in P_i : d_i \leq \sum_{e \in p} d(e)$$

Exponential sized programs?

Answer 1:

Exponential size.

Multicommodity flow.

$$\min \mu$$
$$\forall e : \mu c_e - \sum_{p \ni e} f_p \geq 0$$
$$\forall i : \sum_{p \in P_i} f_p = d_i$$
$$f_p \geq 0$$

Dual is.

$$\max \sum_i D_i d_i$$
$$\forall p \in P_i : d_i \leq \sum_{e \in p} d(e)$$

Exponential sized programs?

Answer 1: We solved anyway!

Exponential size.

Multicommodity flow.

$$\min \mu$$
$$\forall e : \mu c_e - \sum_{p \ni e} f_p \geq 0$$
$$\forall i : \sum_{p \in P_i} f_p = d_i$$
$$f_p \geq 0$$

Dual is.

$$\max \sum_i D_i d_i$$
$$\forall p \in P_i : d_i \leq \sum_{e \in p} d(e)$$

Exponential sized programs?

Answer 1: We solved anyway!

Answer 2:

Exponential size.

Multicommodity flow.

$$\min \mu$$
$$\forall e : \mu c_e - \sum_{p \ni e} f_p \geq 0$$
$$\forall i : \sum_{p \in P_i} f_p = d_i$$
$$f_p \geq 0$$

Dual is.

$$\max \sum_i D_i d_i$$
$$\forall p \in P_i : d_i \leq \sum_{e \in p} d(e)$$

Exponential sized programs?

Answer 1: We solved anyway!

Answer 2: Ellipsoid algorithm.

Exponential size.

Multicommodity flow.

$$\min \mu$$
$$\forall e : \mu c_e - \sum_{p \ni e} f_p \geq 0$$
$$\forall i : \sum_{p \in P_i} f_p = d_i$$
$$f_p \geq 0$$

Dual is.

$$\max \sum_i D_i d_i$$
$$\forall p \in P_i : d_i \leq \sum_{e \in p} d(e)$$

Exponential sized programs?

Answer 1: We solved anyway!

Answer 2: Ellipsoid algorithm.
Find violated constraint $\rightarrow$ poly time algorithm.

Exponential size.

Multicommodity flow.

$$\min \mu$$
$$\forall e : \mu c_e - \sum_{p \ni e} f_p \geq 0$$
$$\forall i : \sum_{p \in P_i} f_p = d_i$$
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Dual is.

$$\max \sum_i D_i d_i$$
$$\forall p \in P_i : d_i \leq \sum_{e \in p} d(e)$$

Exponential sized programs?

Answer 1: We solved anyway!

Answer 2: Ellipsoid algorithm.
Find violated constraint $\rightarrow$ poly time algorithm.

Answer 3: there is polynomial sized formulation.

Exponential size.

Multicommodity flow.

$$\min \mu$$
$$\forall e : \mu c_e - \sum_{p \ni e} f_p \geq 0$$
$$\forall i : \sum_{p \in P_i} f_p = d_i$$
$$f_p \geq 0$$

Dual is.

$$\max \sum_i D_i d_i$$
$$\forall p \in P_i : d_i \leq \sum_{e \in p} d(e)$$

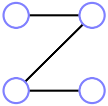
Exponential sized programs?

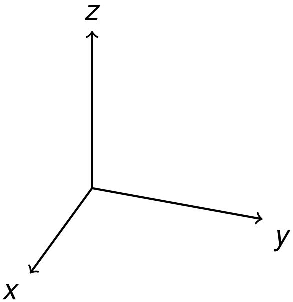
Answer 1: We solved anyway!

Answer 2: Ellipsoid algorithm.
Find violated constraint $\rightarrow$ poly time algorithm.

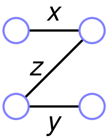
Answer 3: there is polynomial sized formulation.
Question: what is it?

Maximum matching and simplex.

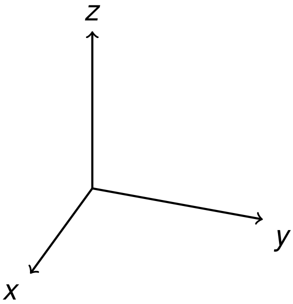




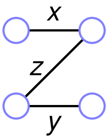
Maximum matching and simplex.



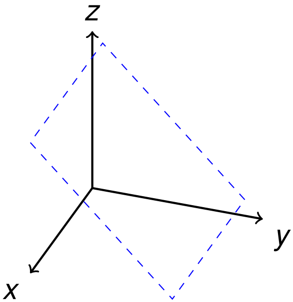
$$\max x + y + z$$
$$x \leq 1$$
$$x + z \leq 1$$
$$z + y \leq 1$$
$$y \leq 1$$
$$x \geq 0$$
$$y \geq 0$$
$$z \geq 0$$



Maximum matching and simplex.

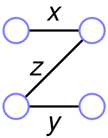


$$\max x + y + z$$
$$x \leq 1$$
$$x + z \leq 1$$
$$z + y \leq 1$$
$$y \leq 1$$
$$x \geq 0$$
$$y \geq 0$$
$$z \geq 0$$

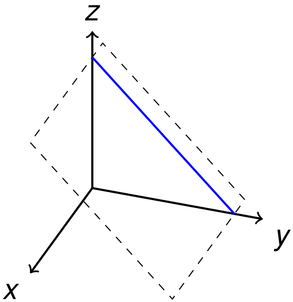


Blue constraints intersect.

Maximum matching and simplex.

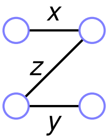


$$
\begin{aligned}
\max x + y + z \\
x \leq 1 \\
x + z \leq 1 \\
{\color{blue} z + y \leq 1} \\
y \leq 1 \\
{\color{blue} x \geq 0} \\
y \geq 0 \\
z \geq 0
\end{aligned}
$$



Blue constraints intersect.

Maximum matching and simplex.



$$\max x + y + z$$
$$x \leq 1$$
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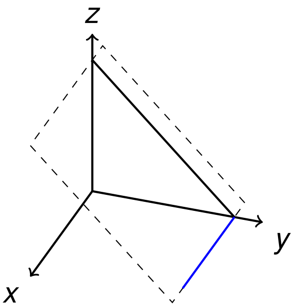
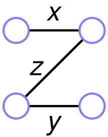
$$z + y \leq 1$$
$$y \leq 1$$
$$x \geq 0$$
$$y \geq 0$$
$$z \geq 0$$



Blue constraints intersect.

Maximum matching and simplex.



$$\max x + y + z$$
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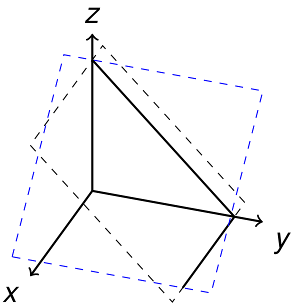
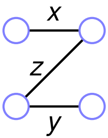
$$x + z \leq 1$$
$$z + y \leq 1$$
$$y \leq 1$$
$$x \geq 0$$
$$y \geq 0$$
$$z \geq 0$$

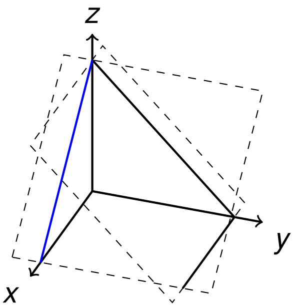


Blue constraints intersect.

Maximum matching and simplex.

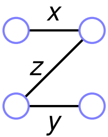


$$\max x + y + z$$
$$x \leq 1$$
$$x + z \leq 1$$
$$z + y \leq 1$$
$$y \leq 1$$
$$x \geq 0$$
$$y \geq 0$$
$$z \geq 0$$

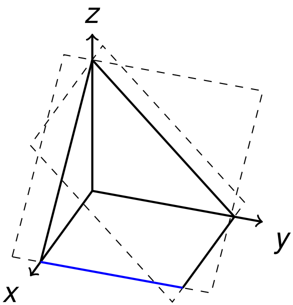


Blue constraints intersect.

