CS270: Lecture 1.

1. Overview
2. Administration
3. Dueling Subroutines: Congestion/Tolls.

Algorithms.

Undergraduate.

Algorithms.

Undergraduate.

1. Classical.

Algorithms.

Undergraduate.

1. Classical.

2. Cleanly Stated Problems. Shortest Paths, max-flow, MST.

Algorithms.

Undergraduate.

1. Classical.
2. Cleanly Stated Problems. Shortest Paths, max-flow, MST.
3. Solutions:

Algorithms.

Undergraduate.

1. Classical.
2. Cleanly Stated Problems. Shortest Paths, max-flow, MST.
3. Solutions: effective

Algorithms.

Undergraduate.

1. Classical.
2. Cleanly Stated Problems. Shortest Paths, max-flow, MST.
3. Solutions: effective precise bounds!

Algorithms.

Undergraduate.

1. Classical.
2. Cleanly Stated Problems. Shortest Paths, max-flow, MST.
3. Solutions: effective precise bounds!
4. Techniques:

Algorithms.

Undergraduate.

1. Classical.
2. Cleanly Stated Problems. Shortest Paths, max-flow, MST.
3. Solutions: effective precise bounds!
4. Techniques: Greedy

Algorithms.

Undergraduate.

1. Classical.
2. Cleanly Stated Problems. Shortest Paths, max-flow, MST.
3. Solutions: effective precise bounds!
4. Techniques: Greedy Dyn. Programming

Algorithms.

Undergraduate.

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3. Solutions: effective precise bounds!
4. Techniques: Greedy Dyn. Programming Linear Programming.

Algorithms.

Undergraduate.

1. Classical.
2. Cleanly Stated Problems. Shortest Paths, max-flow, MST.
3. Solutions: effective precise bounds!
4. Techniques: Greedy Dyn. Programming Linear Programming.
5. Techniques tend to be Combinatorial.

Algorithms.

Undergraduate.
This class.

1. Classical.
2. Cleanly Stated Problems. Shortest Paths, max-flow, MST.
3. Solutions: effective precise bounds!
4. Techniques: Greedy Dyn. Programming Linear Programming.
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Algorithms.

Undergraduate.
This class.

1. Classical.
   Flavor of the week?
2. Cleanly Stated Problems. Shortest Paths, max-flow, MST.
3. Solutions: effective precise bounds!
4. Techniques: Greedy Dyn. Programming Linear Programming.
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Algorithms.

Undergraduate.
This class.

1. Classical.
   Modern.
2. Cleanly Stated Problems. Shortest Paths, max-flow, MST.
3. Solutions: effective precise bounds!
4. Techniques: Greedy Dyn. Programming Linear Programming.
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Algorithms.

Undergraduate.
This class.

1. Classical.
   Modern.
2. Cleanly Stated Problems. Shortest Paths, max-flow, MST.
   Vaguely stated problems!
3. Solutions: effective precise bounds!

4. Techniques: Greedy Dyn. Programming Linear Programming.

5. Techniques tend to be Combinatorial.

Algorithms.

Undergraduate.
This class.

1. Classical.
   Modern.
2. Cleanly Stated Problems. Shortest Paths, max-flow, MST.
   Address problems; messy or not.
3. Solutions: effective precise bounds!

4. Techniques: Greedy Dyn. Programming Linear Programming.

5. Techniques tend to be Combinatorial.

Algorithms.

Undergraduate.
This class.

1. Classical.
   Modern.
2. Cleanly Stated Problems. Shortest Paths, max-flow, MST.
   Address problems; messy or not.
3. Solutions: effective precise bounds!
   Ineffective ..imprecise!
4. Techniques: Greedy Dyn. Programming Linear Programming.

5. Techniques tend to be Combinatorial.

Algorithms.

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This class.

1. Classical.
   Modern.
2. Cleanly Stated Problems. Shortest Paths, max-flow, MST.
   Address problems; messy or not.
3. Solutions: effective precise bounds!
   Analysis sometimes based on modelling world.
4. Techniques: Greedy Dyn. Programming Linear Programming.

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4. Techniques: Greedy Dyn. Programming Linear Programming.
   Heuristic, in practice.
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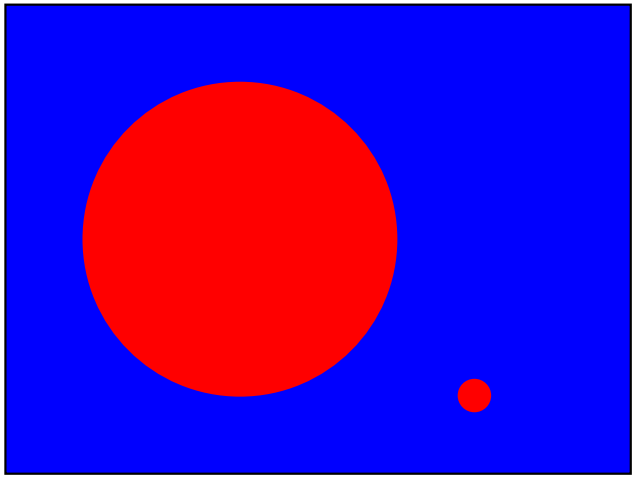
   Address problems; messy or not.
3. Solutions: effective precise bounds!
   Analysis sometimes based on modelling world.
4. Techniques: Greedy Dyn. Programming Linear Programming.
   Heuristic, in practice.
5. Techniques tend to be Combinatorial.
   Probabilistic, linear algebra methods, continuous.

Example Problem: clustering.

- Points: documents, dna, preferences.
- Graphs: applications to VLSI, parallel processing, image segmentation.

Image example.

Image Segmentation



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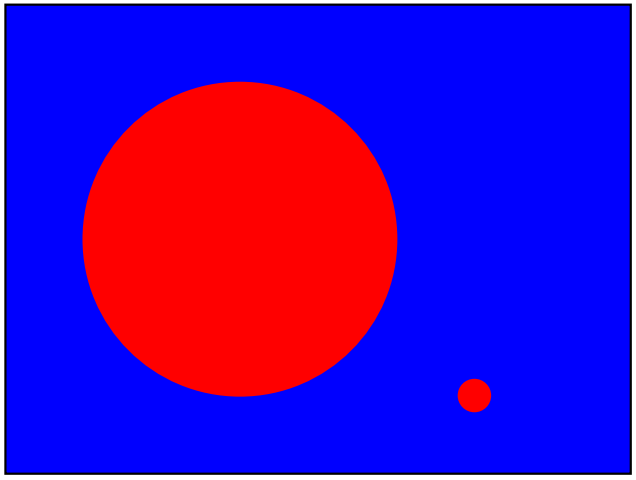

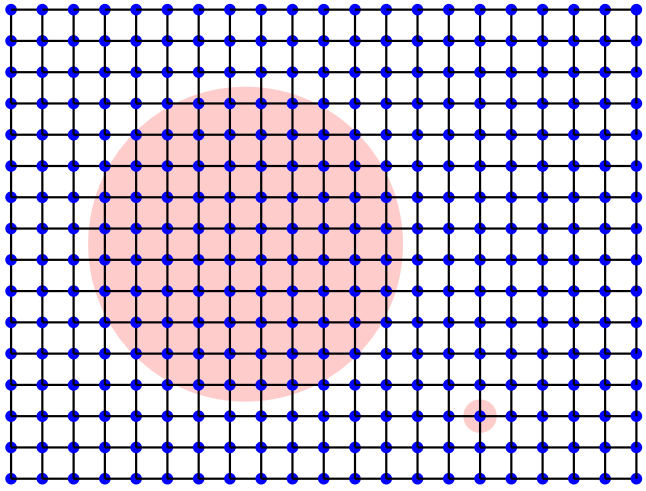

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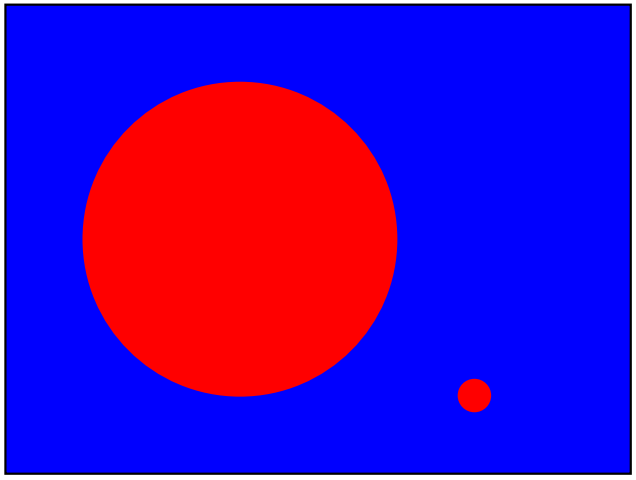
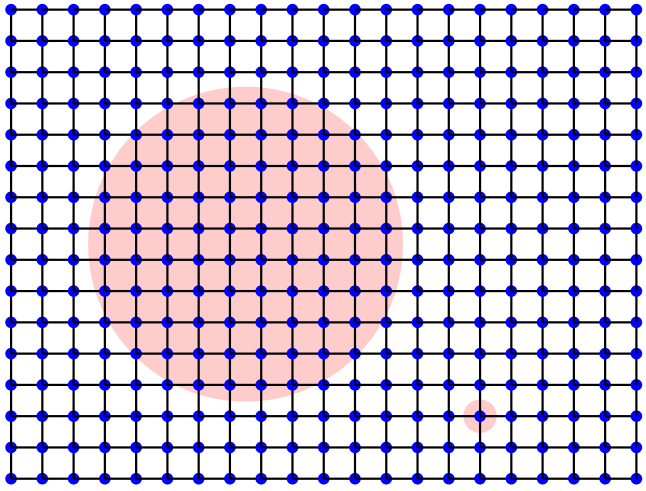
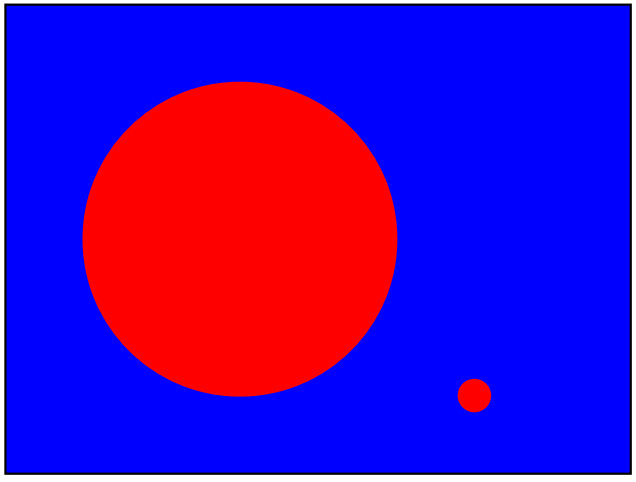
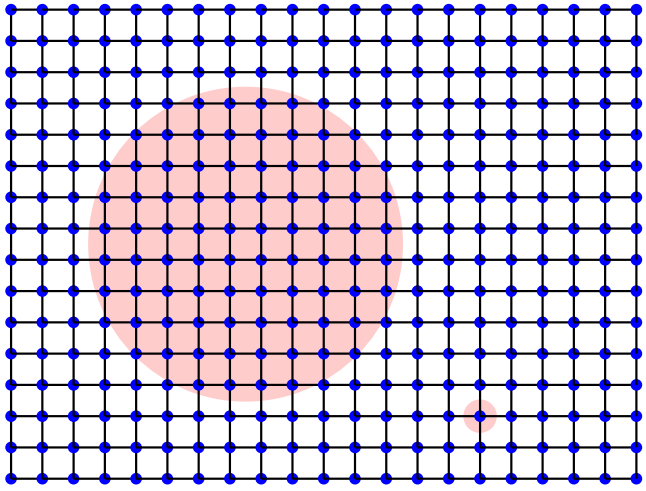




Which region?