Maximum matching and simplex.

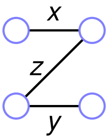


$$\max x + y + z$$
$$x \le 1$$
$$x + z \le 1$$
$$z + y \le 1$$
$$y \le 1$$
$$x \ge 0$$
$$y \ge 0$$
$$z \ge 0$$

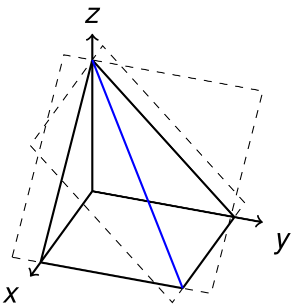


Blue constraints intersect.

Maximum matching and simplex.

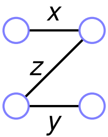


$$\max x + y + z$$
$$x \leq 1$$
$$x + z \leq 1$$
$$z + y \leq 1$$
$$y \leq 1$$
$$x \geq 0$$
$$y \geq 0$$
$$z \geq 0$$

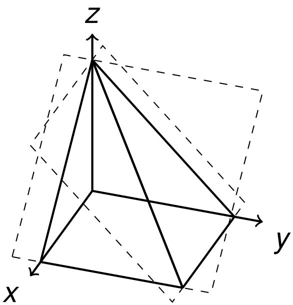


Blue constraints intersect.

Maximum matching and simplex.

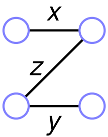


$$
\begin{aligned}
\max x + y + z \\
x \le 1 \\
x + z \le 1 \\
z + y \le 1 \\
y \le 1 \\
x \ge 0 \\
y \ge 0 \\
z \ge 0
\end{aligned}
$$

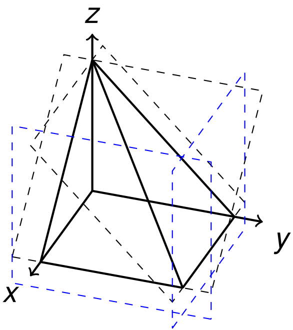


Blue constraints intersect.

Maximum matching and simplex.

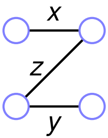


$$\max x + y + z$$
$$x \leq 1$$
$$x + z \leq 1$$
$$z + y \leq 1$$
$$y \leq 1$$
$$x \geq 0$$
$$y \geq 0$$
$$z \geq 0$$



Blue constraints redundant.

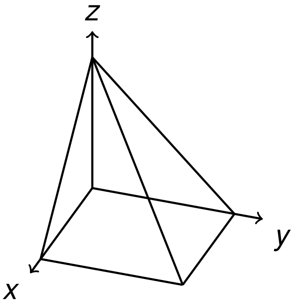
Maximum matching and simplex.



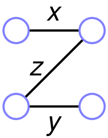
$$\max x + y + z$$

$$x + z \leq 1$$
$$z + y \leq 1$$

$$x \geq 0$$
$$y \geq 0$$
$$z \geq 0$$



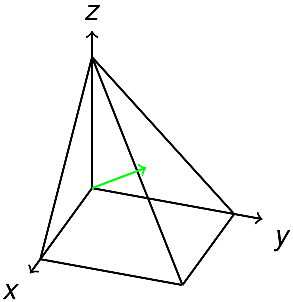
Maximum matching and simplex.



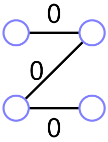
$$\max x + y + z$$

$$x + z \leq 1$$
$$z + y \leq 1$$

$$x \geq 0$$
$$y \geq 0$$
$$z \geq 0$$



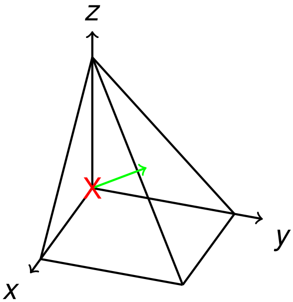
Maximum matching and simplex.



$$\max x + y + z$$

$$x + z \leq 1$$
$$z + y \leq 1$$

$$x \geq 0$$
$$y \geq 0$$
$$z \geq 0$$

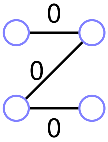


Blue constraints tight.

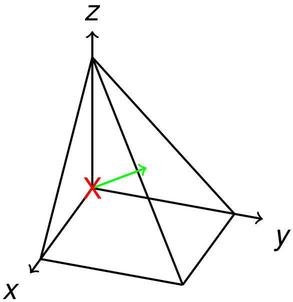
Maximum matching and simplex.

$$\max x + y + z$$

$$x + z \leq 1$$
$$z + y \leq 1$$

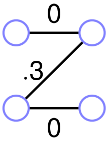


$$x \geq 0$$
$$y \geq 0$$
$$z \geq 0$$



Blue constraints tight.

Maximum matching and simplex.



$\max x + y + z$

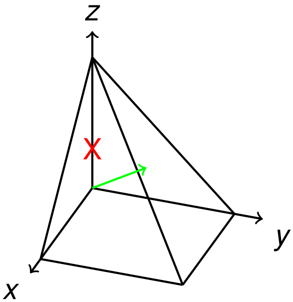
$x + z \leq 1$

$z + y \leq 1$

$x \geq 0$

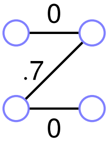
$y \geq 0$

$z \geq 0$



Blue constraints tight.

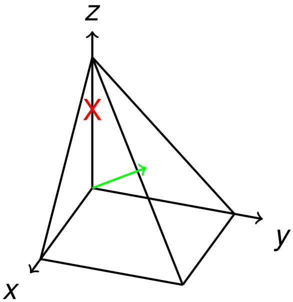
Maximum matching and simplex.



$$\max x + y + z$$

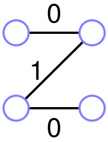
$$x + z \leq 1$$
$$z + y \leq 1$$

$$\color{blue}{x \geq 0}$$
$$\color{blue}{y \geq 0}$$
$$z \geq 0$$



Blue constraints tight.

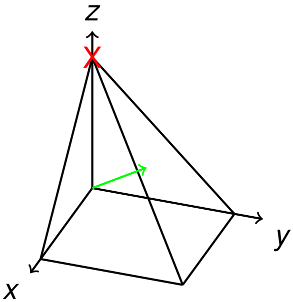
Maximum matching and simplex.



$$\max x + y + z$$

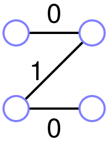
$$x + z \leq 1$$
$$z + y \leq 1$$

$$x \geq 0$$
$$y \geq 0$$
$$z \geq 0$$



Blue constraints tight.

Maximum matching and simplex.



$$\max x + y + z$$

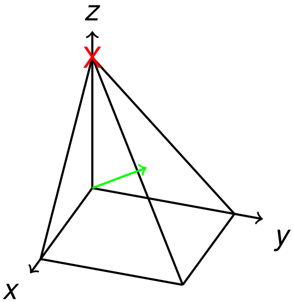
$$x + z \leq 1$$

$$z + y \leq 1$$

$$x \geq 0$$

$$y \geq 0$$

$$z \geq 0$$

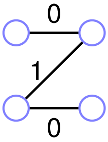


Blue constraints tight.

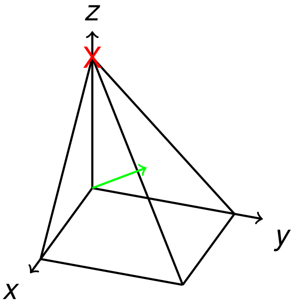
Maximum matching and simplex.

$$\max x + y + z$$

$$x + z \leq 1$$
$$z + y \leq 1$$

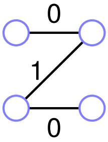


$$x \geq 0$$
$$y \geq 0$$
$$z \geq 0$$



Blue constraints tight.

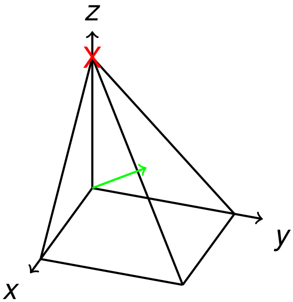
Maximum matching and simplex.



$$\max x + y + z$$

$$x + z \leq 1$$
$$z + y \leq 1$$

$$x \geq 0$$
$$y \geq 0$$
$$z \geq 0$$

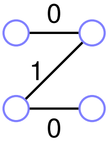


Blue constraints tight.

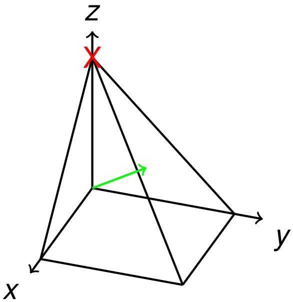
Maximum matching and simplex.

$$\max x + y + z$$

$$x + z \leq 1$$
$$z + y \leq 1$$



$$x \geq 0$$
$$y \geq 0$$
$$z \geq 0$$



Blue constraints tight.

Maximum matching and simplex.

$$\max x + y + z$$

$$x + z \leq 1$$
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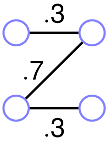
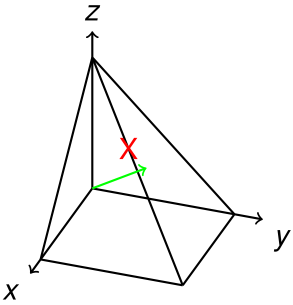
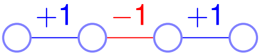


$$x \geq 0$$
$$y \geq 0$$
$$z \geq 0$$



Blue constraints tight.



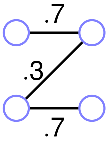
Augmenting Path.

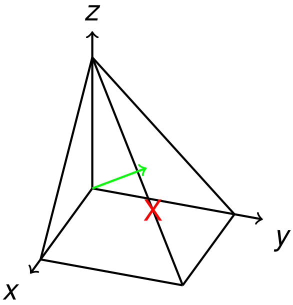
Maximum matching and simplex.

$$\max x + y + z$$

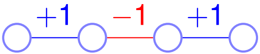
$$x + z \leq 1$$
$$z + y \leq 1$$

$$x \geq 0$$
$$y \geq 0$$
$$z \geq 0$$



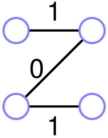


Blue constraints tight.



Augmenting Path. Via Gaussian Elimination!

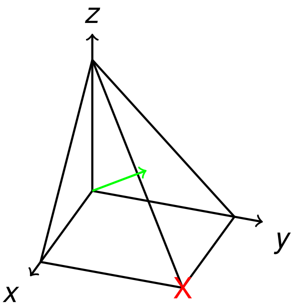
Maximum matching and simplex.



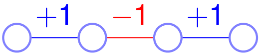
$$\max x + y + z$$

$$x + z \le 1$$
$$z + y \le 1$$

$$x \ge 0$$
$$y \ge 0$$
$$z \ge 0$$

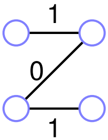


Blue constraints tight.



Augmenting Path. Via Gaussian Elimination!

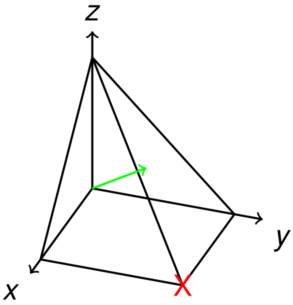
Maximum matching and simplex.



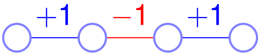
$$\max x + y + z$$

$$x + z \leq 1$$
$$z + y \leq 1$$

$$x \geq 0$$
$$y \geq 0$$
$$z \geq 0$$



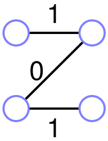
Blue constraints tight.



Augmenting Path. Via Gaussian Elimination!

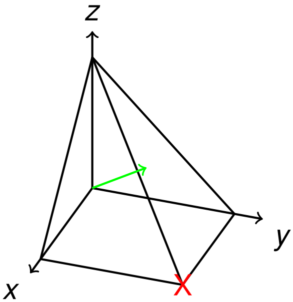
Maximum matching and simplex.

$$\max x + y + z$$

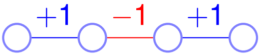


$$x + z \leq 1$$
$$z + y \leq 1$$

$$x \geq 0$$
$$y \geq 0$$
$$z \geq 0$$



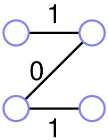
Blue constraints tight.



Augmenting Path. Via Gaussian Elimination!

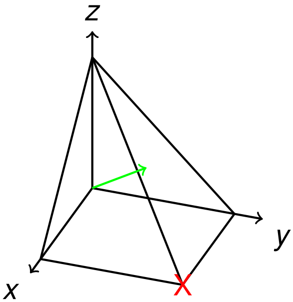
Maximum matching and simplex.

$$\max x + y + z$$

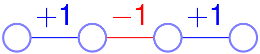


$$x + z \leq 1 \quad a$$
$$z + y \leq 1 \quad b$$

$$x \geq 0$$
$$y \geq 0$$
$$z \geq 0 \quad c$$



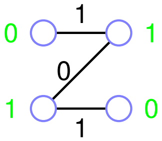
Blue constraints tight.



Augmenting Path. Via Gaussian Elimination!

Maximum matching and simplex.

$$\max x + y + z$$



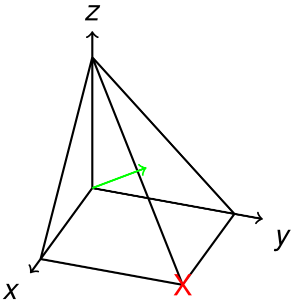
$x + z \leq 1 \quad a = 1$

$z + y \leq 1 \quad b = 1$

$x \geq 0$

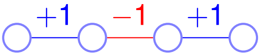
$y \geq 0$

$z \geq 0 \quad c = 1$



Blue constraints tight.

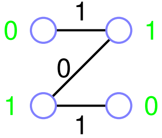
Sum: $x + 2z + y$.



Augmenting Path. Via Gaussian Elimination!

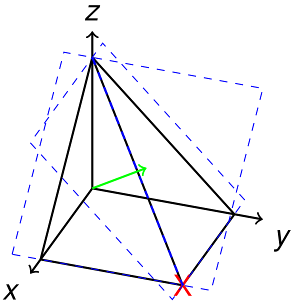
Maximum matching and simplex.

$$\max x + y + z$$

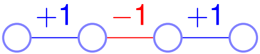


$$x + z \leq 1 \quad a = 1$$
$$z + y \leq 1 \quad b = 1$$

$$x \geq 0$$
$$y \geq 0$$
$$z \geq 0 \quad c = 1$$



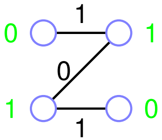
Blue constraints tight.



Augmenting Path. Via Gaussian Elimination!

Maximum matching and simplex.

$$\max x + y + z$$



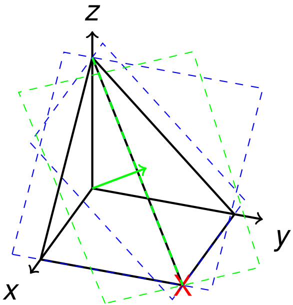
$x + z \leq 1 \quad a = 1$

$z + y \leq 1 \quad b = 1$

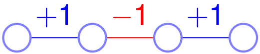
$x \geq 0$

$y \geq 0$

$z \geq 0 \quad c = 1$

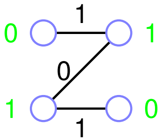


Blue constraints tight.



Augmenting Path. Via Gaussian Elimination!

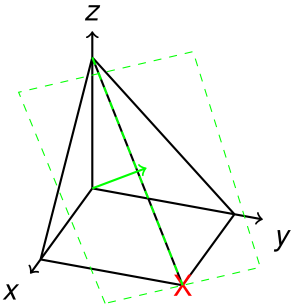
Maximum matching and simplex.