Image Segmentation

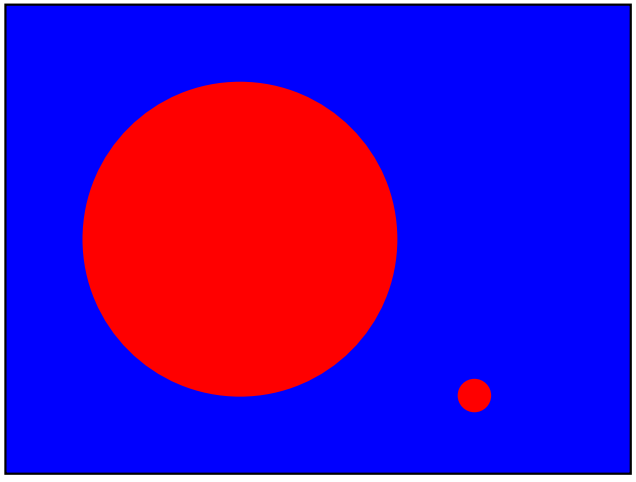


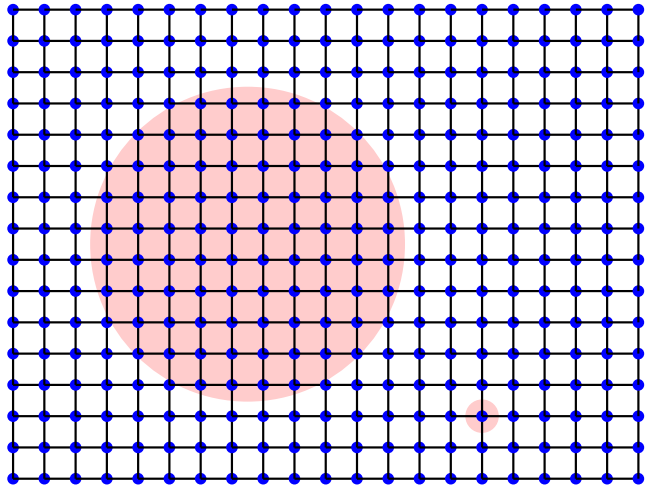


Which region? Normalized Cut: Find S, which minimizes

$$\frac{w(S,\overline{S})}{w(S) \times w(\overline{S})}.$$

Image Segmentation





Which region? Normalized Cut: Find S, which minimizes

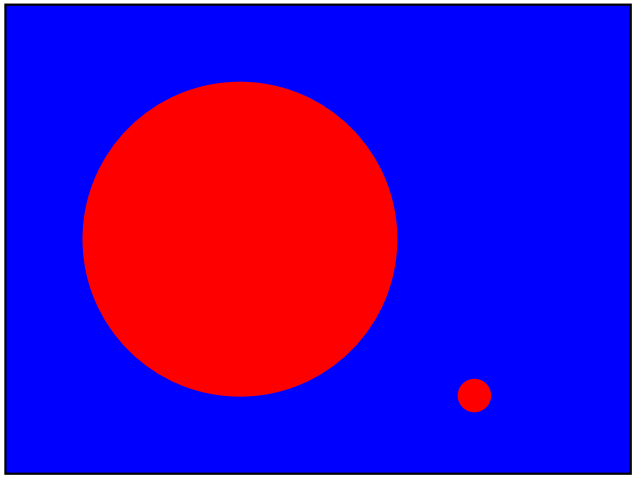
$$\frac{w(S,\overline{S})}{w(S)\times w(\overline{S})}.$$

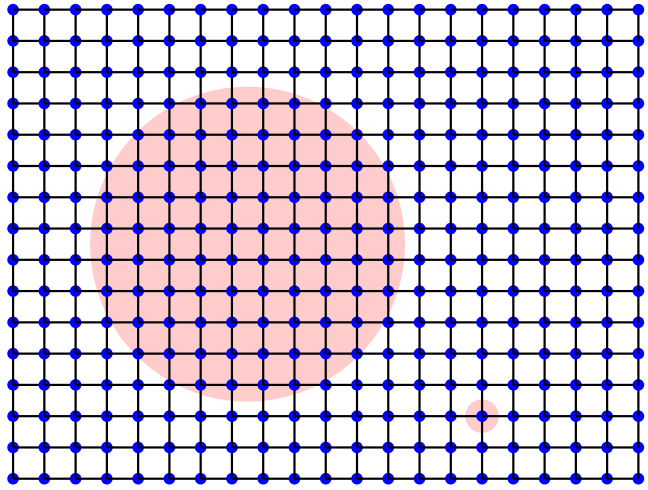
Ratio Cut: minimize

$$\frac{w(S,\overline{S})}{w(S)},$$

$w(S)$ no more than half the weight. (Minimize cost per unit weight that is removed.)

Image Segmentation





Which region? Normalized Cut: Find S, which minimizes

$$\frac{w(S,\overline{S})}{w(S)\times w(\overline{S})}.$$

Ratio Cut: minimize

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$w(S)$ no more than half the weight. (Minimize cost per unit weight that is removed.)
Either is generally useful!

Example: recommendations.

Sarah Palin likes True Grit (the old one.)

Example: recommendations.

Sarah Palin likes True Grit (the old one.)
Sarah Palin doesn't like The Social Network.

Example: recommendations.

Sarah Palin likes True Grit (the old one.)
Sarah Palin doesn't like The Social Network.
Sarah Palin doesn't like Black Swan.

Example: recommendations.

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Hillary Clinton doesn't like True Grit (the old one.)

Example: recommendations.

Sarah Palin likes True Grit (the old one.)
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Hillary Clinton doesn't like True Grit (the old one.)
Hillary Clinton likes The Social Network.

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Should you recommend the discovery channel to Hillary?

Example: recommendations.

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Are you Hillary?

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Are you Hillary? Are you Sarah?

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High dimensional data: dimension for each movie.

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More than three dimensions!

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High dimensional data: dimension for each movie.
More than three dimensions!
Nearest neighbors. Principal Components methods.

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Topic Models.

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What about you?

Are you Hillary? Are you Sarah? A bit of both?

High dimensional data: dimension for each movie.
More than three dimensions!
Nearest neighbors. Principal Components methods.
Topic Models.
Reasoning about these methods.

Linear Systems.

Revolution!

Linear Systems.

Revolution!

Linear Systems.

Revolution!

Linear Systems.

Revolution!

Linear Systems.

Revolution!

Linear Systems.

Revolution!

Physical Simulation.

Linear Systems.

Revolution!

Physical Simulation. Airflow.

Linear Systems.

Revolution!

Physical Simulation. Airflow.

Solve $Ax = b$.

Linear Systems.

Revolution!

Physical Simulation. Airflow.

Solve $Ax = b$.

How long?

Linear Systems.

Revolution!

Physical Simulation. Airflow.

Solve $Ax = b$.

How long?

$n \times n$ matrix A.

Linear Systems.

Revolution!

Physical Simulation. Airflow.

Solve $Ax = b$.

How long?

$n \times n$ matrix A.

Middle School:

Linear Systems.

Revolution!

Physical Simulation. Airflow.

Solve $Ax = b$.

How long?

$n \times n$ matrix A.

Middle School: substitution,

Linear Systems.

Revolution!

Physical Simulation. Airflow.

Solve $Ax = b$.

How long?

$n \times n$ matrix A.

Middle School: substitution, adding equations ...

Linear Systems.

Revolution!

Physical Simulation. Airflow.

Solve $Ax = b$.

How long?

$n \times n$ matrix A.

Middle School: substitution, adding equations ...
Time: $O(n^3)$.

Linear Systems.

Revolution!

Physical Simulation. Airflow.

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How long?

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Middle School: substitution, adding equations ...
Time: $O(n^3)$.

Linear Systems.

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Solve $Ax = b$.

How long?

$n \times n$ matrix A.

Middle School: substitution, adding equations ...
Time: $O(n^3)$.

Now: $\tilde{O}(m)$.

Linear Systems.

Revolution!

Physical Simulation. Airflow.

Solve $Ax = b$.

How long?

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Middle School: substitution, adding equations ...
Time: $O(n^3)$.

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Revolution!

Physical Simulation. Airflow.

Solve $Ax = b$.

How long?

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Middle School: substitution, adding equations ...
Time: $O(n^3)$.

Now: $\tilde{O}(m)$. Hmmm. What's that tilde?

Linear Systems.

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Techniques:

Linear Systems.

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Time: $O(n^3)$.

Now: $\tilde{O}(m)$. Hmmm. What's that tilde?

Techniques:
Relate graph theory to matrix properties.

Linear Systems.

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Middle School: substitution, adding equations ...
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Techniques:
Relate graph theory to matrix properties.
Dense matrix (graph) to sparse matrix (graph).

Linear Systems.

Revolution!

Physical Simulation. Airflow.

Solve $Ax = b$.

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Techniques:
Relate graph theory to matrix properties.
Dense matrix (graph) to sparse matrix (graph).
Approximating distances by trees.

Linear Systems.

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Techniques:
Relate graph theory to matrix properties.
Dense matrix (graph) to sparse matrix (graph).
Approximating distances by trees.
Electrical networks analysis.

Linear Systems.

Revolution!

Physical Simulation. Airflow.

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Techniques:
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Combinatorial Applications:

Linear Systems.

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Techniques:
Relate graph theory to matrix properties.
Dense matrix (graph) to sparse matrix (graph).
Approximating distances by trees.
Electrical networks analysis.

Combinatorial Applications: Better Max Flow!

Other Algorithmic Techiniques

Sketching:

Large stream of data: $a_1, a_2, \ldots$

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Find digest.

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Graphs: Sparse graph.

Other Algorithmic Techiniques

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Large stream of data: $a_1, a_2, \ldots$

Find digest.

Graphs: Sparse graph.

Data: average, statistics.

Other Algorithmic Techiniques

Sketching:

Large stream of data: $a_1, a_2, \ldots$

Find digest.

Graphs: Sparse graph.

Data: average, statistics.

Points: center point, k-medians, .

Other Algorithmic Techiniques

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High Dimensional optimization.

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High Dimensional optimization.

Gradient Descent. Convexity.

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Dueling Subroutines. Duality.

Other Algorithmic Techiniques

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Dueling Subroutines. Duality.
Lower bounding, upper bounder.

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Lower bounding, upper bounder.
Upper uses lower's evidence to find better solutions.

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Lower bounding, upper bounder.
Upper uses lower's evidence to find better solutions.
Lower uses upper's evidence to prove better lower bounds.

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CS270: Administration.

1. Staff:
   Satish Rao

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   Di Wang

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   Satish Rao
   Di Wang
2. Piazza. Log in! Pay attention to “spam everyone” especially.
3. Assessment.
   3.1 Homeworks (40%).

CS270: Administration.

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   Satish Rao
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   3.1 Homeworks (40%).
   Homework 1 out tonight/tomorrow.

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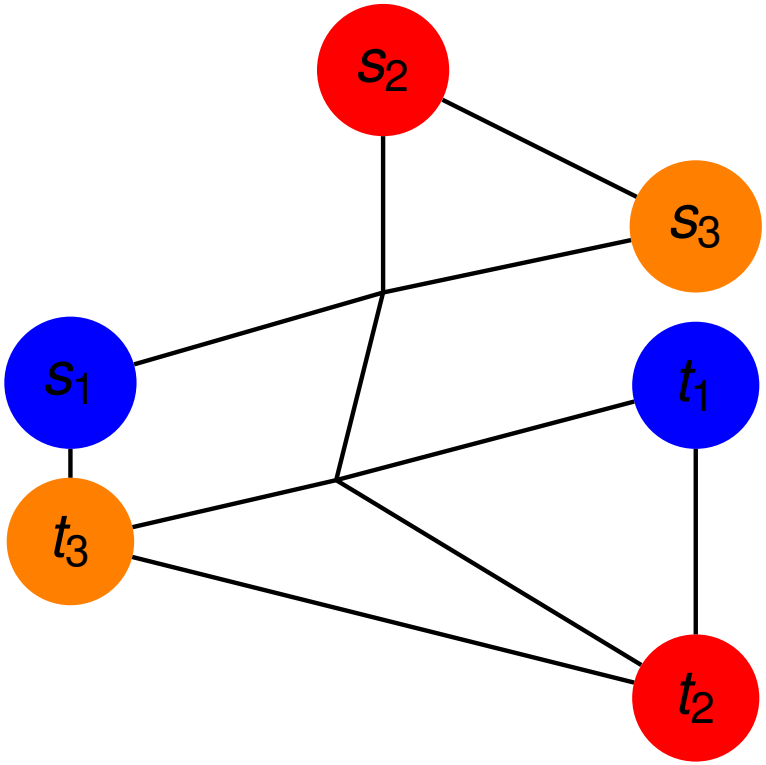
     Groups of 2 or 3.
     Connect research to class.
     Or explore/digest a topic from class.
   - 3.4 No Discussion this week.

Path Routing.

Given $G = (V, E)$, $(s_1, t_1), \ldots, (s_k, t_k)$, find a set of k paths connecting s_i and t_i and minimize max load on any edge.



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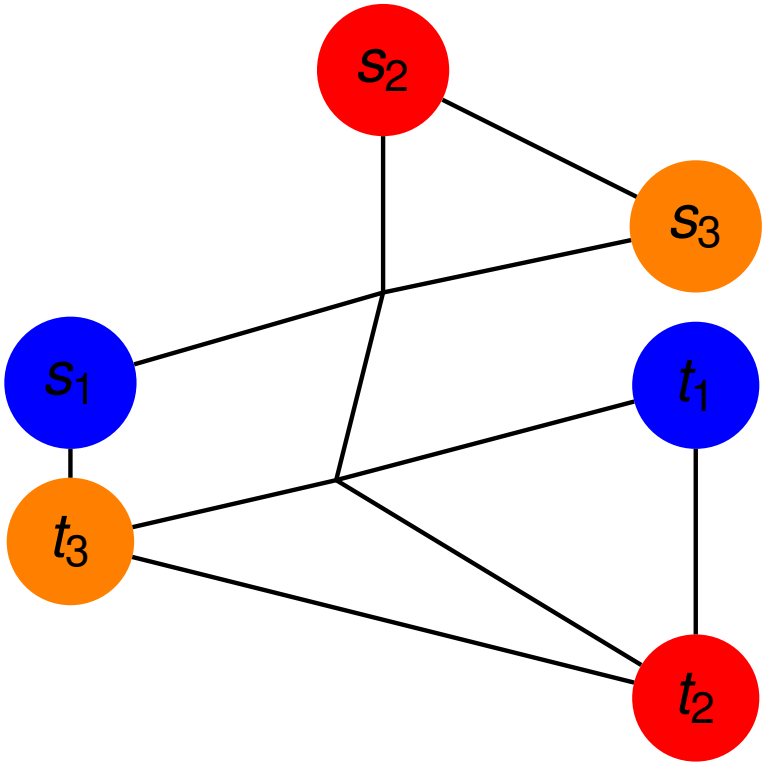

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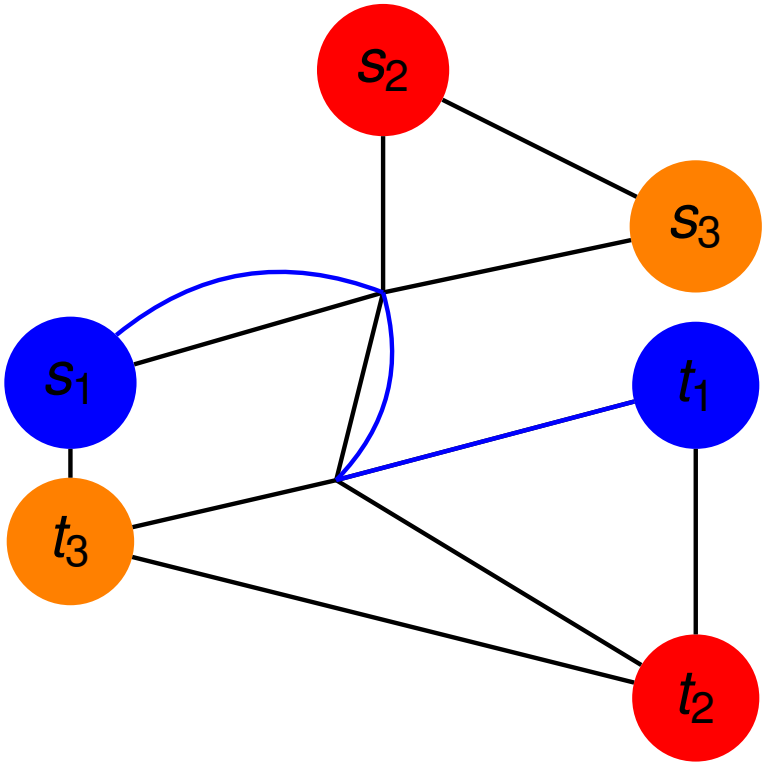

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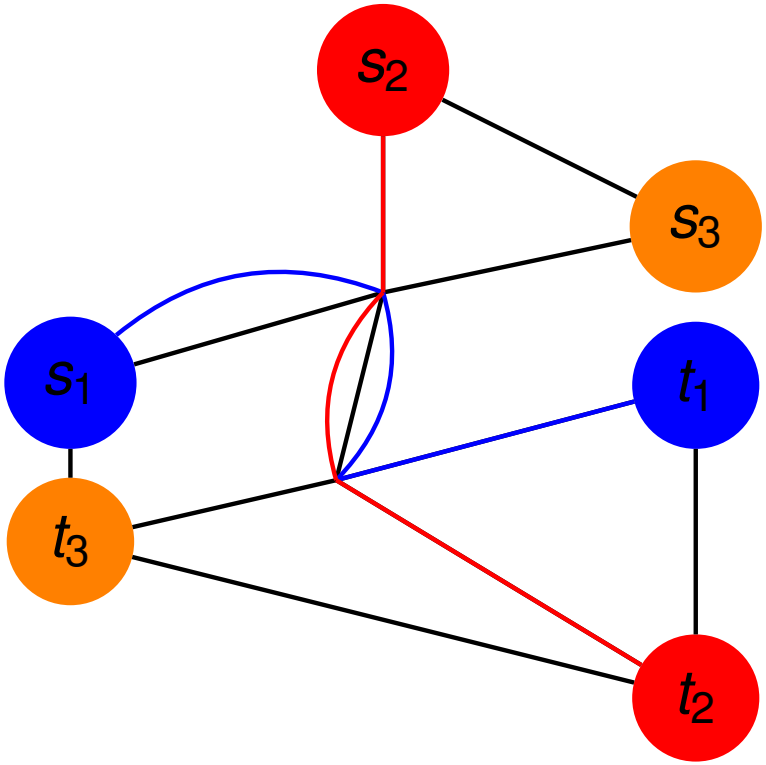

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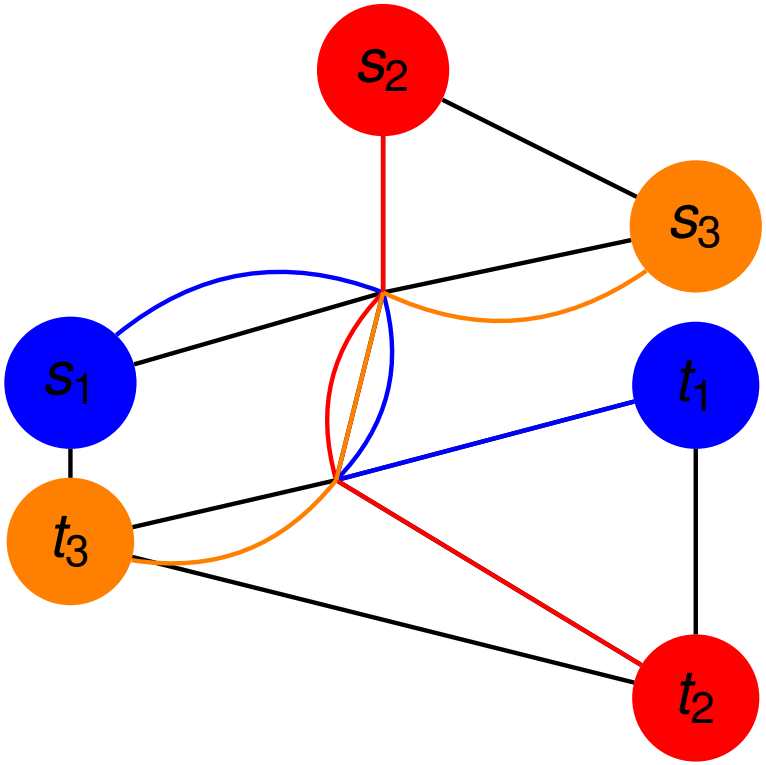

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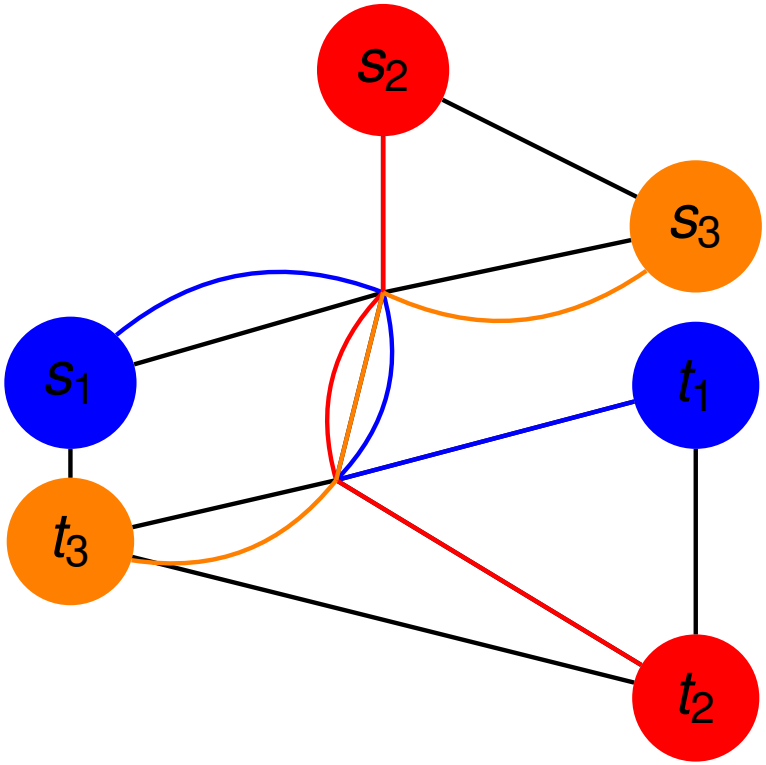

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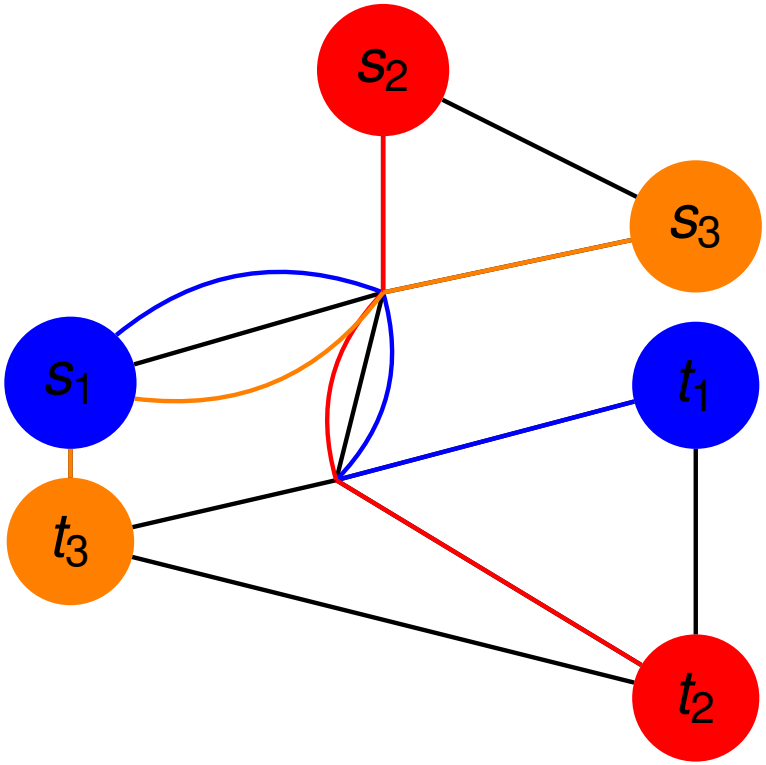

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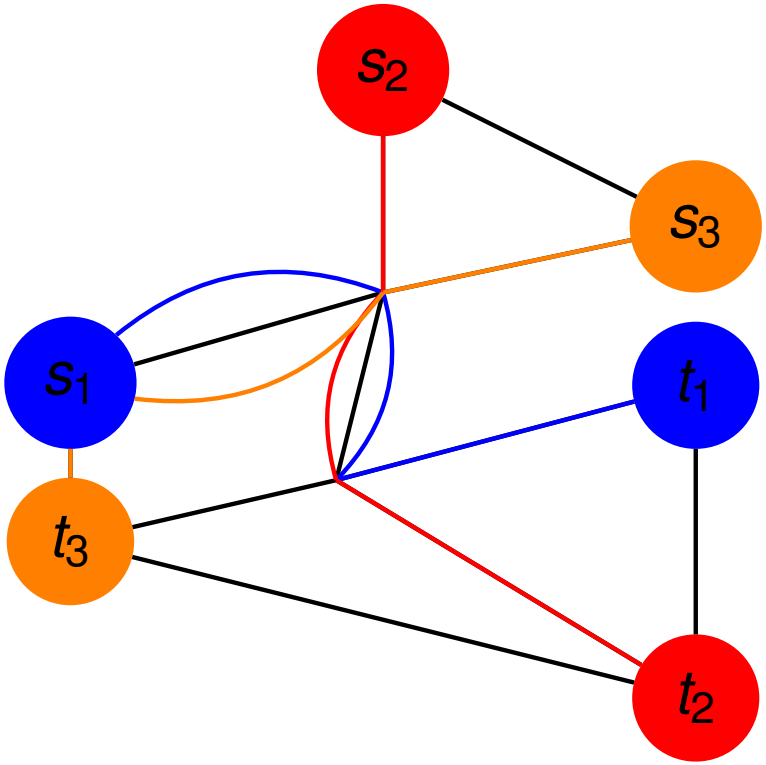

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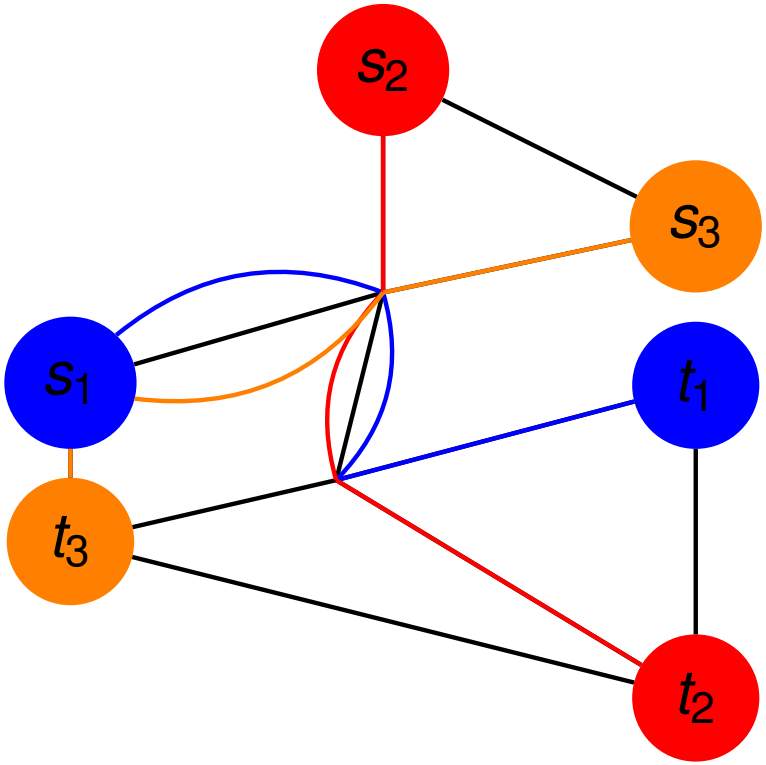

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Value: 2

Terminology

Routing: Paths $p_1, p_2, \ldots, p_k$, p_i connects s_i and t_i.

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Congestion of edge, e: $c(e)$
number of paths in routing that contain e.

Congestion of routing: maximum congestion of any edge.

Find routing that minimizes congestion (or maximum congestion.)

Algorithms?

Route along any path.

Algorithms?

Route along any path.
Feasible...

Algorithms?

Route along any path.
Feasible...but good?

Algorithms?

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How far from optimal could it be?

(A) It is optimal!

(B) A factor of two.

(C) A factor of k, in general.