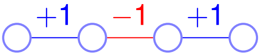
$$\max x + y + z$$

$$\color{blue}{x + z \le 1} \quad a = 1$$
$$\color{blue}{z + y \le 1} \quad b = 1$$

$$x \ge 0$$
$$y \ge 0$$
$$\color{blue}{z \ge 0} \quad c = 1$$



Blue constraints tight.



Augmenting Path. Via Gaussian Elimination!

Facility location

Set of facilities: F, opening cost f_i for facility i

Facility location

Set of facilities: F, opening cost f_i for facility i

Set of clients: D.

Facility location

Set of facilities: F, opening cost f_i for facility i

Set of clients: D.

d_{ij} - distance between i and j.

Facility location

Set of facilities: F, opening cost f_i for facility i

Set of clients: D.

d_{ij} - distance between i and j.
(notation abuse: clients/facility confusion.)

Facility location

Set of facilities: F, opening cost f_i for facility i

Set of clients: D.

d_{ij} - distance between i and j.
(notation abuse: clients/facility confusion.)

Triangle inequality: $d_{ij} \leq d_{ik} + d_{kj}$.

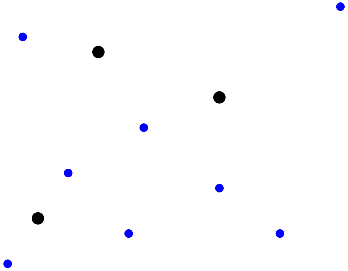
Facility location

Set of facilities: F, opening cost f_i for facility i

Set of clients: D.

d_{ij} - distance between i and j.
(notation abuse: clients/facility confusion.)

Triangle inequality: $d_{ij} \leq d_{ik} + d_{kj}$.



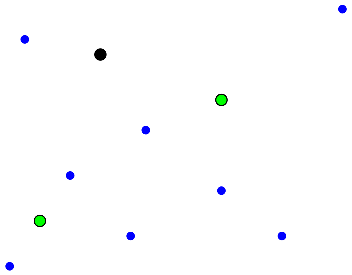
Facility location

Set of facilities: F, opening cost f_i for facility i

Set of clients: D.

d_{ij} - distance between i and j.
(notation abuse: clients/facility confusion.)

Triangle inequality: $d_{ij} \leq d_{ik} + d_{kj}$.



Facility location

Set of facilities: F, opening cost f_i for facility i

Set of clients: D.

d_{ij} - distance between i and j.
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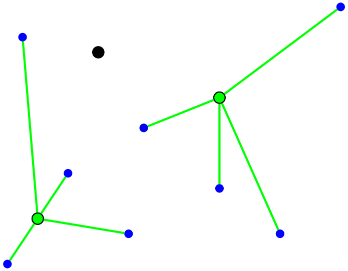
Triangle inequality: $d_{ij} \leq d_{ik} + d_{kj}$.



Facility Location

Linear program relaxation:

Facility Location

Linear program relaxation:

“Decision Variables”.

Facility Location

Linear program relaxation:

"Decision Variables".

y_i - facility i open?

Facility Location

Linear program relaxation:

“Decision Variables”.

y_i - facility i open?

x_{ij} - client j assigned to facility i.

Facility Location

Linear program relaxation:

"Decision Variables".

y_i - facility i open?

x_{ij} - client j assigned to facility i.

Facility Location

Linear program relaxation:

"Decision Variables".

y_i - facility i open?

x_{ij} - client j assigned to facility i.

$$\min \sum_{i \in F} y_i f_i + \sum_{i \in F, j \in D} x_{ij} d_{ij}$$

$$\forall j \in D \quad \sum_{i \in F} x_{ij} \geq 1$$

$$\forall i \in F, j \in D \quad x_{ij} \leq y_i,$$

$$x_{ij}, y_i \geq 0$$

Facility Location

Linear program relaxation:

"Decision Variables".
y_i - facility i open?
x_{ij} - client j assigned to facility i.

$$\min \sum_{i \in F} y_i f_i + \sum_{i \in F, j \in D} x_{ij} d_{ij}$$
$$\forall j \in D \quad \sum_{i \in F} x_{ij} \geq 1$$
$$\forall i \in F, j \in D \quad x_{ij} \leq y_i,$$
$$x_{ij}, y_i \geq 0$$

Facility opening cost.

Facility Location

Linear program relaxation:

"Decision Variables".

y_i - facility i open?

x_{ij} - client j assigned to facility i.

$$\min \sum_{i \in F} y_i f_i + \sum_{i \in F, j \in D} x_{ij} d_{ij}$$

$$\forall j \in D \quad \sum_{i \in F} x_{ij} \geq 1$$

$$\forall i \in F, j \in D \quad x_{ij} \leq y_i,$$

$$x_{ij}, y_i \geq 0$$

Facility opening cost.

Client Connnection cost.

Facility Location

Linear program relaxation:

“Decision Variables”.

y_i - facility i open?

x_{ij} - client j assigned to facility i.

$$\min \sum_{i \in F} y_i f_i + \sum_{i \in F, j \in D} x_{ij} d_{ij}$$

$$\forall j \in D \quad \sum_{i \in F} x_{ij} \geq 1$$

$$\forall i \in F, j \in D \quad x_{ij} \leq y_i,$$

$$x_{ij}, y_i \geq 0$$

Facility opening cost.

Client Connnection cost.

Must connect each client.

Facility Location

Linear program relaxation:

"Decision Variables".
y_i - facility i open?
x_{ij} - client j assigned to facility i.

$$\min \sum_{i \in F} y_i f_i + \sum_{i \in F, j \in D} x_{ij} d_{ij}$$

$$\forall j \in D \quad \sum_{i \in F} x_{ij} \geq 1$$

$$\forall i \in F, j \in D \quad x_{ij} \leq y_i,$$

$$x_{ij}, y_i \geq 0$$

Facility opening cost.
Client Connnection cost.
Must connect each client.
Only connect to open facility.

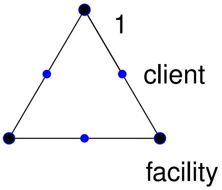
Integer Solution?

$$\min \sum_{i \in F} y_i f_i + \sum_{i \in F, j \in D} x_{ij} d_{ij}$$

$$\forall j \in D \quad \sum_{i \in F} x_{ij} \geq 1$$

$$\forall i \in F, j \in D \quad x_{ij} \leq y_i,$$

$$x_{ij}, y_i \geq 0$$



$x_{ij} = \frac{1}{2}$ edges.
$y_i = \frac{1}{2}$ edges.

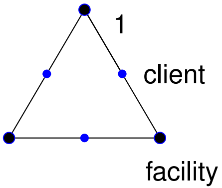
Integer Solution?

$$\min \sum_{i \in F} y_i f_i + \sum_{i \in F, j \in D} x_{ij} d_{ij}$$

$$\forall j \in D \quad \sum_{i \in F} x_{ij} \geq 1$$

$$\forall i \in F, j \in D \quad x_{ij} \leq y_i,$$

$$x_{ij}, y_i \geq 0$$



$x_{ij} = \frac{1}{2}$ edges.
$y_i = \frac{1}{2}$ edges.

Facility Cost: $\frac{3}{2}$

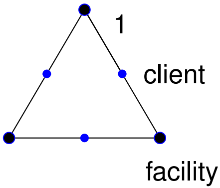
Integer Solution?

$$\min \sum_{i \in F} y_i f_i + \sum_{i \in F, j \in D} x_{ij} d_{ij}$$

$$\forall j \in D \quad \sum_{i \in F} x_{ij} \geq 1$$

$$\forall i \in F, j \in D \quad x_{ij} \leq y_i,$$

$$x_{ij}, y_i \geq 0$$



$x_{ij} = \frac{1}{2}$ edges.
$y_i = \frac{1}{2}$ edges.

Facility Cost: $\frac{3}{2}$ Connection Cost: 3

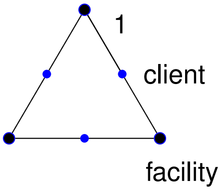
Integer Solution?

$$\min \sum_{i \in F} y_i f_i + \sum_{i \in F, j \in D} x_{ij} d_{ij}$$

$$\forall j \in D \quad \sum_{i \in F} x_{ij} \geq 1$$

$$\forall i \in F, j \in D \quad x_{ij} \leq y_i,$$

$$x_{ij}, y_i \geq 0$$

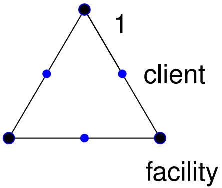


$x_{ij} = \frac{1}{2}$ edges.
$y_i = \frac{1}{2}$ edges.

Facility Cost: $\frac{3}{2}$ Connection Cost: 3
Any one Facility:
Facility Cost: 1

Integer Solution?

$$\min \sum_{i \in F} y_i f_i + \sum_{i \in F, j \in D} x_{ij} d_{ij}$$
$$\forall j \in D \quad \sum_{i \in F} x_{ij} \geq 1$$
$$\forall i \in F, j \in D \quad x_{ij} \leq y_i,$$
$$x_{ij}, y_i \geq 0$$

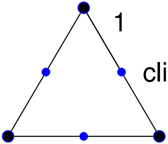


$x_{ij} = \frac{1}{2}$ edges.
$y_i = \frac{1}{2}$ edges.

Facility Cost: $\frac{3}{2}$ Connection Cost: 3
Any one Facility:
Facility Cost: 1 Client Cost: 3.7

Integer Solution?

$$\min \sum_{i \in F} y_i f_i + \sum_{i \in F, j \in D} x_{ij} d_{ij}$$
$$\forall j \in D \quad \sum_{i \in F} x_{ij} \geq 1$$
$$\forall i \in F, j \in D \quad x_{ij} \leq y_i,$$
$$x_{ij}, y_i \geq 0$$



$x_{ij} = \frac{1}{2}$ edges.
$y_i = \frac{1}{2}$ edges.

Facility Cost: $\frac{3}{2}$ Connection Cost: 3
Any one Facility:
Facility Cost: 1 Client Cost: 3.7
Make it worse?

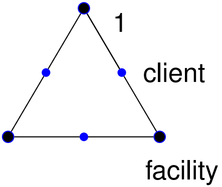
Integer Solution?

$$\min \sum_{i \in F} y_i f_i + \sum_{i \in F, j \in D} x_{ij} d_{ij}$$

$$\forall j \in D \quad \sum_{i \in F} x_{ij} \geq 1$$

$$\forall i \in F, j \in D \quad x_{ij} \leq y_i,$$

$$x_{ij}, y_i \geq 0$$



$x_{ij} = \frac{1}{2}$ edges.
$y_i = \frac{1}{2}$ edges.

Facility Cost: $\frac{3}{2}$ Connection Cost: 3
Any one Facility:
Facility Cost: 1 Client Cost: 3.7
Make it worse? Sure.

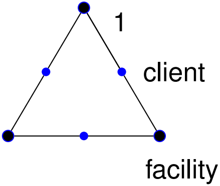
Integer Solution?

$$\min \sum_{i \in F} y_i f_i + \sum_{i \in F, j \in D} x_{ij} d_{ij}$$

$$\forall j \in D \quad \sum_{i \in F} x_{ij} \geq 1$$

$$\forall i \in F, j \in D \quad x_{ij} \leq y_i,$$

$$x_{ij}, y_i \geq 0$$



$x_{ij} = \frac{1}{2}$ edges.
$y_i = \frac{1}{2}$ edges.

Facility Cost: $\frac{3}{2}$ Connection Cost: 3
Any one Facility:
Facility Cost: 1 Client Cost: 3.7
Make it worse? Sure. Not as pretty!

Round solution?

$$\begin{aligned} \min \sum_{i \in F} y_i f_i + \sum_{i \in F, j \in D} x_{ij} d_{ij} \\ \forall j \in D \quad \sum_{i \in F} x_{ij} \geq 1 \\ \forall i \in F, j \in D \quad x_{ij} \leq y_i, \\ x_{ij}, y_i \geq 0 \end{aligned}$$

Round solution?

$$\min \sum_{i \in F} y_i f_i + \sum_{i \in F, j \in D} x_{ij} d_{ij}$$

$$\forall j \in D \quad \sum_{i \in F} x_{ij} \geq 1$$

$$\forall i \in F, j \in D \quad x_{ij} \leq y_i,$$

$$x_{ij}, y_i \geq 0$$

Round independently?

Round solution?

$$\min \sum_{i \in F} y_i f_i + \sum_{i \in F, j \in D} x_{ij} d_{ij}$$
$$\forall j \in D \quad \sum_{i \in F} x_{ij} \geq 1$$
$$\forall i \in F, j \in D \quad x_{ij} \leq y_i,$$
$$x_{ij}, y_i \geq 0$$

Round independently?

y_i and x_{ij} separately?

Round solution?

$$\min \sum_{i \in F} y_i f_i + \sum_{i \in F, j \in D} x_{ij} d_{ij}$$

$$\forall j \in D \quad \sum_{i \in F} x_{ij} \geq 1$$

$$\forall i \in F, j \in D \quad x_{ij} \leq y_i,$$

$$x_{ij}, y_i \geq 0$$

Round independently?

y_i and x_{ij} separately? Assign to closed facility!

Round solution?

$$\min \sum_{i \in F} y_i f_i + \sum_{i \in F, j \in D} x_{ij} d_{ij}$$

$$\forall j \in D \quad \sum_{i \in F} x_{ij} \geq 1$$

$$\forall i \in F, j \in D \quad x_{ij} \leq y_i,$$

$$x_{ij}, y_i \geq 0$$

Round independently?

y_i and x_{ij} separately? Assign to closed facility!

Round x_{ij} and open facilities?

Round solution?

$$\min \sum_{i \in F} y_i f_i + \sum_{i \in F, j \in D} x_{ij} d_{ij}$$
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Round independently?

y_i and x_{ij} separately? Assign to closed facility!

Round x_{ij} and open facilities?
Different clients force different facilities open.

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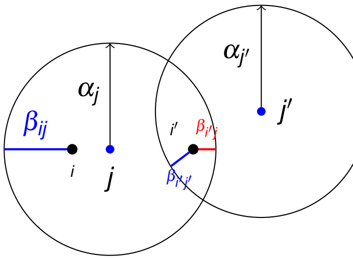

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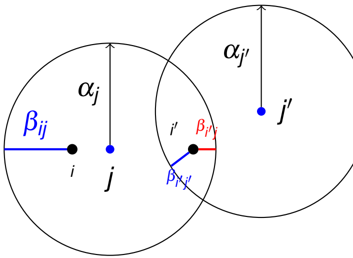

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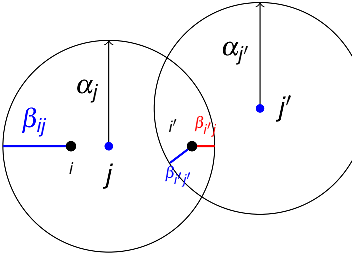

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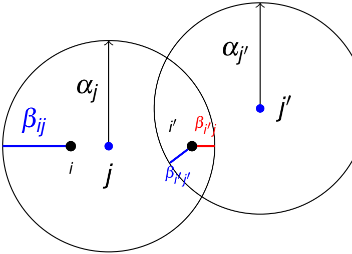

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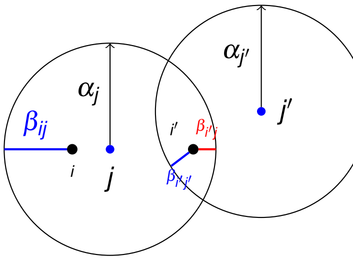
only assign client to "paid to" facilities.



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3. Removed assigned clients, goto 2.

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Claim: Total facility cost is at most $\sum_i f_i y_i$.