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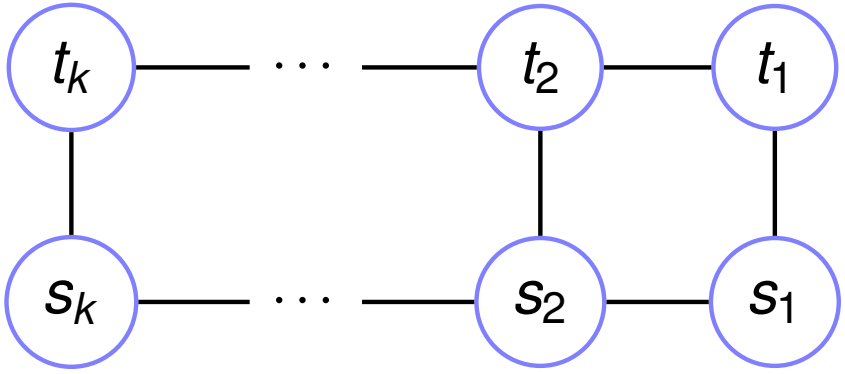

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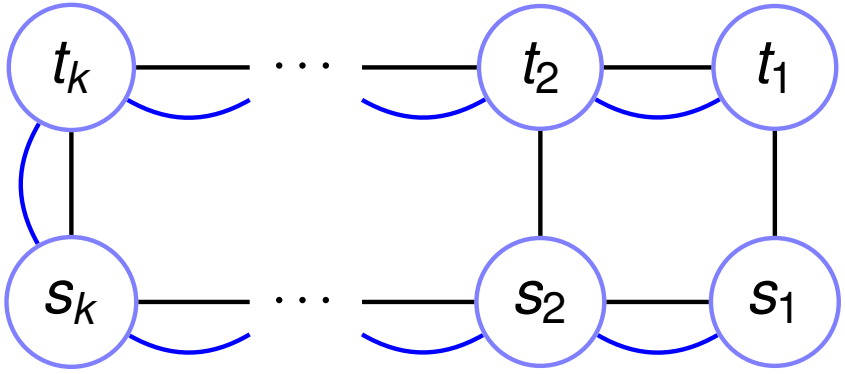

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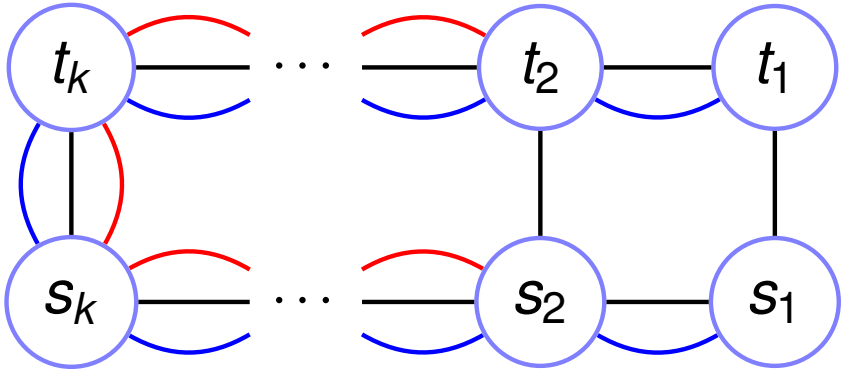

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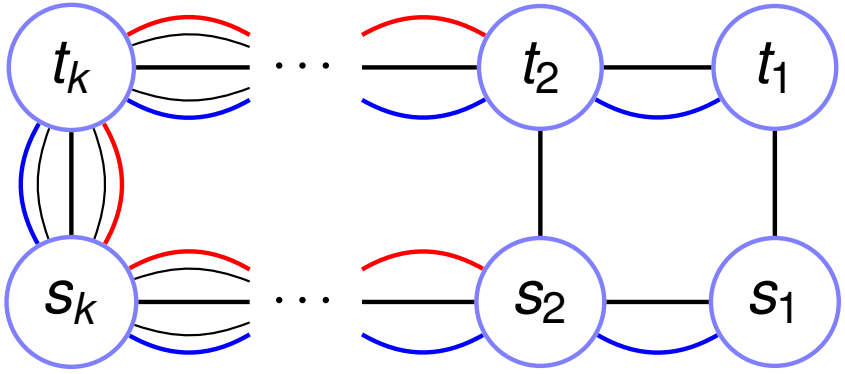

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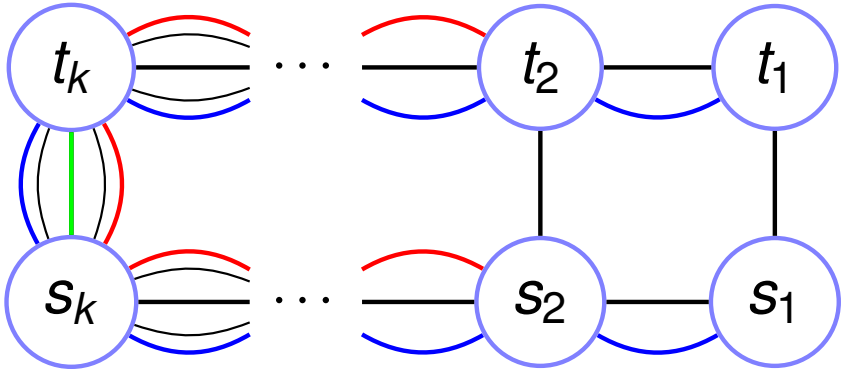

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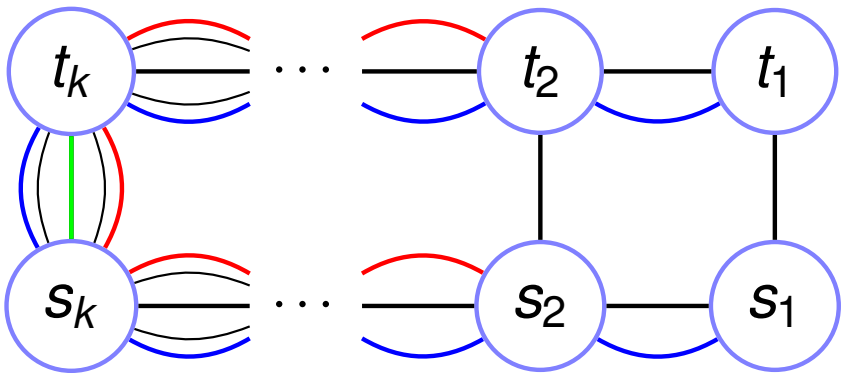

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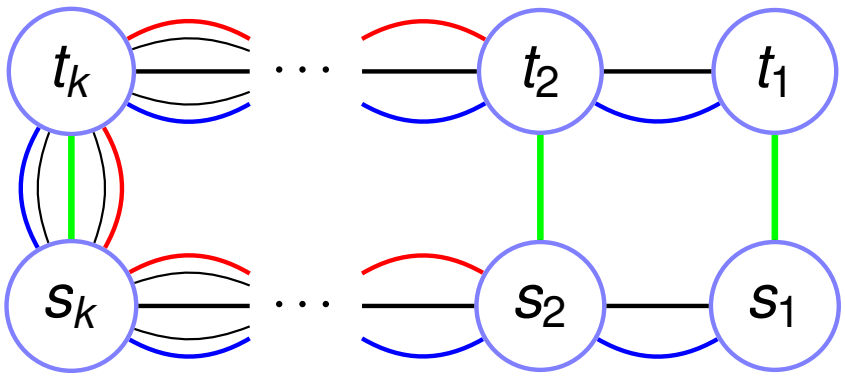

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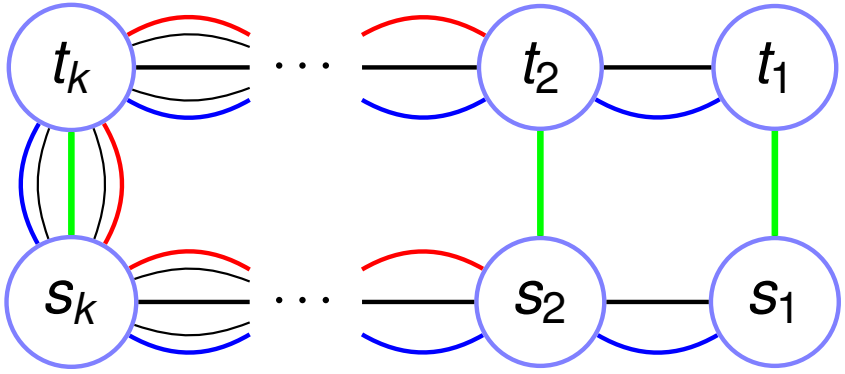

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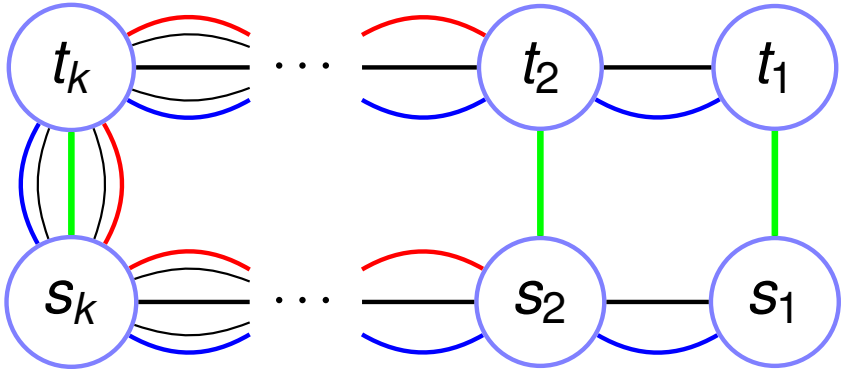

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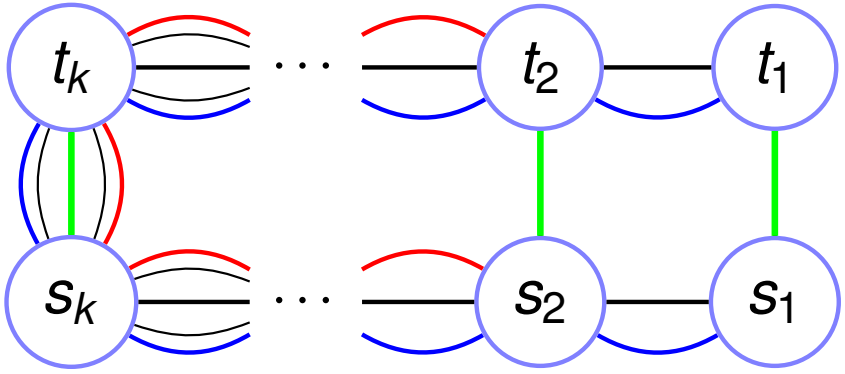

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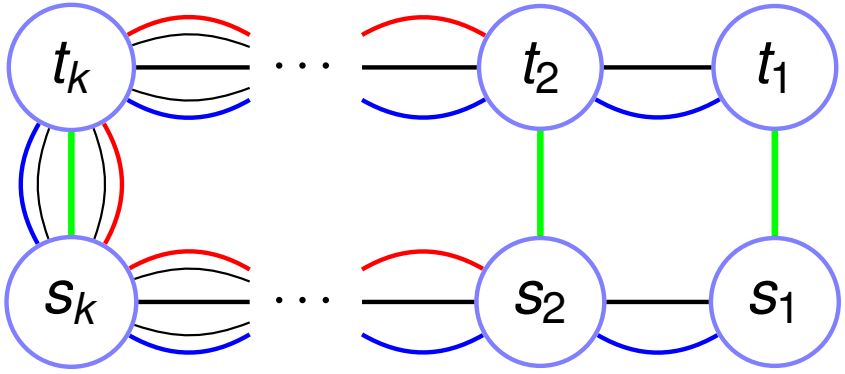

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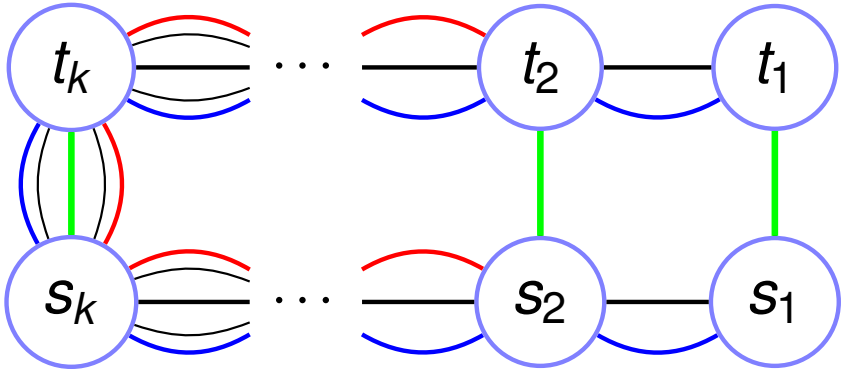

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Shortest Path Routing and Congestion.

Minimize each path length minimizes total congestion.

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Average load is lower bound on the lowest max congestion!

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Average load is lower bound on the lowest max congestion!

Shortest path routing minimizes average load.

Shortest Path Routing and Congestion.

Minimize each path length minimizes total congestion.

Also minimizes average: $\frac{1}{m}\sum_e c(e)$. Just a scaling!

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Does it minimize maximum load?

One problem...

How far from optimal?

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Optimal?

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Optimal? Factor 2?

One problem...

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Optimal? Factor 2? Factor k?

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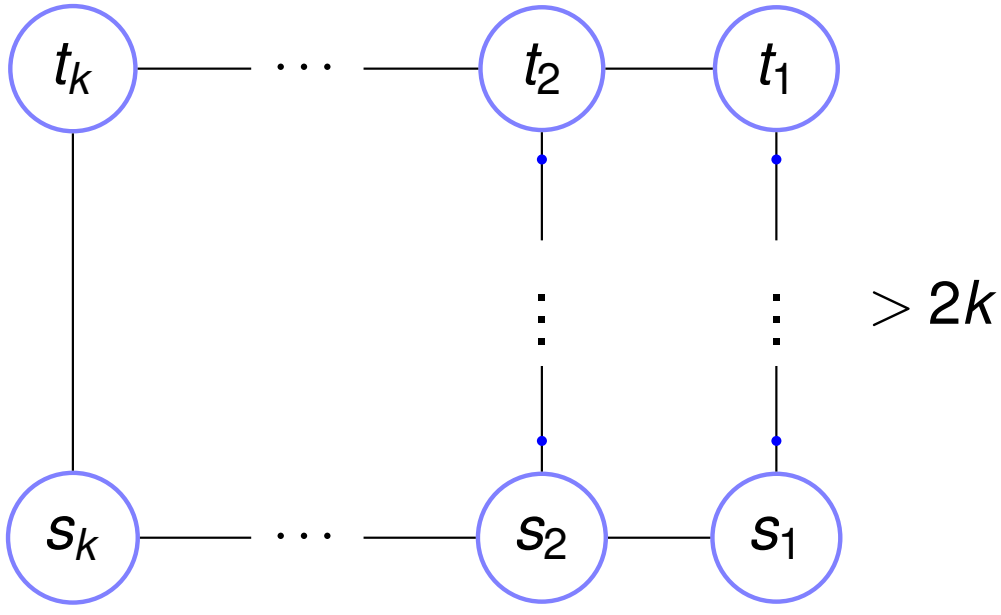

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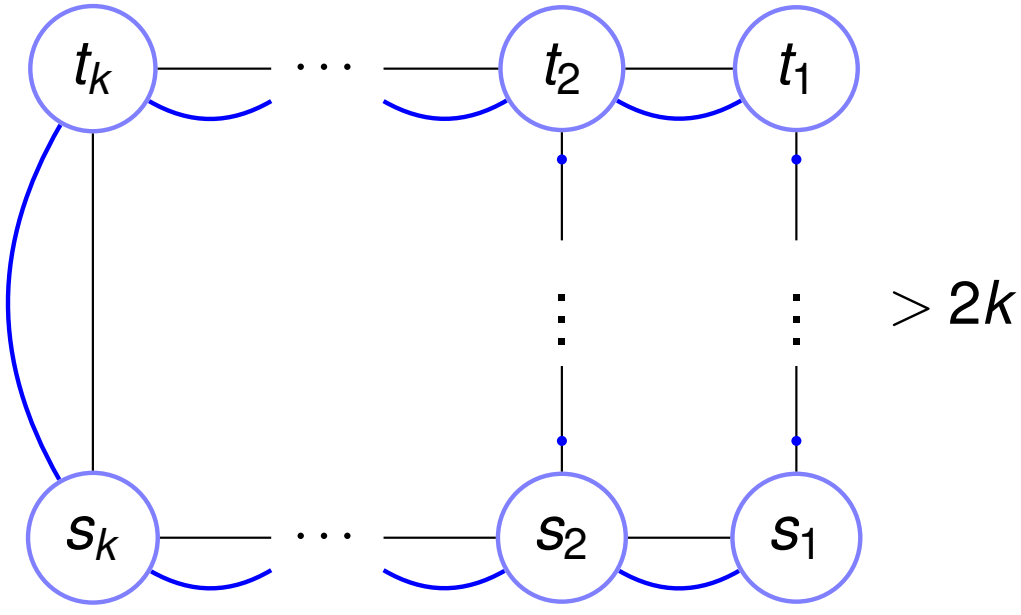

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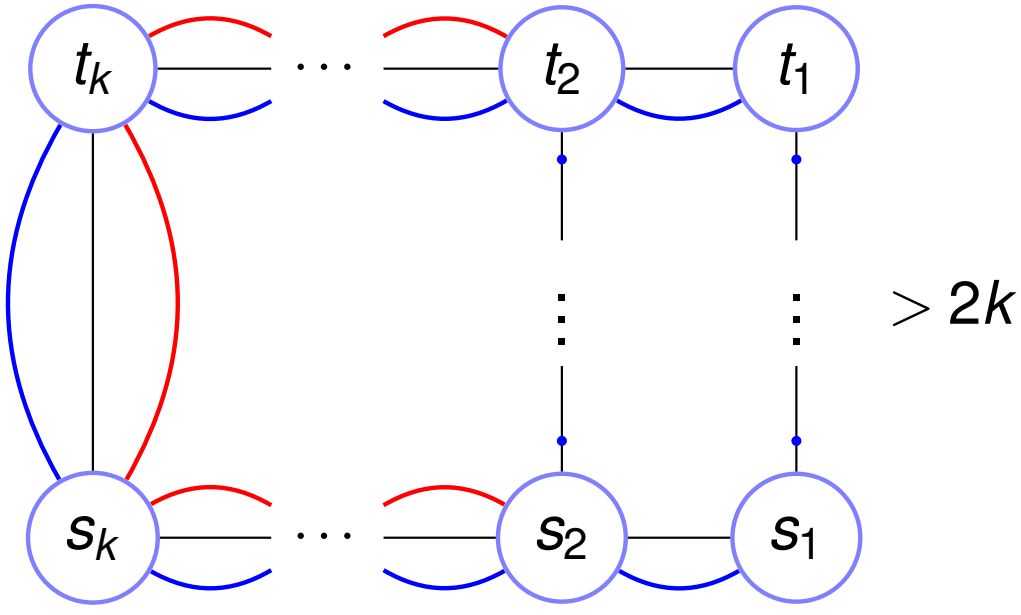

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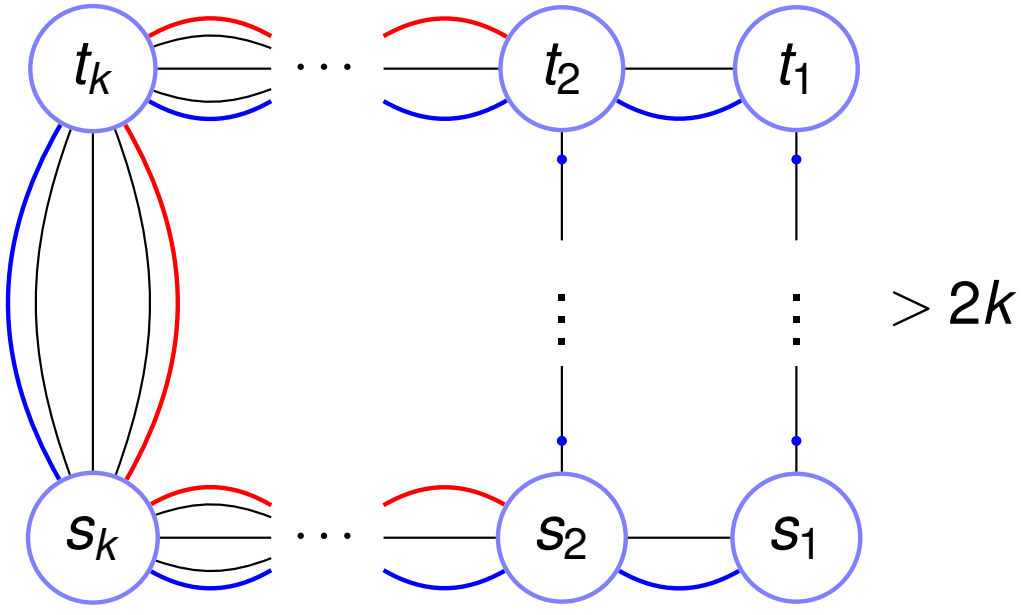

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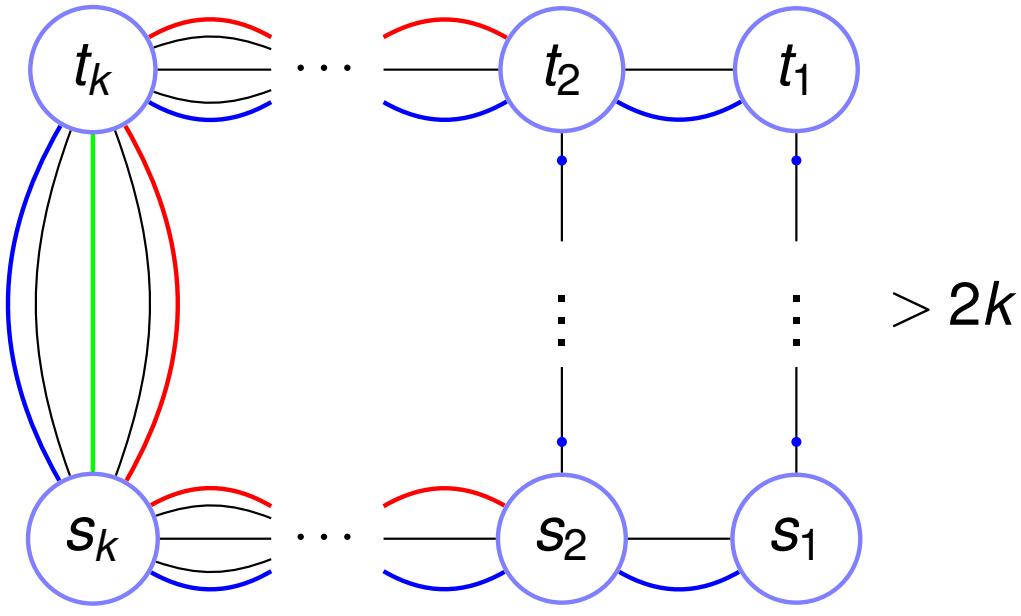

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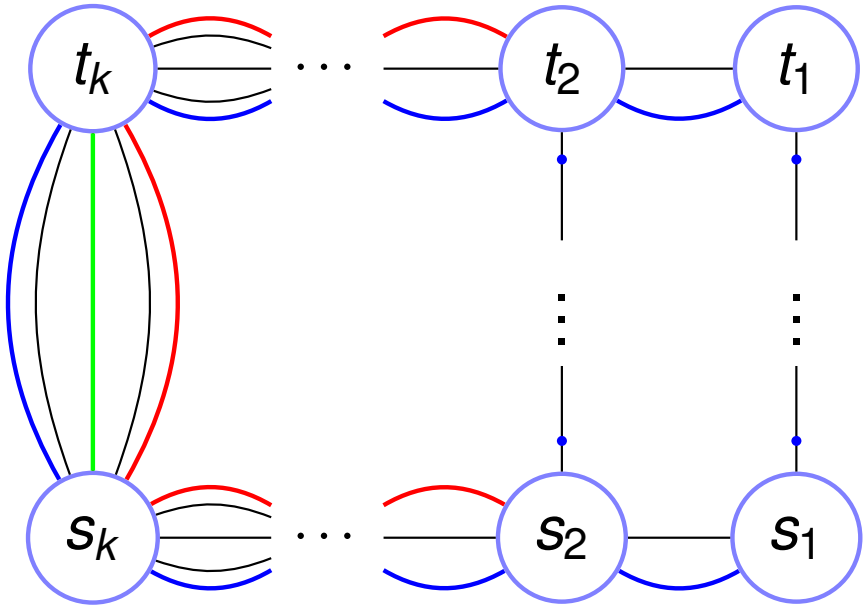

Value: k.

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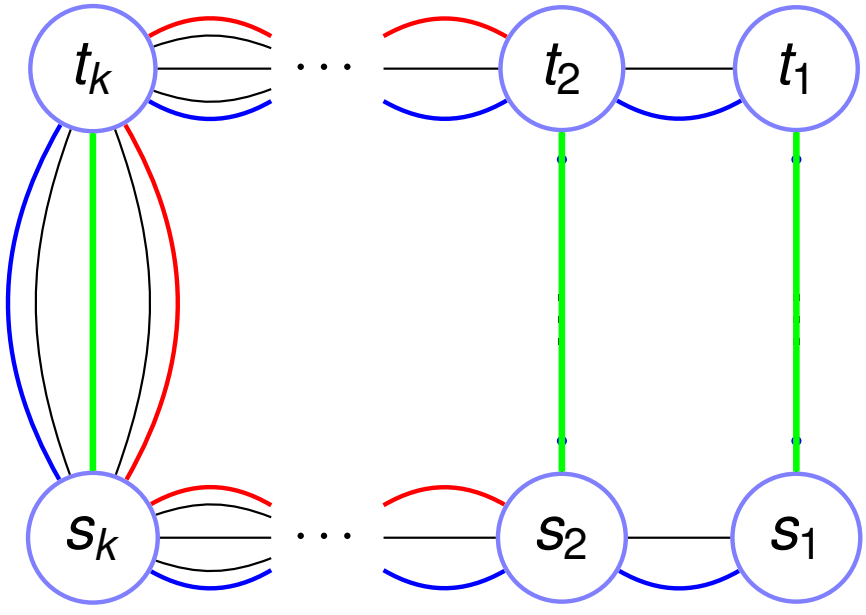