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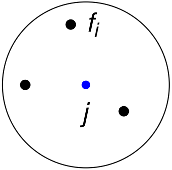

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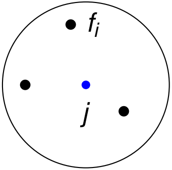

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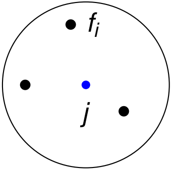

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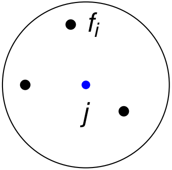

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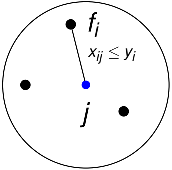

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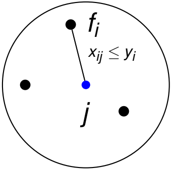

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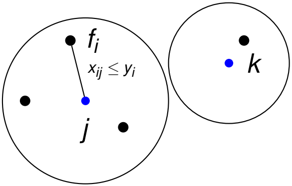

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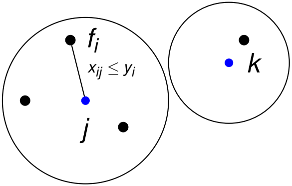

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$f_{\min} \leq f_{\min} \cdot \sum_{i \in N_j} x_{ij} \leq f_{\min} \sum_{i \in N_j} y_i \leq \sum_{i \in N_j} y_i f_i$.

For k used in Step 2.
$N_j \cap N_k = \emptyset$ for j and k in step 2.

Integral facility cost at most LP facility cost.

Claim: Total facility cost is at most $\sum_i f_i y_i$.

2. For smallest (remaining) α_j,
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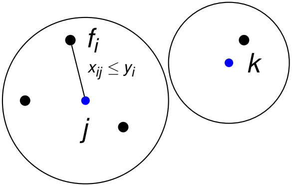

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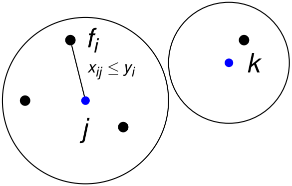

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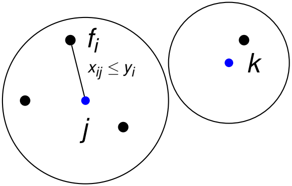

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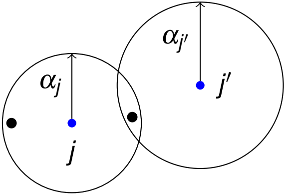

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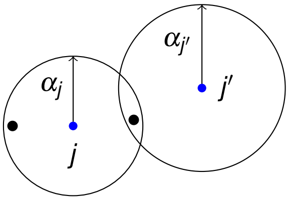

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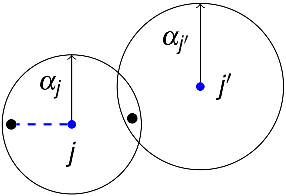

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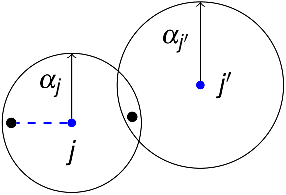
Connection Cost of j: $\leq \alpha_j$.



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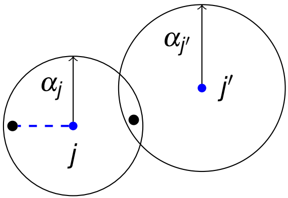

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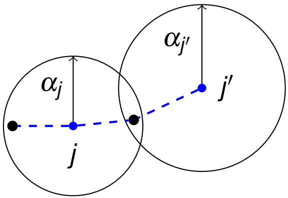

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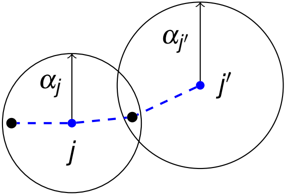

Connection Cost of j: $\leq \alpha_j$.
Connection Cost of j':
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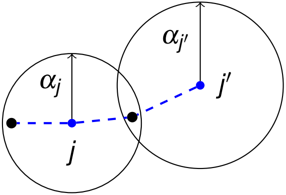

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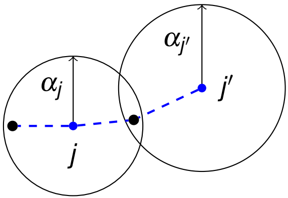

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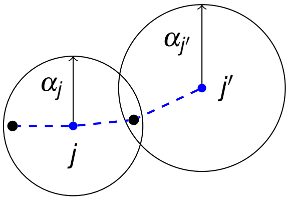

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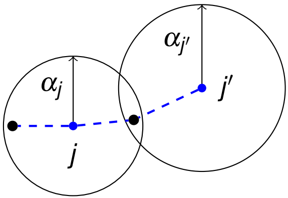

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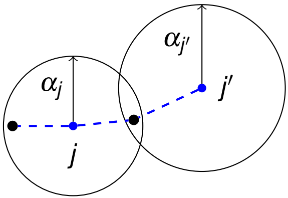

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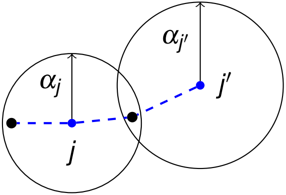

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Total Cost: 4 OPT.

Twist on randomized rounding.

Client j:

Twist on randomized rounding.

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Probability distribution!

Twist on randomized rounding.

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Expected opening cost:

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$D_j = \sum_i x_{ij} d_{ij}$

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$D_j = \sum_i x_{ij} d_{ij}$ Connection cost of primal for j.

Twist on randomized rounding.

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$D_j = \sum_i x_{ij} d_{ij}$ Connection cost of primal for j.

Expected connection cost j' $\alpha_j + \alpha_{j'} + D_j$.

Twist on randomized rounding.

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In step 2: pick in increasing order of $\alpha_j + D_j$.

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$\rightarrow$ Expected cost is $(2\alpha_{j'} + D_{j'})$.   Connection cost: $2\sum_j \alpha_j + \sum_j D_j$.
$2OPT(D)$ plus connection cost or primal.

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Total expected cost:
Facility cost is at most facility cost of primal.
Connection cost at most $2OPT$ + connection cost of prmal.
$\rightarrow$ at most $3OPT$.

Primal dual algorithm.

1. Feasible integer solution.

Primal dual algorithm.

1. Feasible integer solution.
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Primal dual algorithm.

1. Feasible integer solution.
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3. Cost of integer solution $\leq \alpha$ times dual value.

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Just did it.

Primal dual algorithm.

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Just did it. Used linear program.

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Just did it. Used linear program. Faster?

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Typically.

Primal dual algorithm.

1. Feasible integer solution.
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Just did it. Used linear program. Faster?

Typically.
  Begin with feasible dual.

Primal dual algorithm.

1. Feasible integer solution.
2. Feasible dual solution.
3. Cost of integer solution $\leq \alpha$ times dual value.

Just did it. Used linear program. Faster?

Typically.
  Begin with feasible dual.
  Raise dual variables until tight constraint.

Primal dual algorithm.

1. Feasible integer solution.
2. Feasible dual solution.
3. Cost of integer solution $\leq \alpha$ times dual value.

Just did it. Used linear program. Faster?

Typically.

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Raise dual variables until tight constraint.
Set corresponding primal variable to an integer.

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Recall Dual:

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Recall Dual:

$$\max \sum_j \alpha_j$$

$$\forall i \in F \quad \sum_{j \in D} \beta_{ij} \leq f_i$$

$$\forall i \in F, j \in D \quad \alpha_j - \beta_{ij} \leq d_{ij}$$

$$\alpha_j, \beta_{ij} \leq 0$$

Facility location primal dual.

Phase 1:

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Connect all tight ji clients j to i.

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Connect facilities that were paid by same client.
Permanently open an independent set of facilities.

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For client j, connected facility i is opened.

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For client j, connected facility i is opened. Good.
Connected facility not open
$\rightarrow$ exists client j' paid i and connected to open facility.