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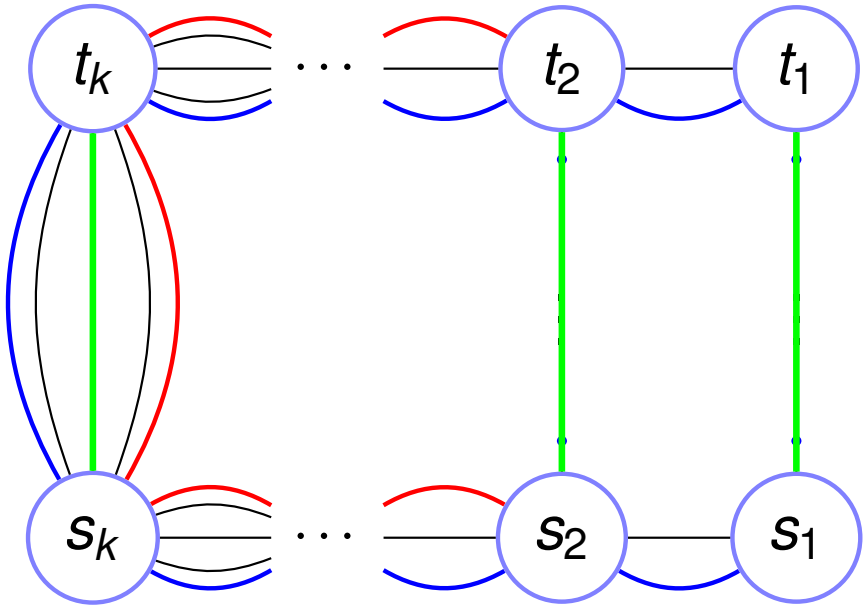

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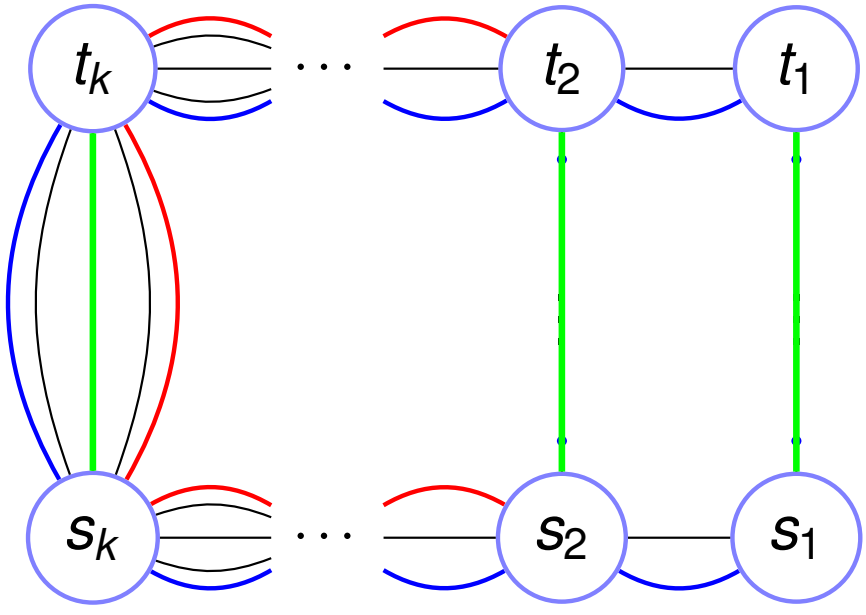

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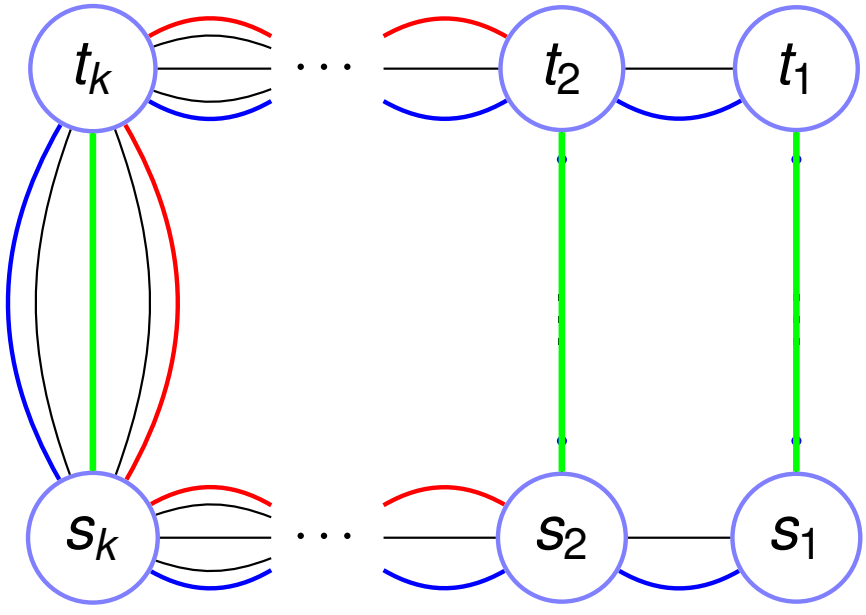

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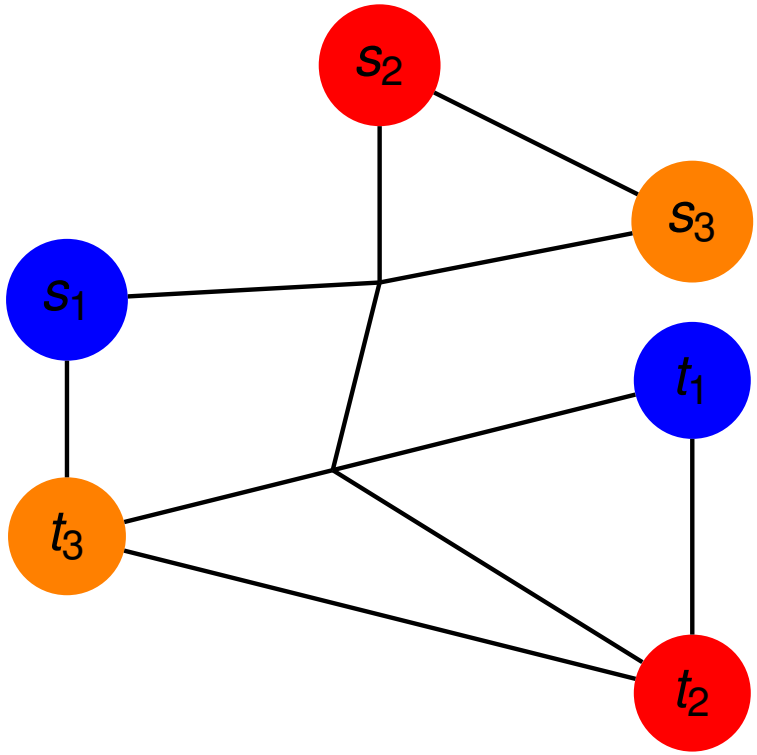

Assign $\frac{1}{11}$ on each of 11 edges.

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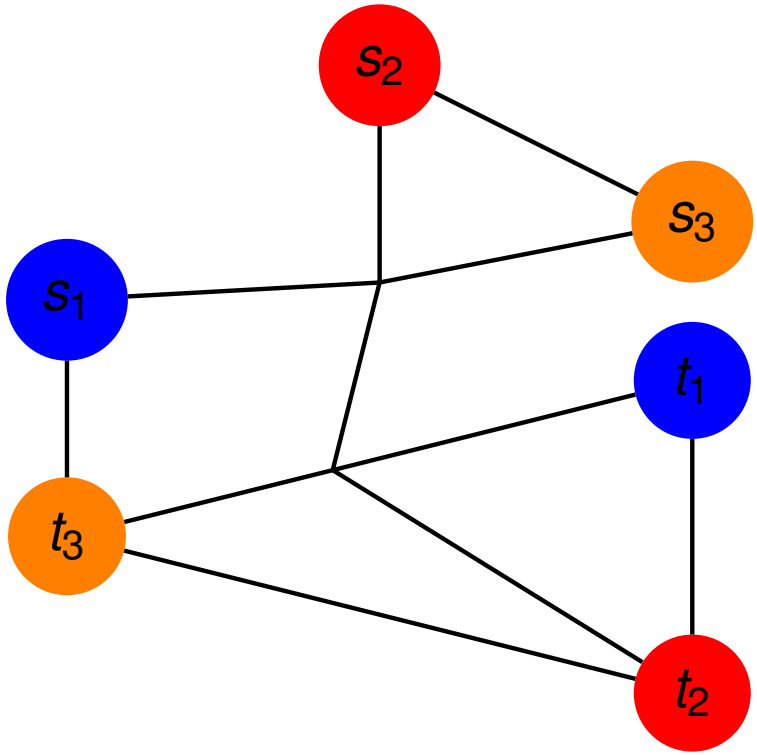

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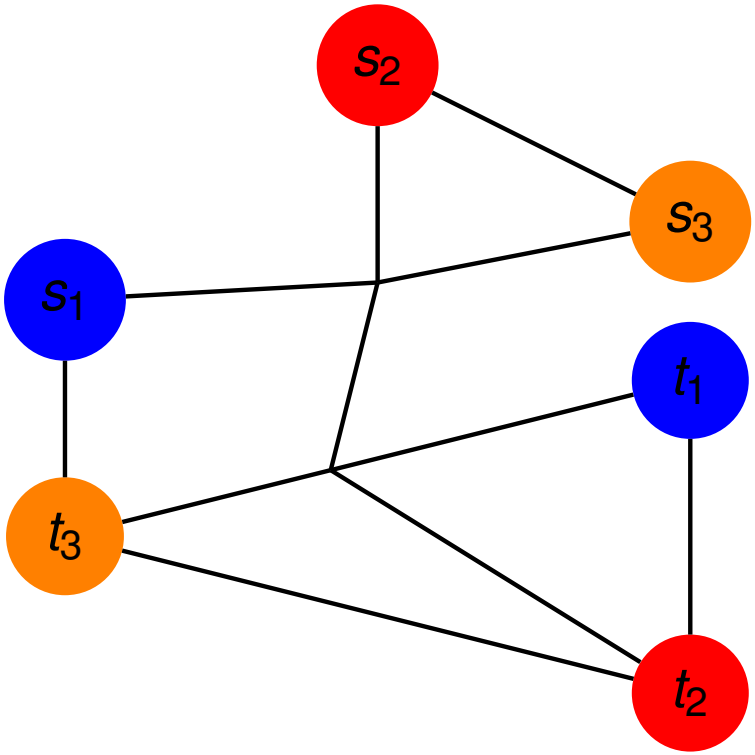

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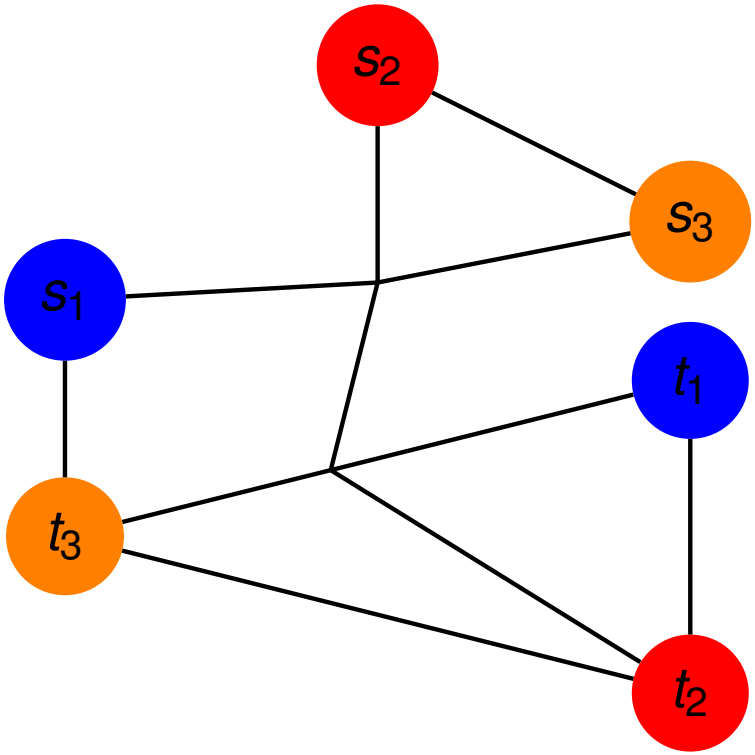

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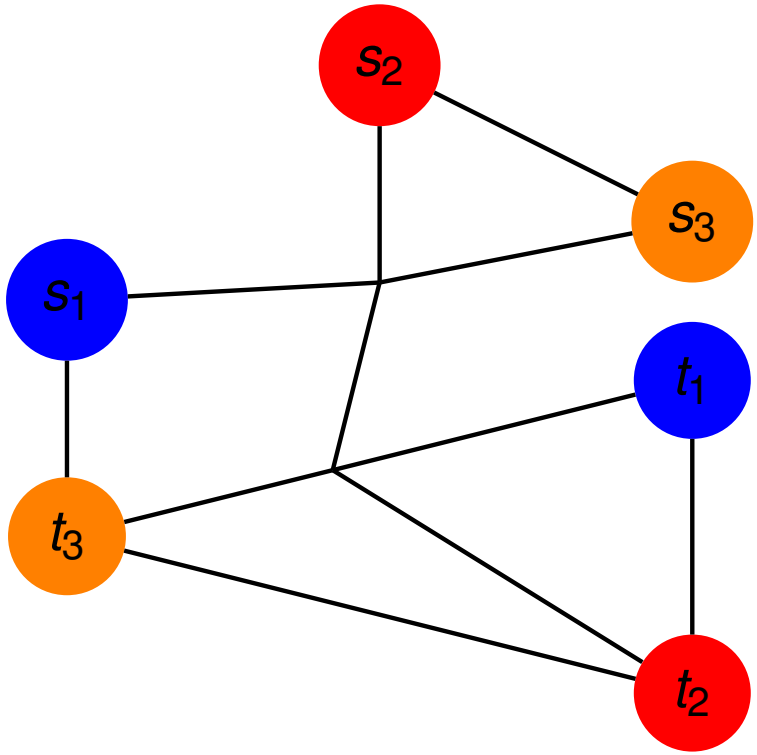

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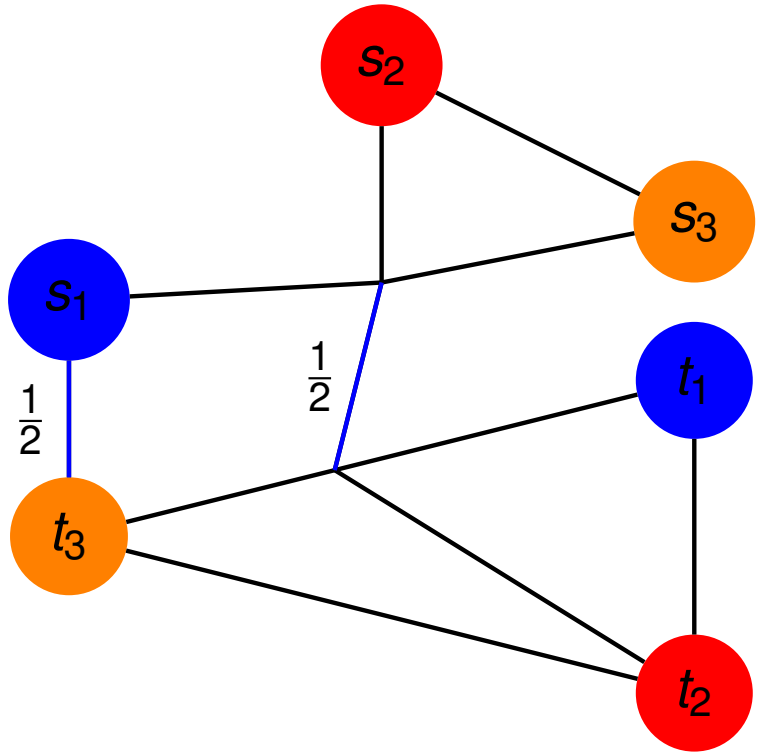

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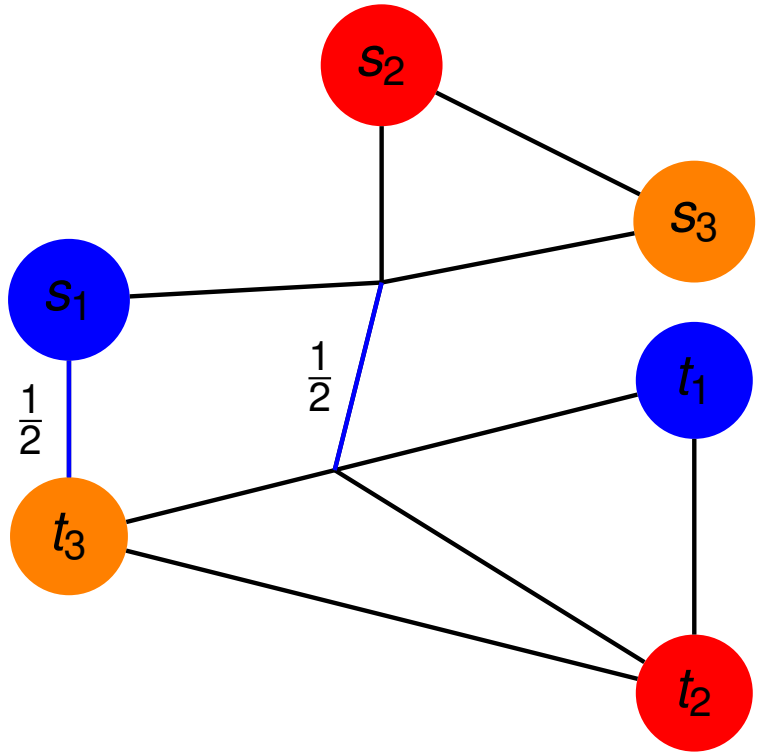

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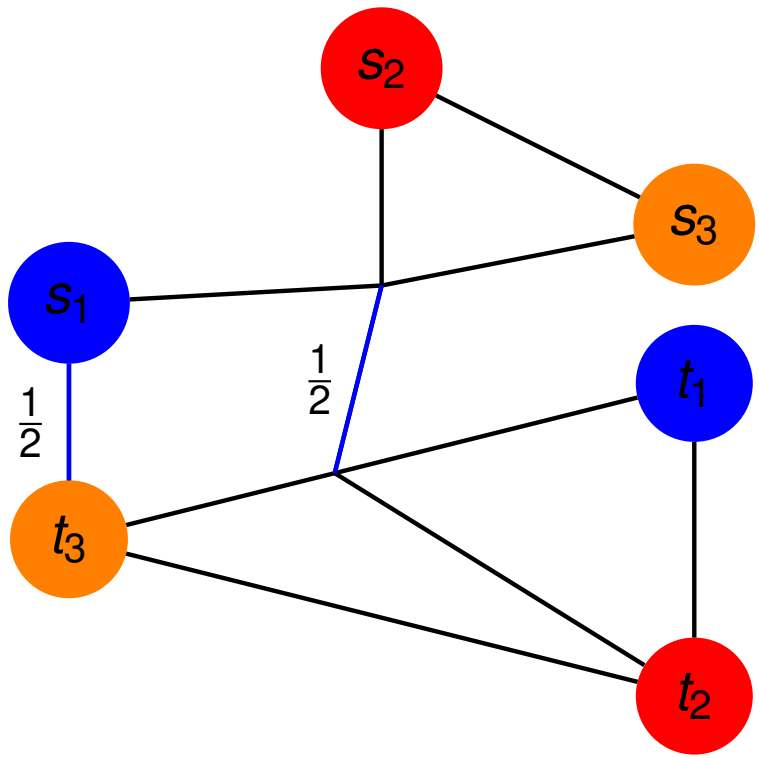

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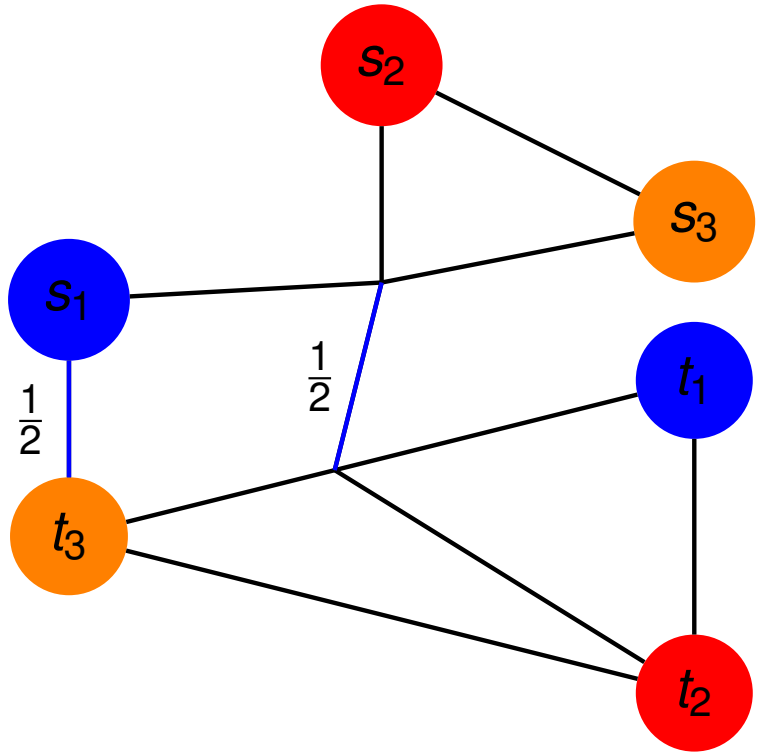

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$$\sum_i d(s_i, t_i).$$

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Optimal toll solution (weighted average congestion) is lower bound on congestion.

Proving lower bound: notation.

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Proving lower bound.

Routing solution: p_i connects (s_i, t_i) and has length $d(p_i)$.

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A path uses "volume" $d(p_i)$.

Volume on edge is $d(e)c(e)$.

$\sum_i d(p_i) = \sum_e d(e)c(e)$.

$\max_e c(e) \geq \sum_e d(e)c(e) = \sum_i d(p_i)$

Proving lower bound.

Routing solution: p_i connects (s_i, t_i) and has length $d(p_i)$.

$c(e)$ - congestion on edge e under routing.
Max $c(e)$?
$\max_e c(e) \geq \sum_e c(e)d(e)$ since $\sum_e d(e) = 1$.

$$\sum_e c(e)d(e) = \sum_i d(p_i)$$

$$\begin{aligned}
\sum_i d(p_i) &= \sum_i \sum_{e \in p_i} d(e) \\
&= \sum_e \sum_{i : e \ni p_i} d(e) \\
&= \sum_e d(e) \sum_{i : e \ni p_i} 1 \\
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$\max_e c(e) \geq \sum_e d(e)c(e) = \sum_i d(p_i) \geq \sum_i d(s_i, t_i)$.
Routing solution cost $\geq$ Any toll solution cost.

Toll is lower bound.

From before:
Max bigger than minimum weighted average:
$\max_e c(e) \geq \sum_e c(e)d(e)$
Total length is total congestion: $\sum_e c(e)d(e) = \sum_i d(p_i)$

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A toll solution is lower bound on any routing solution.
Any routing solution is an upper bound on a toll solution.

Algorithm.

Assign tolls.

Algorithm.

Assign tolls.
How to route?

Algorithm.

Assign tolls.
How to route? Shortest paths!

Algorithm.

Assign tolls.
How to route? Shortest paths!
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How to assign tolls?

Algorithm.

Assign tolls.
How to route? Shortest paths!
Assign routing.
How to assign tolls? Higher tolls on congested edges.

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Toll: $d(e) \propto 2^{c(e)}$.

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Algorithm.

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How to route? Shortest paths!
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How to assign tolls? Higher tolls on congested edges.
Toll: $d(e) \propto 2^{c(e)}$.

Equilibrium:
The shortest path routing has has $d(e) \propto 2^{c(e)}$.

The routing does not change, the tolls do not change.

How good is equilibrium?

Path is routed along shortest path and $d(e) \propto 2^{c(e)}$.

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For e with $c(e) \leq c_{max} - 2\log m$; $2^{c(e)} \leq 2^{c_{\max} - 2\log m} = \frac{2^{c_{max}}}{m^2}$.

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Or $c_{max} \leq (1+\frac{1}{m})c_{opt} + 2\log m$.

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Or $c_{max} \leq (1+\frac{1}{m})c_{opt} + 2\log m$.
(Almost) within $2\log m$ of optimal!

The end: sort of.

Got to here in class. Feel free to continue reading.

Getting to equilibrium.

Maybe no equilibrium!

Getting to equilibrium.

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Approximate equilibrium:

Getting to equilibrium.

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Each path is routed along a path with length within a factor of 3 of the shortest path and $d(e) \propto 2^{c(e)}$.

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Lose a factor of three at the beginning.

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Lose a factor of three at the beginning.
$c_{opt} \geq \sum_i d(s_i, t_i) \geq \frac{1}{3} \sum_e d(p_i)$.

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Maybe no equilibrium!

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We obtain $c_{max} = 3(1 + \frac{1}{m}) c_{opt} + 2 \log m$.

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This is worse!
What do we gain?

An algorithm!

Algorithm: reroute paths that are off by a factor of three.
(Note: $d(e)$ recomputed every rerouting.)

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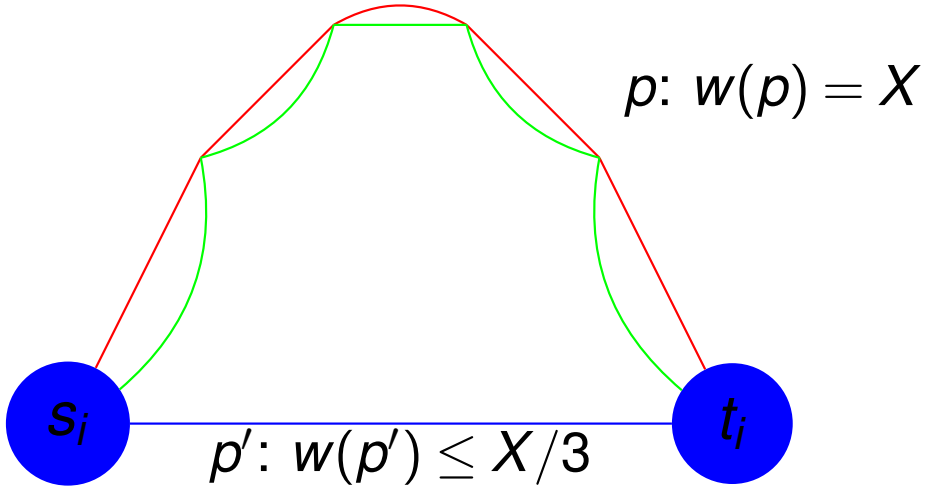

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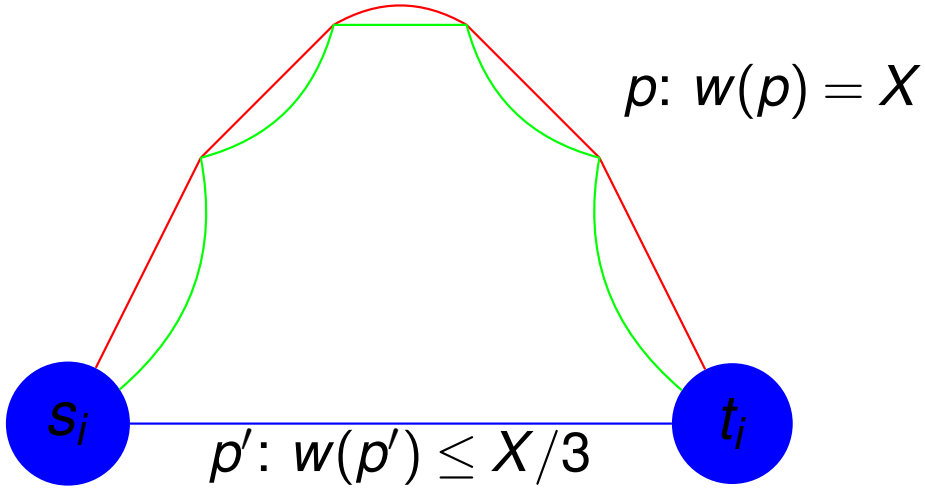