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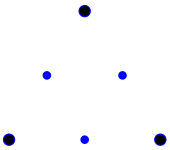
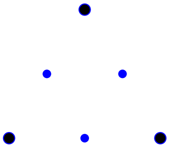
For client j, connected facility i is opened. Good.
Connected facility not open
$\rightarrow$ exists client j' paid i and connected to open facility.
Connect j to j''s open facility.

Constraints for dual.



Constraints for dual.

$\sum_j \beta_{ij} \leq f_i$



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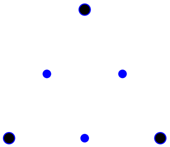

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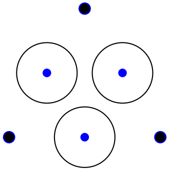
Grow α_j.



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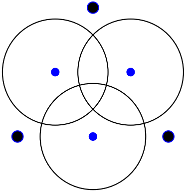

Constraints for dual.

$\sum_j \beta_{ij} \leq f_i$

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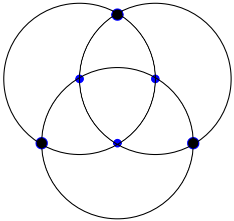
Grow α_j.

$\alpha_j = d_{ij}$!



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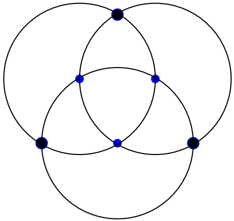
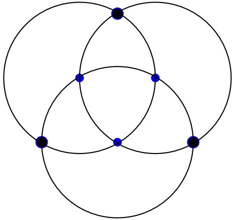
Tight constraint:





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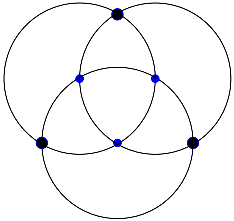

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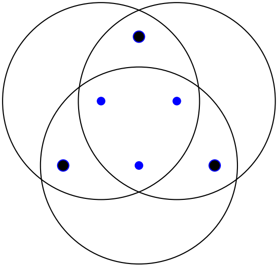

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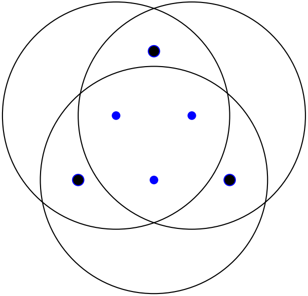

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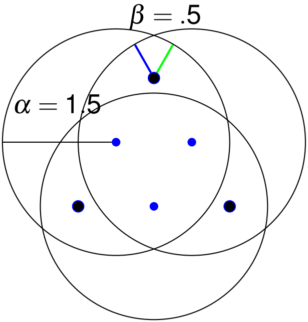

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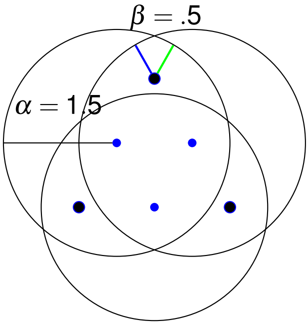
$\sum_j \beta_{ij} = f_i$ for all facilities.



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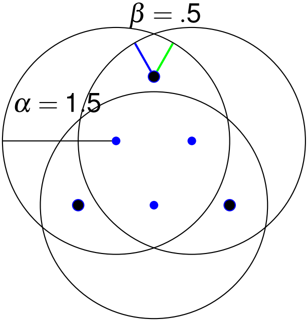

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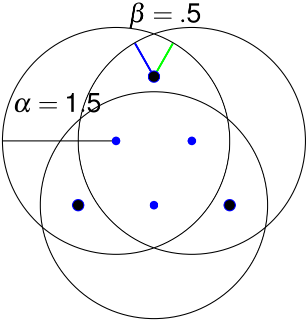

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$\sum_j \beta_{ij} = f_i$ for all facilities.

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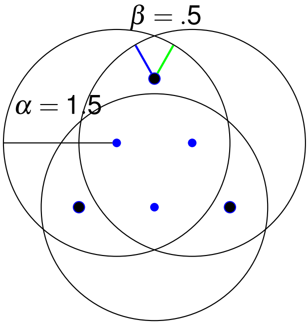
LP Cost: $\sum_j \alpha_j$



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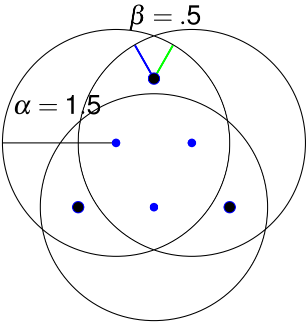
$\sum_j \beta_{ij} = f_i$ for all facilities.

Tight: $\sum_j \beta_{ij} \leq f_i$

LP Cost: $\sum_j \alpha_j = 4.5$



Constraints for dual.

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$\sum_j \beta_{ij} = f_i$ for all facilities.

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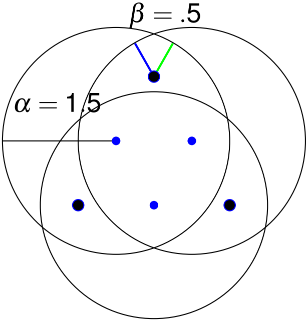
LP Cost: $\sum_j \alpha_j = 4.5$

Temporarily open all facilities.



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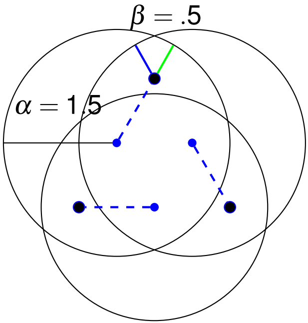

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Grow β_{ij} (and α_j).

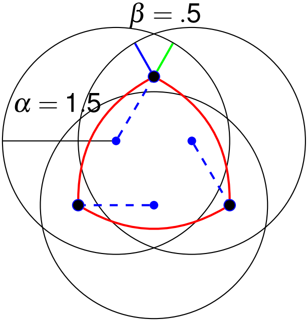
$\sum_j \beta_{ij} = f_i$ for all facilities.

Tight: $\sum_j \beta_{ij} \leq f_i$

LP Cost: $\sum_j \alpha_j = 4.5$

Temporarily open all facilities.

Assign Clients to “paid to” open facility.



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$\alpha_i - \beta_{ij} \leq d_{ij}$.

Grow α_j.

$\alpha_j = d_{ij}$!

Tight constraint: $\alpha_j - \beta_{ij} \leq d_{ij}$.

Grow β_{ij} (and α_j).

$\sum_j \beta_{ij} = f_i$ for all facilities.

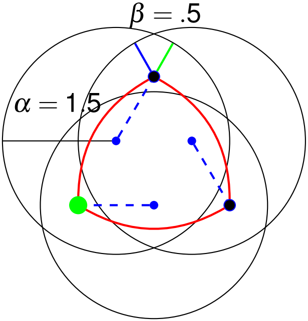
Tight: $\sum_j \beta_{ij} \leq f_i$

LP Cost: $\sum_j \alpha_j = 4.5$

Temporarily open all facilities.

Assign Clients to “paid to” open facility.

Connect facilities with client that pays both.



Constraints for dual.

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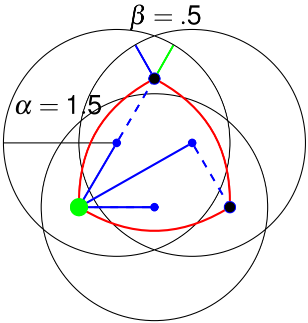
Open independent set.



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Assign Clients to “paid to” open facility.

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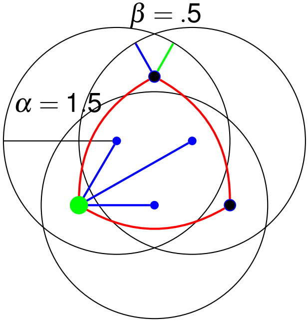
Open independent set.

Connect to “killer” client’s facility.



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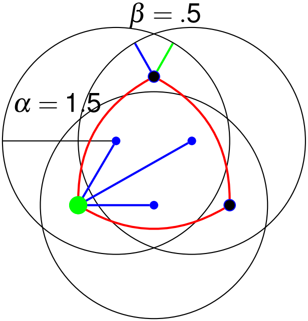
Connect to “killer” client’s facility.