Potential function: $\sum_e w(e)$, $w(e) = 2^{c(e)}$

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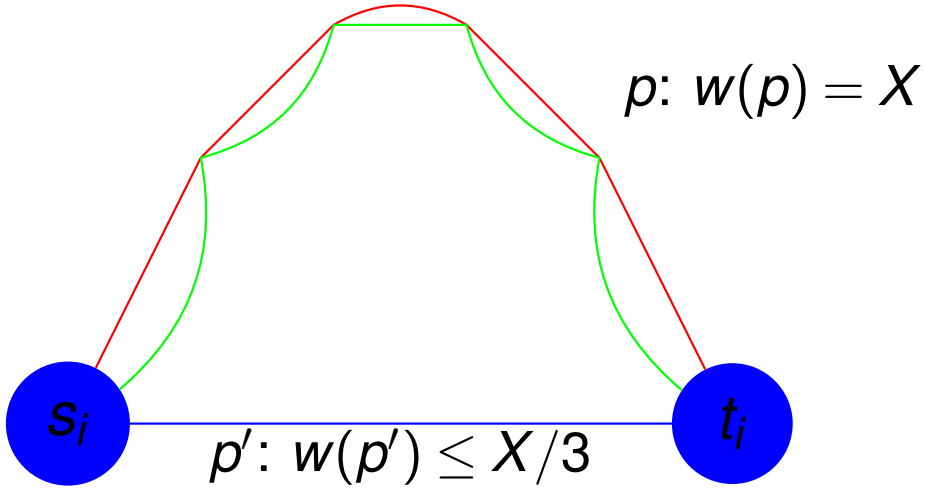

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Moving path:

An algorithm!

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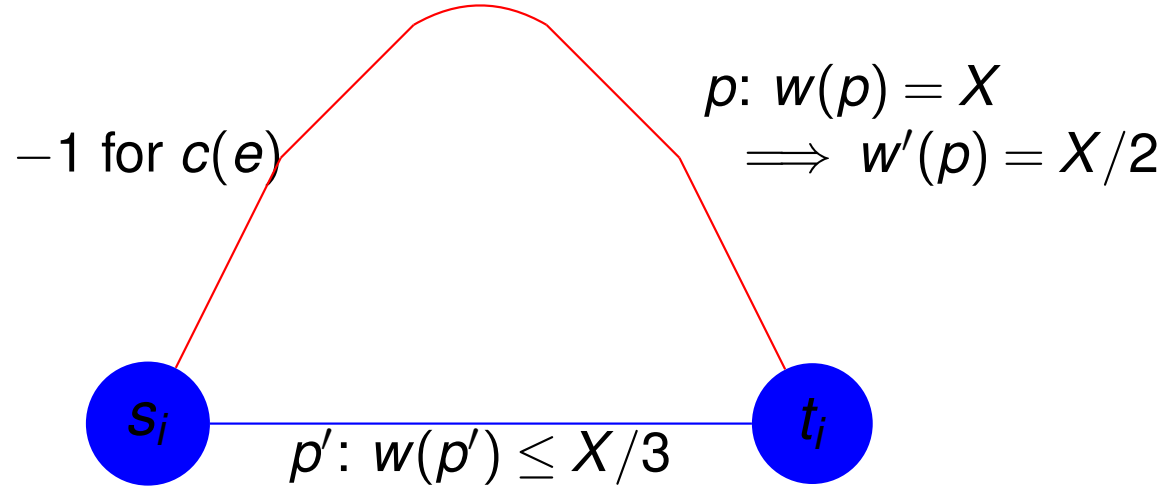

Potential function: $\sum_e w(e)$, $w(e) = 2^{c(e)}$
Moving path:
Divides $w(e)$ along long path (with $w(p)$ of X) by two.

An algorithm!

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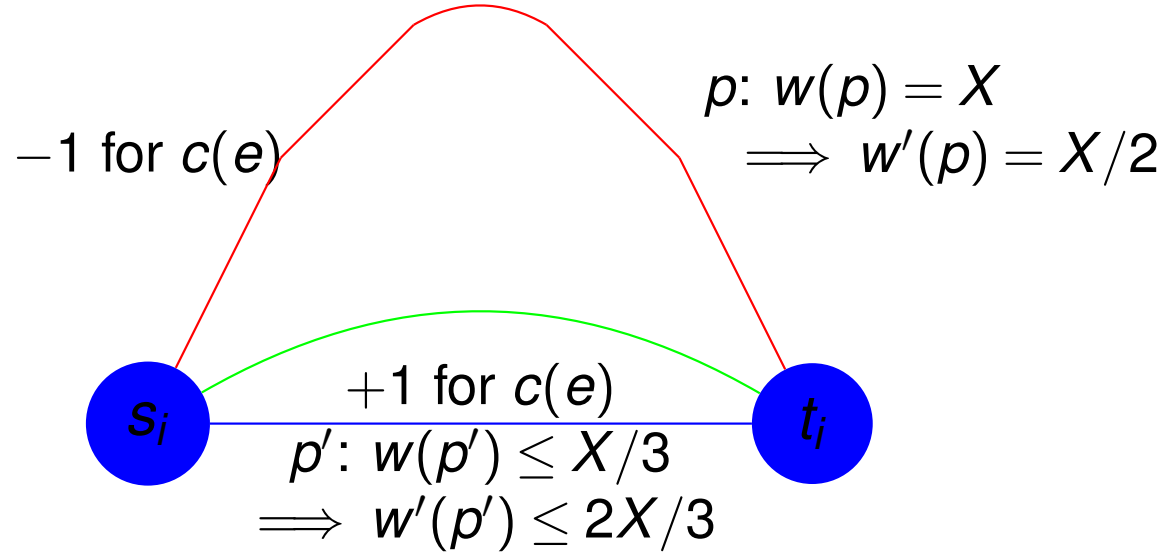

Potential function: $\sum_e w(e)$, $w(e) = 2^{c(e)}$
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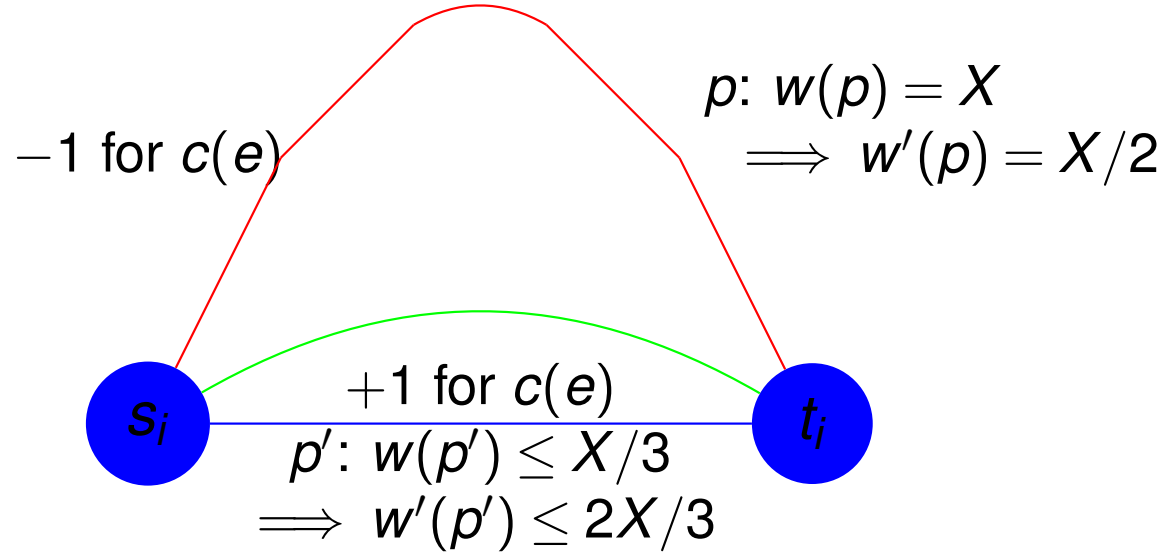

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$$-\frac{X}{2} + \frac{X}{3} = -\frac{X}{6}.$$

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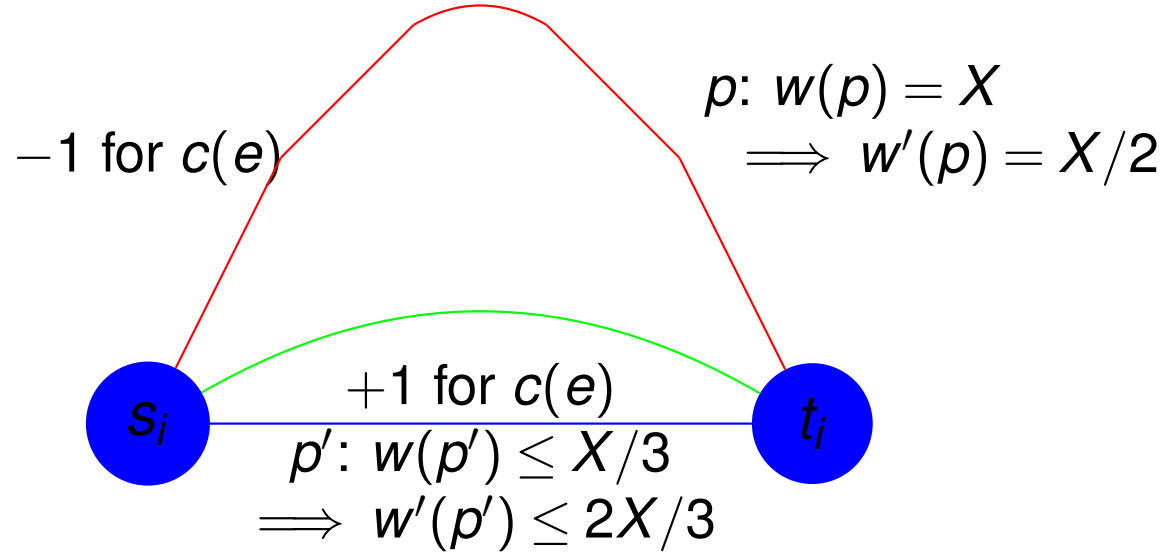

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Potential function decreases.

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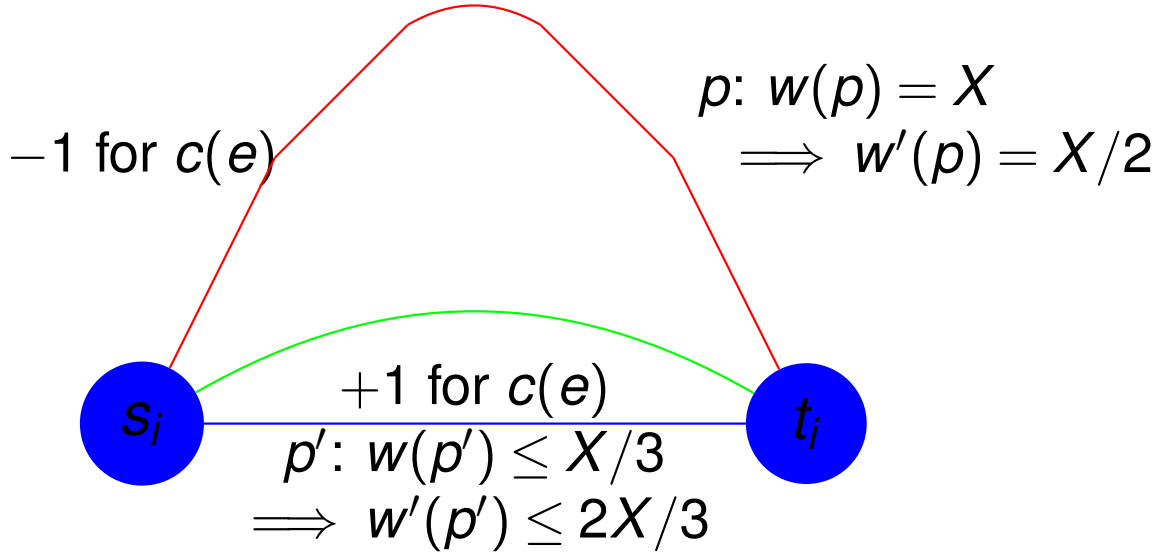

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Potential function decreases. $\implies$ termination and existence.

Tuning...

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Replace $d(e) = (1+\varepsilon)^{c(e)}$.

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$c_{max} \leq (1+2\varepsilon)c_{opt} + 2\log m/\varepsilon$.. (Roughly)

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Replace factor of 3 by $(1+2\varepsilon)$

$c_{max} \leq (1+2\varepsilon)c_{opt} + 2\log m/\varepsilon$.. (Roughly)

Fractional paths?

Wrap up.

Dueling players:

Wrap up.

Dueling players:
Toll player raises tolls on congested edges.

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Toll player raises tolls on congested edges.
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Wrap up.

Dueling players:
Toll player raises tolls on congested edges.
Congestion player avoids tolls.

Converges to near optimal solution!

A lower bound is “necessary” (natural),
and helpful (mysterious?)!

Done for the day.....

...see you on Thursday.