Cost: 1 + 3.7



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<u>Temporarily open all facilities.</u>

Assign Clients to "paid to" open facility.

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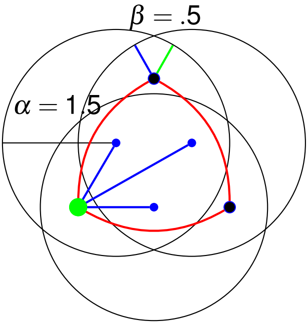
Open independent set.

Connect to "killer" client's facility.

Cost: 1 + 3.7 = 4.7.



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LP Cost: $\sum_j \alpha_j = 4.5$

Temporarily open all facilities.

Assign Clients to “paid to” open facility.

Connect facilities with client that pays both.

Open independent set.

Connect to “killer” client’s facility.

Cost: 1 + 3.7 = 4.7.

A bit more than the LP cost.

Analysis

Claim: Client only pays one facility.

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Independent set of facilities.

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$f_i + \sum_{j \in S_i} d_{ij} \leq \sum_j \alpha_j$.

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Claim: Client only pays one facility.

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Proof:

Analysis

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Claim: S_i - directly connected clients to open facility i.
$f_i + \sum_{j \in S_i} d_{ij} \leq \sum_j \alpha_j$.

Proof:
$f_i = \sum_{j \in S_i} \beta_{ij}$

Analysis

Claim: Client only pays one facility.

Independent set of facilities.

Claim: S_i - directly connected clients to open facility i.
$f_i + \sum_{j \in S_i} d_{ij} \leq \sum_j \alpha_j$.

Proof:
$f_i = \sum_{j \in S_i} \beta_{ij} = \sum_{j \in S_i} \alpha_j - d_{ij}$.
Since directly connected: $\beta_{ij} = \alpha_j - d_{ij}$. □

Analysis.

Claim: Client j is indirectly connected to i

Analysis.

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$\rightarrow d_{ij} \leq 3\alpha_j$.

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Directly connected to (temp open) i'
conflicts with i.

Analysis.

Claim: Client j is indirectly connected to i
$\rightarrow d_{ij} \leq 3\alpha_j$.

Directly connected to (temp open) i'
conflicts with i.
exists j' with $\alpha_{j'} \geq d_{ij'}$ and $\alpha_j \geq d_{i'j'}$.

Analysis.

Claim: Client j is indirectly connected to i $\rightarrow d_{ij} \leq 3\alpha_j$.

Directly connected to (temp open) i' conflicts with i.
exists j' with $\alpha_{j'} \geq d_{ij'}$ and $\alpha_j \geq d_{i'j'}$.
When i' opens, stops both α_j and α_j'.

Analysis.

Claim: Client j is indirectly connected to i
$\rightarrow d_{ij} \leq 3\alpha_j$.

Directly connected to (temp open) i'
conflicts with i.
exists j' with $\alpha_{j'} \geq d_{ij'}$ and $\alpha_j \geq d_{i'j'}$.
When i' opens, stops both α_j and α_j'.
α_j' stopped no later

Analysis.

Claim: Client j is indirectly connected to i $\rightarrow d_{ij} \leq 3\alpha_j$.

Directly connected to (temp open) i'
conflicts with i.
exists j' with $\alpha_{j'} \geq d_{ij'}$ and $\alpha_j \geq d_{i'j'}$.
When i' opens, stops both α_j and α_j'.
α_j' stopped no later (..maybe earlier..)

Analysis.

Claim: Client j is indirectly connected to i
$\rightarrow d_{ij} \leq 3\alpha_j$.

Directly connected to (temp open) i'
conflicts with i.
exists j' with $\alpha_{j'} \geq d_{ij'}$ and $\alpha_j \geq d_{i'j'}$.
When i' opens, stops both α_j and α'_j.
α'_j stopped no later (..maybe earlier..)
$\alpha_j \leq \alpha'_j$.

Analysis.

Claim: Client j is indirectly connected to i
$\rightarrow d_{ij} \leq 3\alpha_j$.

Directly connected to (temp open) i'
conflicts with i.
exists j' with $\alpha_{j'} \geq d_{ij'}$ and $\alpha_j \geq d_{i'j'}$.
When i' opens, stops both α_j and α_j'.
α_j' stopped no later (..maybe earlier..)
$\alpha_j \leq \alpha_j'$.
Total distance from j to i'.

Analysis.

Claim: Client j is indirectly connected to i $\rightarrow d_{ij} \leq 3\alpha_j$.

Directly connected to (temp open) i' conflicts with i.
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$d_{ij} +$

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Directly connected to (temp open) i' conflicts with i.

exists j' with $\alpha_{j'} \geq d_{ij'}$ and $\alpha_j \geq d_{i'j'}$.

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α_j' stopped no later (..maybe earlier..)

$\alpha_j \leq \alpha_j'$.

Total distance from j to i'.

$d_{ij} + d_{ij'} +$

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Claim: Client j is indirectly connected to i
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Directly connected to (temp open) i'
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Total distance from j to i'.
$d_{ij} + d_{ij'} + d_{i'j'}$

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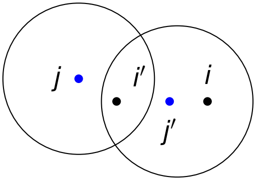
When i' opens, stops both α_j and α_j'.
α_j' stopped no later (..maybe earlier..)
$\alpha_j \leq \alpha_j'$.
Total distance from j to i'.
$d_{ij} + d_{ij'} + d_{i'j'} \leq 3\alpha_j$ □



Putting it together!

Claim: Client only pays one facility.
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Total Cost:

Putting it together!

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Total Cost:
direct clients dual (α_j) pays for facility and own connections.

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Claim: Client j is indirectly connected to i
$\rightarrow d_{ij} \leq 3\alpha_j$.

Total Cost:
direct clients dual (α_j) pays for facility and own connections.
plus no more than 3 times indirect client dual.

Putting it together!

Claim: Client only pays one facility.
Claim: S_i - directly connected clients to open facility i.
$f_i + \sum_{j \in S_i} d_{ij} \leq \sum_j \alpha_j$.
Claim: Client j is indirectly connected to i
$\rightarrow d_{ij} \leq 3\alpha_j$.

Total Cost:
direct clients dual (α_j) pays for facility and own connections.
plus no more than 3 times indirect client dual.
Total Cost: 3 times dual.

Putting it together!

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Claim: S_i - directly connected clients to open facility i.
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Total Cost:
direct clients dual (α_j) pays for facility and own connections.
plus no more than 3 times indirect client dual.
Total Cost: 3 times dual.
feasible dual upper bounds fractional (and integer) primal.

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3 OPT.

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Total Cost:
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Total Cost: 3 times dual.
feasible dual upper bounds fractional (and integer) primal.

3 OPT.

Fast!

Putting it together!

Claim: Client only pays one facility.
Claim: S_i - directly connected clients to open facility i.
$f_i + \sum_{j \in S_i} d_{ij} \leq \sum_j \alpha_j$.
Claim: Client j is indirectly connected to i
$\rightarrow d_{ij} \leq 3\alpha_j$.

Total Cost:
direct clients dual (α_j) pays for facility and own connections.
plus no more than 3 times indirect client dual.
Total Cost: 3 times dual.
feasible dual upper bounds fractional (and integer) primal.

3 OPT.

Fast! Cheap!

Putting it together!

Claim: Client only pays one facility.
Claim: S_i - directly connected clients to open facility i.
$f_i + \sum_{j \in S_i} d_{ij} \leq \sum_j \alpha_j$.
Claim: Client j is indirectly connected to i
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Total Cost:
direct clients dual (α_j) pays for facility and own connections.
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Total Cost: 3 times dual.
feasible dual upper bounds fractional (and integer) primal.

3 OPT.

Fast! Cheap! Safe!

See you on Thursday.